A unified framework for Schelling's model of segregation

Tim Rogers

Theoretical Physics, University of Manchester

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Intro

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The aim is to separate the fundamental mechanisms from the peripheral detail.

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- An equal number of agents of types A and B are placed randomly on a grid, leaving a small fraction of spaces empty.
- An agent is unhappy if fewer than T of the eight squares surrounding it contain agents of the same type.
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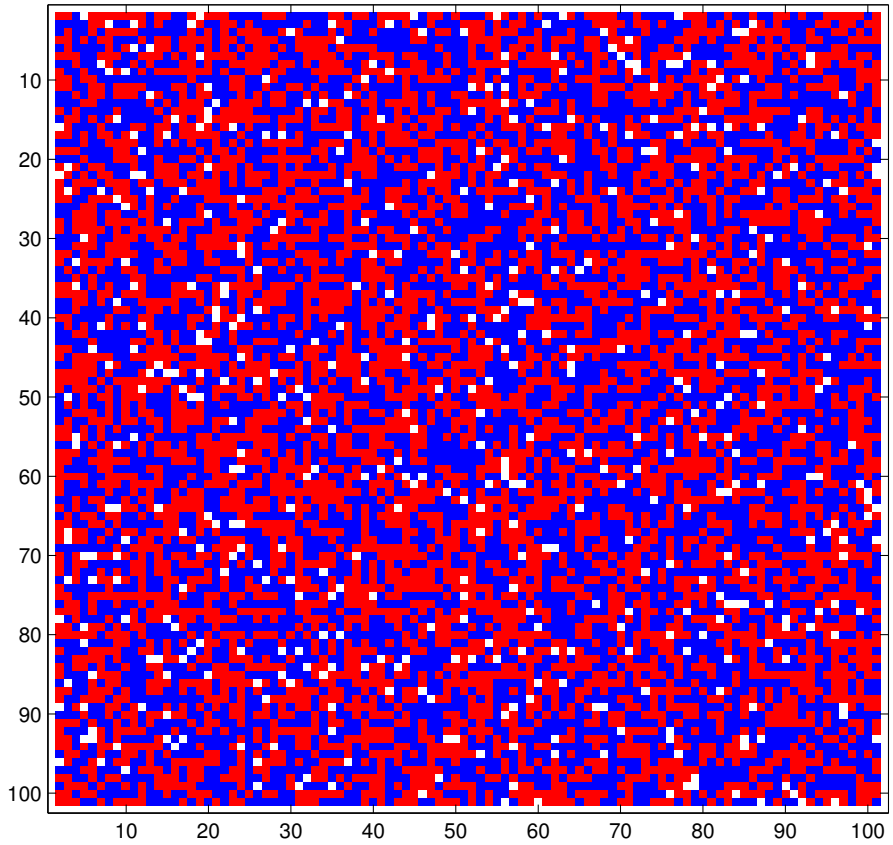
Even for low T, the agents split themselves into large homogeneous groups.

