6   Philosophical Applications

6.1   Complexity and Relevance

Since the presence of irrelevance allows the top-down decomposition of complex models into sub-models without any loss of descriptive power, the pattern of relevance relations (or rather their absence) amongst the parts of a model limit the complexity of their interactions. Thus a first-cut at understanding the emergence of complex behaviour in models can be approached via relevance (in its widest sense). Of course, to gain a complete understanding one has to go beyond *whether* something is relevant, or even *to what extent* it is relevant, on to *how* it is relevant.

As argued by Hitchcock [224] relevance is properly considered a ternary relation relative to a framework or situation (much as I have relativised complexity). Thus in syntactic models of relevance which level of syntax the relevance relation is relative to is important. For example, there are several different kinds of relevance in logics: R and its related logics are concerned with relevance in the proof theory [12], while the relevance of Woods [479] is a relevance between formula, and the relevance of content (as described by Epstein [154]) is a relevance relation between the objects in the logic's semantics.

Changing the relevance relation will affect the language of representation of the model and hence its complexity. For example the analytic complexity of the proof in some formal logic might be different depending on whether one is considering the relevance relation to hold between repeated uses of rules and axioms or between formulas that refer to the same ground term. The relevance is inherent to the type of difficulty one is concerned with.

Reversing this, complexity could be used to define relevance, as happens when mutual information (section 8.25 on page 151) is defined in terms of algorithmic information (section 8.2 on page 136).

6.2   Complexity and Emergence

The definition given above (section 4 on page 72) implies that a complex system will be difficult to model in a top-down fashion. Thus for many very complex system we will not be able to find complete top-down models. In particular any models that are derived from the bottom-up description of the system will be unlikely to capture all the

top-level behaviour. So models that are used to try and capture some of a complex system's behaviour will be necessarily imperfect. That is to say that there is the potential for behaviour that one would not be able to predict given the models available. This behaviour is thus *emergent* in a meaningful way - it is not reducible to the bottom-up description of the system given the models available, as postulated by Cottrell in [116].

Conversely if we have a system that exhibits such emergent behaviour, that would mean that our model(s) of it did not completely capture such behaviour[59]. This may be because there is some fundamental reason why we are unable to produce such models, or merely that we happen not to have found such a model. In the first case our definition identifies the system as complex relative to the framework from which the fundamental reason comes; the more fundamental the reason the wider the framework. In the second case the system is complex relative to our current modelling techniques.

Only rarely are we in a position where we know that we have a fundamentally complex system rather than one which is complex relative to our current knowledge, because it is difficult to show conclusively that one has the best possible model. One generally can only show such things in artificially restricted abstract classes of models. In effect modelling is complex; it is almost impossible to ascribe optimality to a model given what one knows about a class of models relative to the goal of completely modelling most systems.

Such emergence is not restricted to the behaviour of natural systems. Chaitin [102] has exhibited a polynomial with a parameter which has inherently random behaviour, in that it is essentially random whether it will have a finite or infinite number of solutions for any given parameter value (where randomness is defined as algorithmic incompressibility). That is to say one can not predict via a theory whether there will be a finite or infinite number of solutions short of doing the calculation.

This modelling gap can be extended to one of many layers of description, each of which is the top-down level to the one below, as does Heylighen [219, 220]. This sort of structure prompts hierarchical approaches to complexity (e.g. Gougen [182]).

59.Darley [133] defines emergence as a difference in complexity at different levels.

6.3   Complexity and Language

From the definition I give and the fact that one typically wants to retain one's goals when tackling a problem, it is evident that one of the most powerful ways of tackling complexity is to change the modelling language (or framework). In other words, the choice of language is often critically important to determining the complexity relative to a type of difficulty.

This is something that we, as humans, are so good at that we are frequently unaware of it. For example, logicians working on a Hilbert style axiomatic system will frequently extend their mental model of the proof theory to include many derived rules of inference in order to tackle more complex proofs - it is not that the proofs have become simpler in the original "bare" axiomatic system, but simpler in this extended language.

Intuitively there is a qualitative difference between the simplification achievable by searching for equivalent expressions within a language and searching for equivalent languages to simplify the corresponding expressions (or even equivalent languages to simplify the search procedure for the simplest corresponding expressions in that language). For example, while a Hilbert style proof system may be parsimonious as to its inference rules, proving theorems using a Fitch style proof system is much easier because it is adapted to match key features of proofs (see section 5.6.2 on page 117).

There are interesting dynamics between complexity and the choice of language. One naturally seeks a language which will simplify the modelling of any particular subject matter, but when one finds such a language (and hence deem it a good representation, as below in section 6.4 on page 128) one is sometimes surprised by unexpected new behaviour in closely connected models - a serendipity of emergence that can tell you something genuinely novel.

