



# Multiple equilibria in economic systems and stochastic dynamics

or: prices as conventions in agent-based models of growing economies

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Satellite meeting: Social Complexity of Informal Value Exchange

# Outline

Background

SCIVE

Ideas in progress

# Economics and Climate Change

- ▶ Standard: Theory of general equilibrium
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- ▶ deviation from optimal path has a cost
- ▶ climate problem = problem of distributing mitigation costs
- ▶ e.g. COP 15, Copenhagen 2009, deadlock situation

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- ▶ agent-based economic models  
aims: investigate system behaviour and generate more complete theory

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- ▶ Bilancini and Petri [2008] caution: capital used in production is 'land', equilibrium notion becomes irrelevant when using capital

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- ▶ Lagom models based on Gintis' model, add heterogeneous capital goods, economic growth
- ▶ sector structure, several regions, labour productivity grows with investment, financial system determines interest rate etc.
- ▶ “proof of concept”: large-scale agent-based macro-model with capital accumulation
- ▶ simulations suggest that agent-based dynamics can drive the economic system to a stochastically stable state with equilibrium features [Mandel et al., 2010]
- ▶ again, theory seems out of reach right now



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- ▶ work in progress in the spirit of the Dahlem Conference “New Approaches to Economics after the Financial Crisis” (Berlin August, 2010)
- ▶ central idea: insights from the social sciences, a variety of models, and judgement needed for “better” economics, in particular concerning policy making

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Mandel and Botta [2009]: exchange economy
- ▶ Ormerod et al. [2009]: cluster analysis of US, British and German data (1871-2009)  
identify different regimes in inflation/unemployment space  
major shifts between regimes and small fluctuations within

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- ▶ “next-best try” towards some understanding, replacing welfare function optimization
- ▶ stochastic dynamic model as a simpler pre-step towards agent-based model that could represent the observed dynamics
- ▶ get an idea of which details might be the important ones from the micro level that influence the macro level

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- ▶ chartist agents use (alleged) correlation between some information (index) and stock price to predict stock price; their behaviour impacts the price
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estimation procedure used by chartist agents  
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combined into an Itô-Langevin equation, with double-well potential function
- ▶ stable conventions appear for correlations of price change and index change (positive/negative)  
quick switches from one to the other

# Goals

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- ▶ not completely scrapping general equilibrium theory:  
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transitions between different equilibria
- ▶ potential function model
- ▶ very basic Lagom model, i.e. agent-based but tractable

# References

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## Thank You!