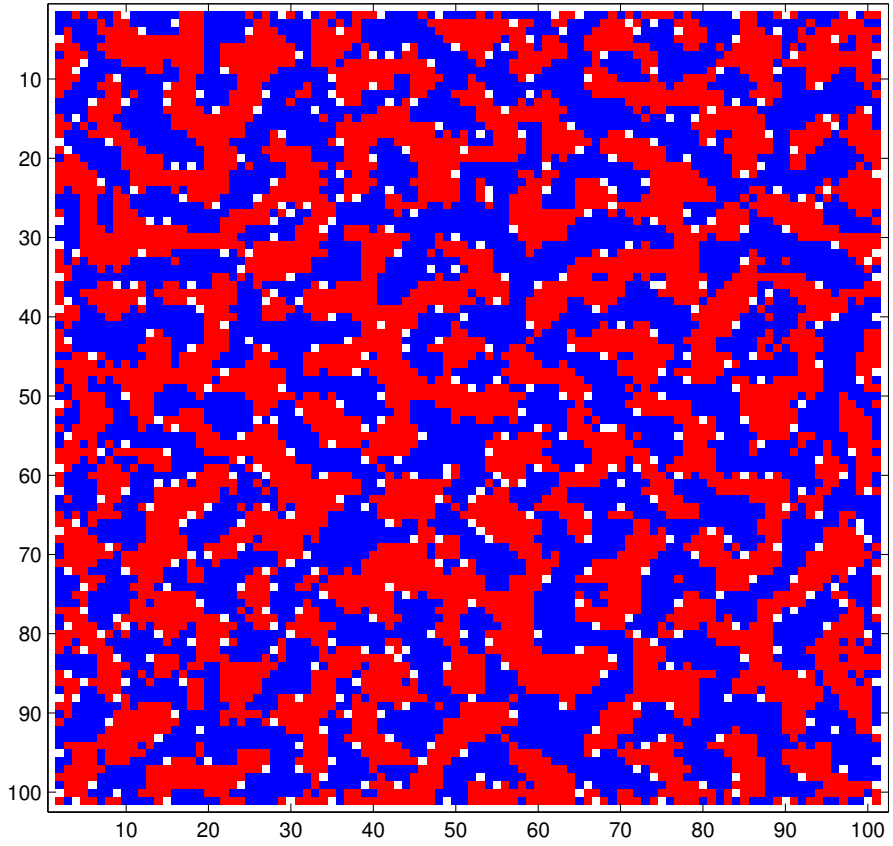
The Schelling model

Initial state



The Schelling model

After 10 time-steps ($T = 3$)



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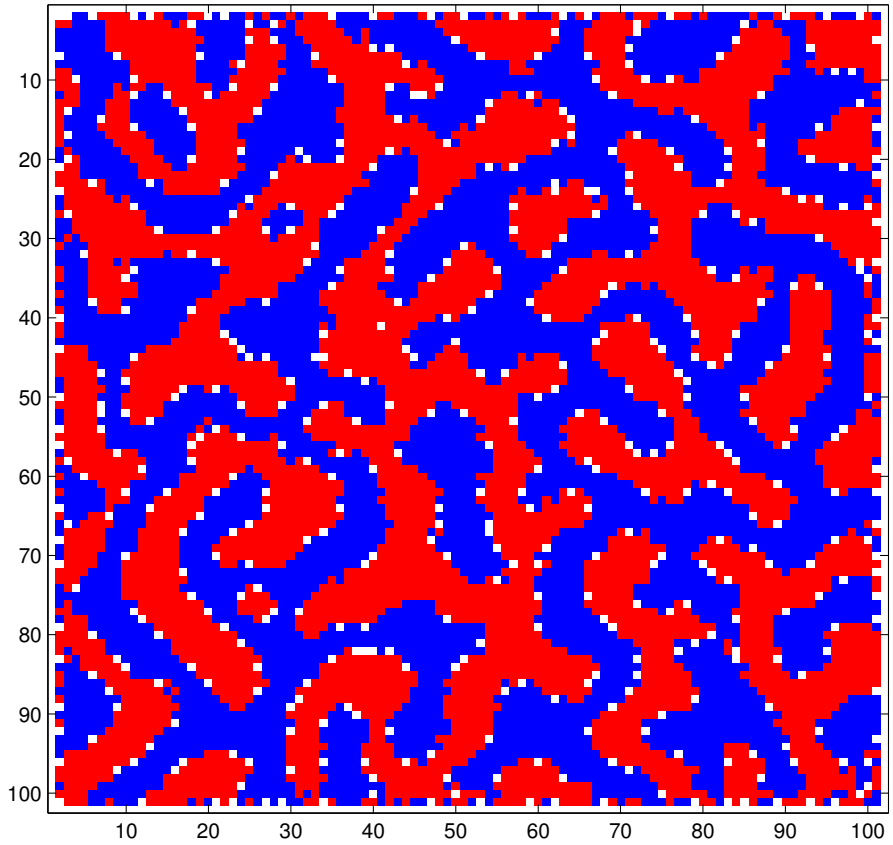
After 100 time-steps ($T = 3$)