6.4   Complexity and Representation

Peter Lupton [301] has claimed that the problem of *mis*representation can be explicated by considerations of the complexity of representations. Clearly, whether or not one thinks that this is the critical feature for the selection of representations, simplicity is one of the criteria for *useful* representation. This depends somewhat on the purpose of such a representation; clearly a simpler representation is preferable for beings of limited mental capacity, but further, even if one is seeking a representation which is as empirically

accurate as possible and one is limited to a particular extensible language then some heuristic to limit the mere elaboration of representations in favour of a more wider search is useful (see section 6.5 on page 129 below).

6.5   Complexity and "Simplicity"

"Simplicity" has a long philosophical history (e.g. [4, 78, 186, 253, 254, 339]) stretching back to Occam's famous razor. It is defined by tradition rather than formally. It could be defined as "that property which leads one to select one theory rather than another with equal empirical support", since at various stages almost every property of theories not directly related to its evidential support has been associated with the term, including: number of parameters [141], extensional plurality [186, 254], falsifiability [358], likelihood [390, 364], stability [447], logical expressive power [343] and content [185]. The idea started as parsimony being truth indicative. This is has now come full circle - Sober [418] defined "simplicity" in terms of relative informativeness; here the extent to which the answer is informative of the truth gave a measure of its simplicity.

Quine [364] could not see any a priori reason why a simpler theory should be more likely to be correct and Bunge [79] thought that there were too many types of simplicity for such a principle to be coherent. It would seem to presume that our language of modelling was inherently attuned to the universe (or even vice versa) such that there was a tendency for the more convenient simpler expressions also happened to be more likely to be correct.

This connection between "simplicity" and a lack of complexity seems to come from a dynamic account of theory development where an old theory being continually elaborated in an effort to maintain its consistency with discovered facts is replaced by a more coherent and powerful theory. The "simplicity" seems to be in contrast with the elaboration of the old theory.

These positions can be reconciled by considering the process of searching for acceptable theories, and what happens when they are unsuccessful. If the search process tends to start with simpler theories before trying more complicated ones, and will try elaborating theories before trying more radically different alternatives, then the fact that a theory has been elaborated will indicate that it has been unsuccessful - in such

circumstances another newer (less elaborated) equally supported theory might be a more productive choice.

Thus although a lack of complexity is no *a priori* indication of its truth, in circumstances where the process producing the theories is known to be an open-ended evolutionary process starting simply then limiting depth-first searches by some measure is a useful heuristic. Pearl [349] notes that this does not have to be a measure of complexity, but that any measure that limits the space of competing theories (for example sheer size) will do. According to my approach to complexity any such a measure could be a complexity measure, depending on how the search process worked. See also the discussion in Appendix 6 - Complexity and Scientific Modelling.

6.6 Complexity and Evolution

It is often assumed that complexity increases with evolution. If this assumption is based on the observation of the increase in the *maximum* complexity over all species (as in [71]) then this is assured merely by the fact that life (presumably) started simply and favoured some level of variety afterwards. That is complexity has increase not due to an active tendency but merely via a passive one due to the fact that there is a lower bound to it.

Evidence that either life as-a-whole or particular evolutionary branches have an active evolutionary tendency towards complexity is more mixed. It is uncertain that the average complexity over all species has significantly increased, since as well as the appearence of larger and more sophisticated organisms, many new simple organisms have appeared (partly to exploit the niches created by these larger organisms). In the case of individual evolutionary lines the evidence is that sometimes they evolve to be more complex but that certainly sometimes it goes in the opposite direction (as documented by McShea [316, 317]).

More deeply worrying for this assumption is that it is unclear why evolution should inherently favour the direction of the more complex rather than the simpler, especially since it is plausible that the simpler is cheaper and easier to maintain (as Martinez points out in [310]). There are some proposed mechanisms for complexity increase by evolution: Kauffman suggests that there is an inherent tendency towards order in large inter-related systems by mechanisms of self-organisation [249]; Arthur suggests that the competitive

co-evolution of species will result in such an increase [19, 21], Wimsatt points out that the evolution of multiple purposes for existing internal structures will tend to make the workings of an organism more complex [468] and Dawkins argues that evolvability itself will evolve [137].

A lot of the problems with this debate stem from the assumption that there is a single obvious notion of complexity that distinguishes us from "lower" species. Both Ho in [399] and myself in [147] attempt to separate out some of these different strands.

