

ADVANCING LEARNING AND EVOLUTIONARY GAME  
THEORY WITH AN APPLICATION TO SOCIAL DILEMMAS

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*To Segis,*

*for finding me whenever I've been lost,  
for lifting me up whenever I've fallen down,  
for using up all his bright light  
to get me through my darkest nights,  
and fill with colour and joy  
every minute of my life.*



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## **ABSTRACT**

This thesis advances game theory by formally analysing the implications of replacing some of its most stringent assumptions with alternatives that –at least in certain contexts– have received greater empirical support. Specifically, this thesis makes two distinct contributions in the field of learning game theory and one in the field of evolutionary game theory. The method employed has been a symbiotic combination of computer simulation and mathematical analysis. Computer simulation has been used extensively to enhance our understanding of various formal systems beyond the current limits of mathematical tractability, and also to illustrate, complement and extend various analytical derivations.

The two extensions to learning game theory presented here abandon the orthodox assumption that players are fully rational, and assume instead that players follow one of two alternative decision-making processes –case-based reasoning or reinforcement learning– that have received strong support from cognitive science research. The formal results derived in this part of the thesis add to the growing body of work in learning game theory that supports the general principle that the stability of outcomes in games depends not only on how unilateral deviations affect the deviator but also, and crucially, on how they affect the non-deviators. Outcomes where unilateral deviations hurt the deviator (strict Nash) but not the non-deviators (protected) tend to be the most stable.

The contribution of this thesis to evolutionary game theory is a systematic study of the extent to which the assumptions made in mainstream evolutionary game theory for the sake of tractability are affecting its conclusions. Our results show that the type of strategies that are likely to emerge and be sustained in evolutionary contexts is strongly dependent on assumptions that traditionally have been thought to be unimportant or secondary (e.g. number of players, continuity of the strategy space, mutation rate, population structure...). This latter contribution is focused on the evolutionary emergence of cooperation.

Following the presentation of the main results and the discussion of their implications, this thesis provides some guidance on how the models analysed here could be parameterised and validated.





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