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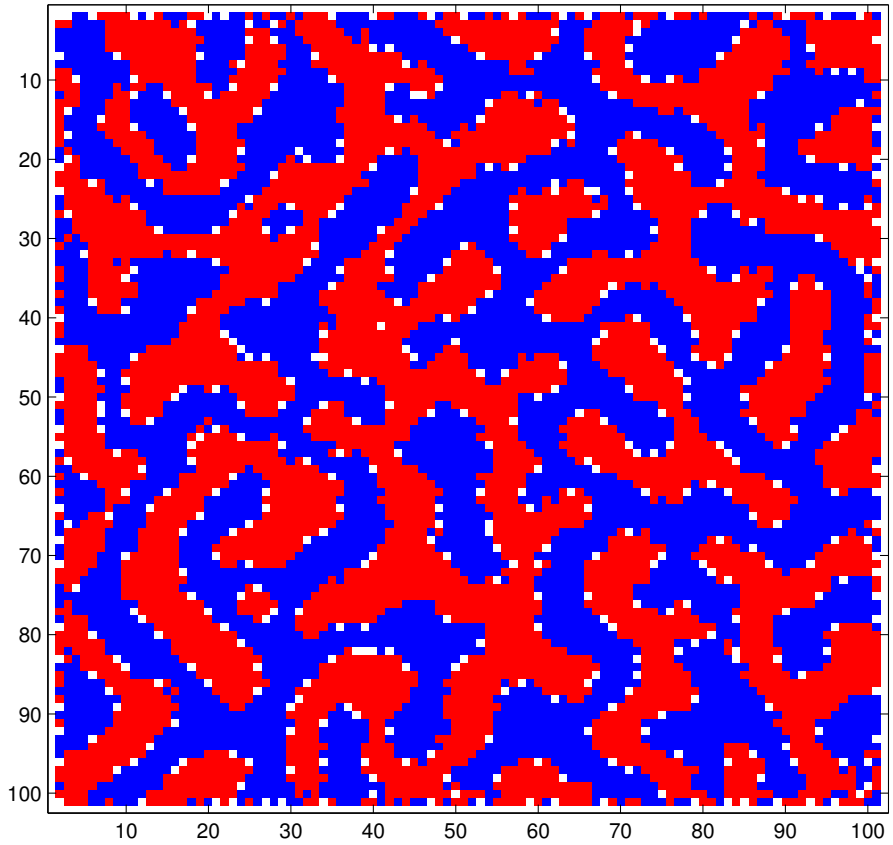

The model illustrates a simple message: a tiny homophilic bias is enough to cause wholescale segregation.