6.7   Complexity and Holism

Several holists (e.g. Rosen [384, 385, 386, 387, 388, 389], Morin [330, 331] or Mikulecky [323] - see section 8.17 on page 145) have laid claim to the word "complexity" to signify systems (or aspects of systems) that are irreducible. What is commonly called complexity is renamed as mere "complication". This is not a practical claim that many systems are in practice irreducible, but a fundamental distinction between syntactically based mechanisms and complex systems with an essentially semantic nature (e.g. living organisms). Modelling *aspects* of such complex systems is not impossible, merely inevitably partial and not unique.

Such an approach to complexity can be seen as a special case of my approach namely that *complexity is that property of systems where it is impossible to model satisfactorily in a top-down manner given almost complete information about its atomic components and their interactions.* The fact that they do not recognize any *intermediate* states of complexity seems to be more political than analytical since it impedes analysis of the transition from systems that might be labelled "simple" to those that might be called "complex" (as in the evolution of life example in section 3.2.1 on page 47).

I would prefer to make an analogy with the idea and use of "infinity". One can not say that any demonstrable thing is infinite (as opposed to potentially unbounded), but not withstanding this, infinity remains a useful abstraction linked to the credible extrapolation of real processes (as Rotman argues in [391]). The fact that models upon which one makes actual decisions are finite (either in representation or calculation) does not prevent the attribution of infinity to things being a useful guide to action. Similarly just because there needs to be a qualitatively different step to get from finite steps to the infinite does not prevent there being meaningful comparisons of size between different finite numbers. The

same applies to the sort of ultimate complexity holists posit. The fact that such ultimate complexity is unprovable for demonstrable models does not prevent its attribution being a useful guide to action, but on the other hand just because such complexity might be qualitatively different from normal scales of complexity does not mean that one can't make meaningful comparisons between them.

6.8   Complexity and System Identity

One of the questions that I avoided in posing my approach to complexity is that of system identity, namely how and why one identifies a certain set of parts and interactions as part of a unitary *system*. After all, every natural system (except, by definition, the universe) can be seen as merely a component of another system, and, (if one extrapolates from the progress of atomic physics) every natural system can be viewed as having sub-systems. On the other hand it is clear that not every collection can be said to have a meaningful *identity* as a system, for example the collection of a fish's gills, the constitution of the U.S. and a Martian rock.

One part of this conundrum can be illuminated by the observation that a system often is seen to have a "tighter" set of internal relationships and processes than external ones. This, however, is not a matter of the number or strength of such relationships. A useful example is a simple but much used module of communication software. It could be used by an unlimited number of different packages and components in a computer system, so the external relationships vastly outnumbered the internal ones, but this would not stop it being considered as an identifiable system.

Rather it is the complexity of the system, viewed as a relationship between its components as compared to the complexity of the system it was a part of that might indicate what could be usefully separated out as a system worth identifying. To use the above example, if the module was used repeatedly by only one process or component (or closely integrated set of processes) then one might well say that the module is more naturally categorised as a component of the system it is a part of, whereby if it was called by many different unrelated processes or components then its identification as a system in its own right becomes more natural.

One problem with using complexity in this way to aid system identification is that there is a danger of circularity: complexity is defined in terms of properties of models of

systems and systems are identified in terms of their complexity. What prevents such circularity making the definition tautologous or even contradictory is that at each stage the complexity of a system is grounded in an observer's model of it along with the type of difficulty that is relevant to the observer's goals, and the identification of systems is not completely determined by considerations of complexity except those that are relevant to the observer's modelling and goals.

Greenberg, in a related approach, uses an axiomatisation of identity with respect to indiscernible and complexes [198].

6.9 Complexity and Society

One of the significant facts that must influence the way an individual inter-acts with society is that the complexity of that society will (at least nowadays and in the developed world) be beyond that individual's capacity to deal with (as pointed out by Beer [51]) or make plans about (Chadwick [98]). Luhman postulates that we create institutions precisely to filter out some of this complexity (as summarised by Bednarz in [50]) and further that even meaning might be a mechanism to compensate for this complexity [299]. Elsewhere I argue that the complexity of modelling the society one inhabits necessitates certain strategies on the part of the society's members as well as effecting the approach an external modeller might need to take [152].

Many (such as Casti) see the increasing complexity of society as a major problem, based on an inequality between the complexity of the system to be controlled (society) and the complexity of our models of it [88]. Some specify very general programmes of remedial action in terms of design (e.g. Galbraith in [166]) or type of approach [157, 158]. Others suggest smaller and more concrete steps for the reduction of legal complexity [82, 248]. A few are more optimistic, envisioning the emergence of a new society suited to such internal complexity (e.g. [132]).

Some holists stress the difficulty of modelling any aspect of society (e.g. Lyon stresses the caution needed with applying the new techniques of the "sciences of complexity" here [302]), but others (e.g. McIntyre in [314]) argue that this is overstated.