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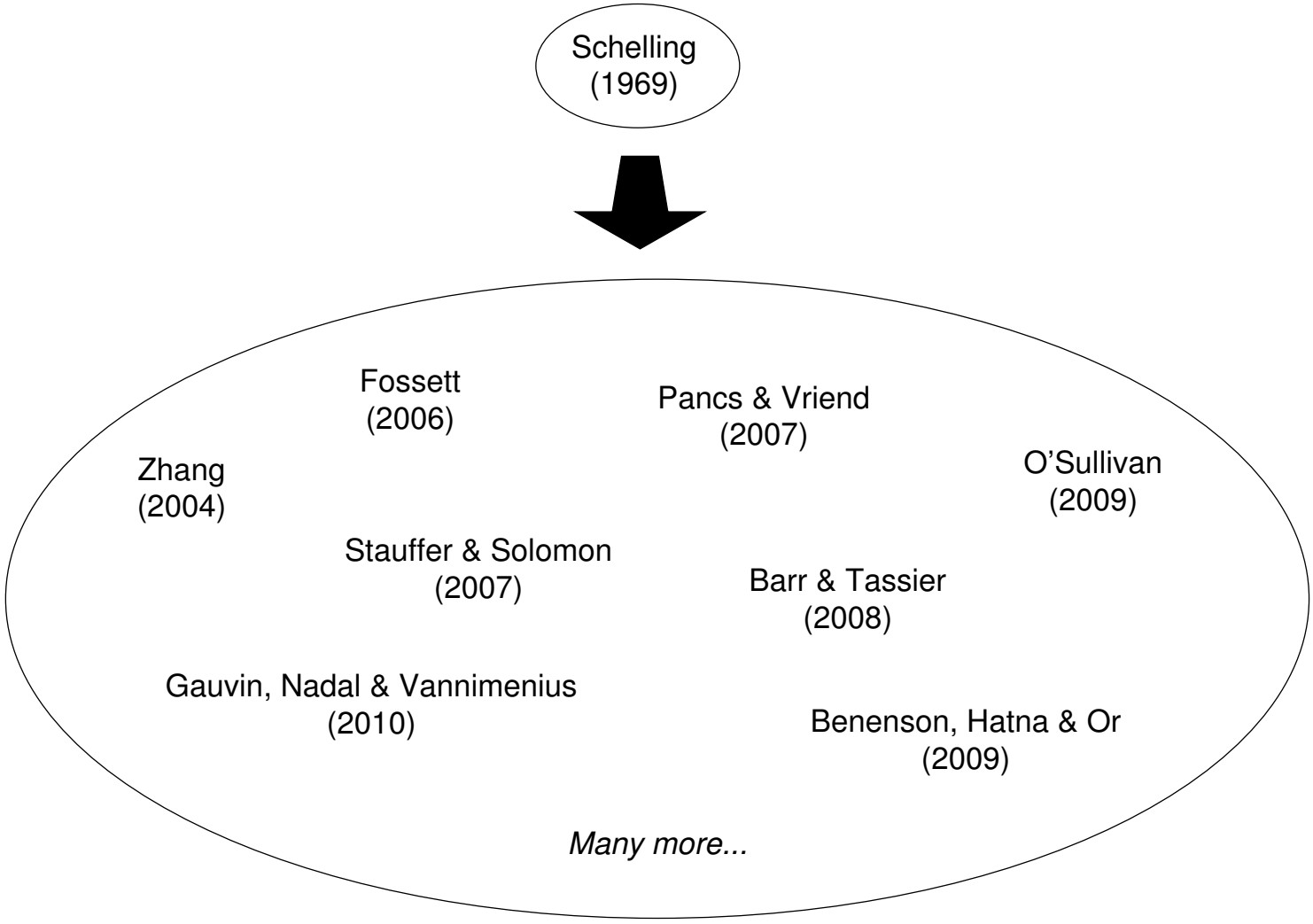

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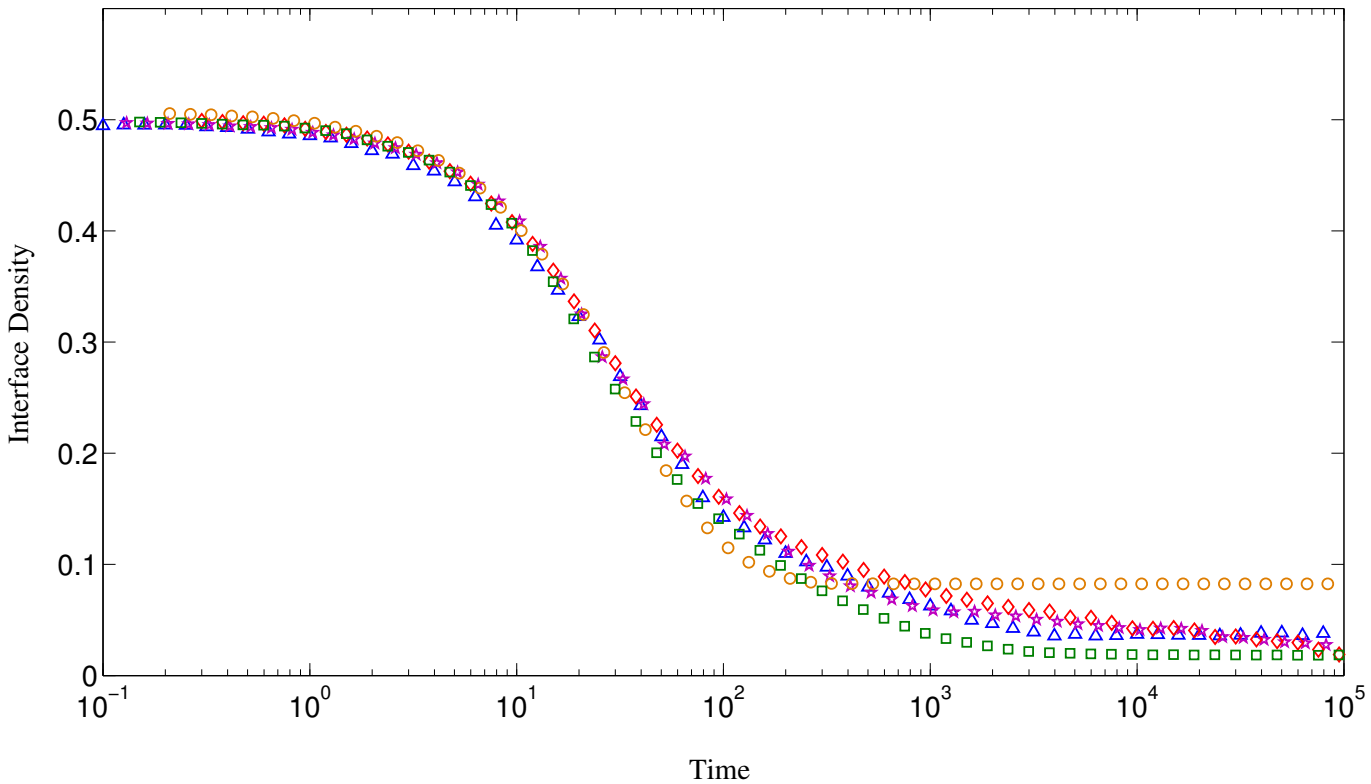

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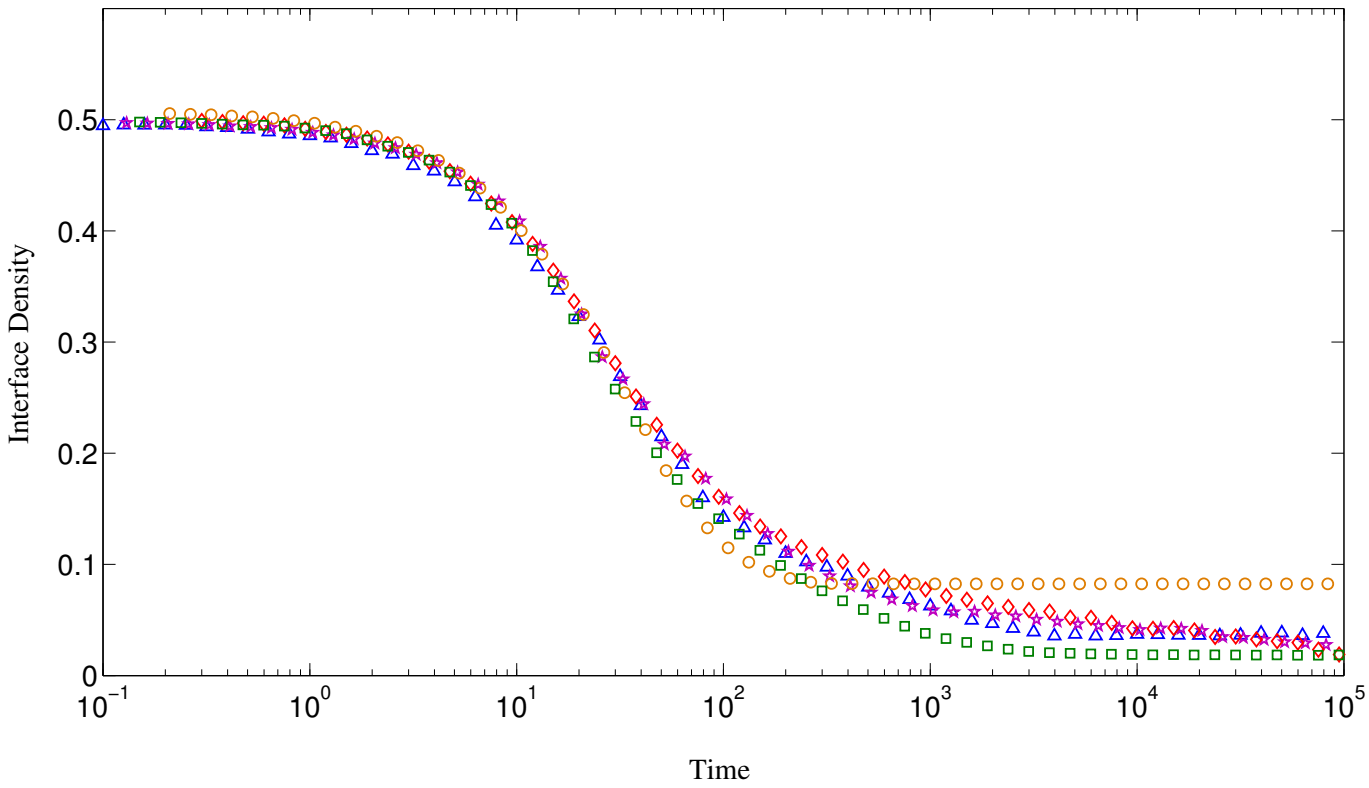

This a sign that the underlying mechanism is the same, and might be simple to describe mathematically.

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- Agents of two types are allowed to move around in some space, without being created or destroyed; agents are motivated to move on the basis of their level of happiness, which in turn is decided by the racial makeup of their neighbourhood.

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1. City layout
2. Initial state
3. Happiness function
4. Transfer probabilities (rules)

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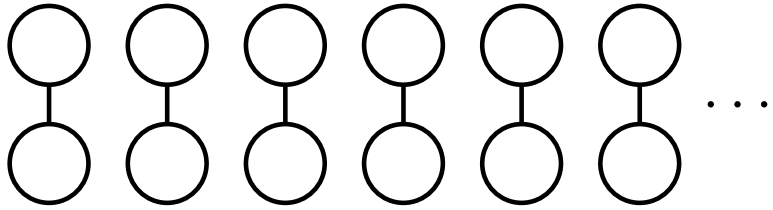
City: 

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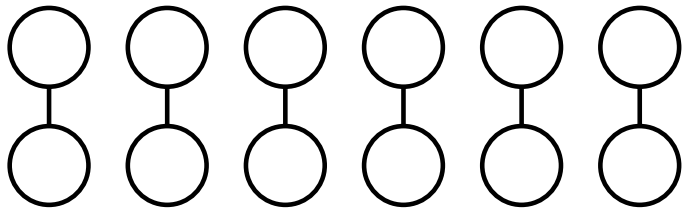
City:  ...

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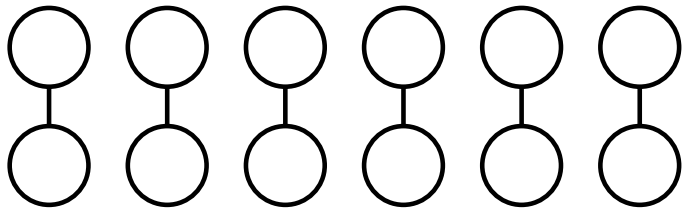
City:  ...

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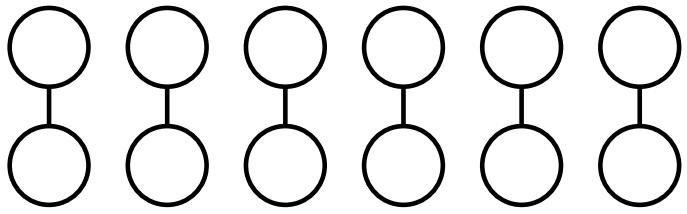
City:  ...

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Rules:

1. Pick two guys, say i and j
2. Swap them with probability $(1 - h_i)h_i^{(ij)}$
3. Repeat

Prediction for interface density

This model is so simple it can be solved exactly:

$$\begin{array}{c}\text{average interface}\\ \text{density at time } t\end{array} = \frac{\sqrt{ab} + a\tanh\left(t\sqrt{ab}\right)}{2\sqrt{ab} + \left(a+b\right)\tanh\left(t\sqrt{ab}\right)}\,.$$

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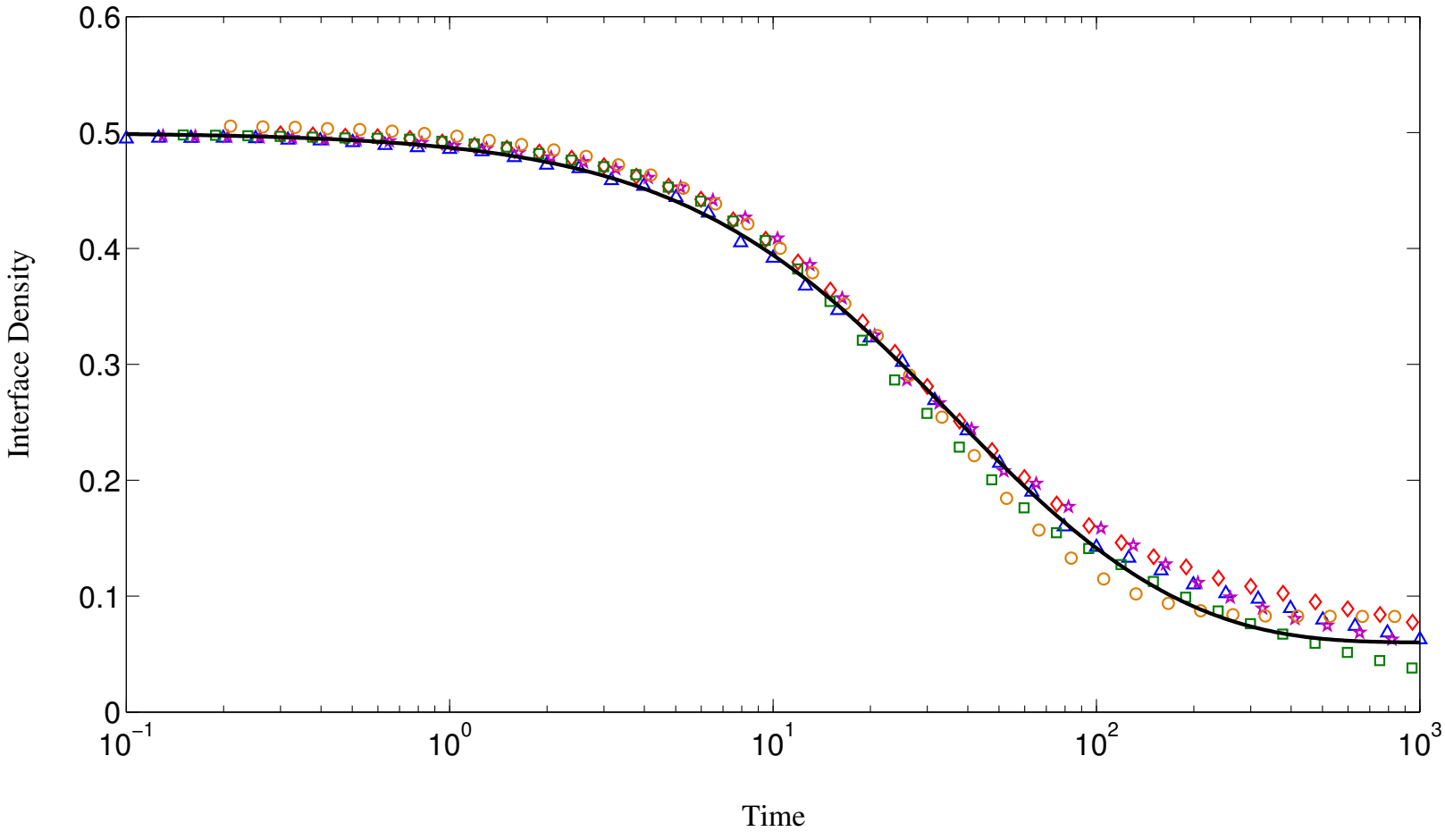

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We hope to achieve something similar in our study of the agent-based model currently in development.