# **Competition in Intermediated Markets: Statistical signatures and critical densities**

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## **1** Introduction

In conventional economic theory, competition is an unmodelled process that is claimed to drive all economic actors to behave *as if* they were constrained optimisers. What is actually modelled in conventional economic theory is a competitive equilibrium that is said to capture the result of the unmodelled competitive process. General equilibrium models prove sufficient conditions for an equilibrium to exist. Computable general equilibrium models simulate a process in which there is a single "representative" firm and a single "representative" household. If price is equal to marginal cost, it is because of the unmodelled competitive process that is claimed to enforce that equality. In the textbook partial equilibrium models, it is claimed that competition among a large number of firms will force them all to produce the output that equates price and marginal cost because each firm faces an effectively infinitely elastic demand function and, so, cannot affect the price. This implies that customers will always shift their demands to any seller that sets its price even a negligibly lower than any competing seller's price.

There is a large literature going back to Alchian [1] arguing that one evolutionary process or another will drive out those firms that do not produce the output at which price is equal to marginal cost because they will be less fit (because they do not optimise) than firms that do produce where price is equal to marginal cost.

Game theory is said to provide the process for imperfect or monopolistic competition. The classic cases are the Cournot and Bertrand models in which there are two competing sellers engaged in a game which is canonically equivalent to the Prisoners' Dilemma. The process leading to equilibrium can be described in some detail. However, a survey of all 14 game theoretic papers<sup>1</sup> published in 1999 in the prestigious *Journal of Economic Theory* showed that seven of the papers proved the existence of a Nash or similar equilibrium for an n-person game, six papers reported models and results for two-person games (sometimes in round robin tournaments) and

<sup>&</sup>lt;sup>1</sup> A paper was judged to be concerned with game theory if "game theor\*" appeared in its title, abstract or among its key words. See [18].

one paper reported results for a three-person game. There were no papers reporting the process of any game with more than three players.

The common element in all of these approaches to the analysis of competitive economic systems is that there is no consideration given to the effect of interaction among any number of economic actors greater than three. This is despite the natural presumption that such interaction is essential to any process of competition.

The purpose of this paper is to demonstrate that the interaction ignored or excluded by conventional economic theory gives rise to the distributions of data observed in real markets. These distributions have much higher peaks than normal distributions and their tails are very much fatter. They are *leptokurtotic* distributions.

If they are stable, these distributions are known as Lévy [9] flights or stable Pareto distributions. In the latter case this is because Pareto [21] discovered that personal income distributions are power law distributed in that the frequency of observations of incomes of size s or greater is proportional to  $s^{-a}$  where a is a positive constant. Whether they are stable or not, leptokurtosis implies that most observations are clustered about a mean value and that there are a few extreme values. It is customary in econometrics and perhaps other branches of statistics to the dismiss extreme values as "outliers" due to some exogenous cause. However, recent work in statistical mechanics [3], [7] indicates that these "outliers" are just extreme events generated by any system in which components change their positions or behaviour only when some threshold of stimulus is reached and where there is a dense pattern of interaction among such components. They characterise not only income distributions but city sizes, earthquakes, traffic jams, solar flares, avalanches and a host of other physical and social phenomena. The existence of extreme events and the consequent leptokurtosis is evidence for systems of dense interaction patterns among *metastable* components.

Mandelbrot [12] demonstrated that prices in the organised stock exchanges have leptokurtotic distributions. More recently, Lux [10], [11] has shown that even if the "fundamental" values of assets are normally distributed, simulated markets with interaction among metastable agents generate leptokurtotic price distributions.

This paper extends the Mandelbrot-Lux finding to competitive, intermediated markets more generally. It will be demonstrated in section 2 below that market shares and changes in sales volumes in competitive retail trades also have leptokurtotic distributions. These are all markets in which there are intermediaries

who buy the assets or goods for the sole purpose of selling them at a profit. If these markets are competitive, then buyers compare the prices and services offered by the various sellers or the qualities of the competing brands to determine which intermediary to buy from or which brand to buy. If buyers do not switch among sellers or brands in response to negligible differences in price then the buyers are metastable and if they communicate among one another then it might be that a sort of critical mass builds up and buyers occasionally switch in large numbers even if only a few normally change their suppliers or brands at any one time.

Having demonstrated that a range of real intermediated markets are characterised by leptokurtotic distributions, the implementation of a model of an intermediated market is reported in section 3. The producers and users in that market can "see" only a limited number of other producers, users and, when they exist, intermediaries. They decide whether to buy from a visible producer or intermediary on the basis of their experience with them. The buyers also share information which enables them to identify suppliers that are not directly visible. There is effectively word-of-mouth communication among the buyers. Results from simulation experiments with this model are reported in section 4 to include leptokurtosis of the same variables that are found to be leptokurtotic in real markets.

# 2 The statistical signature of competitive intermediated markets

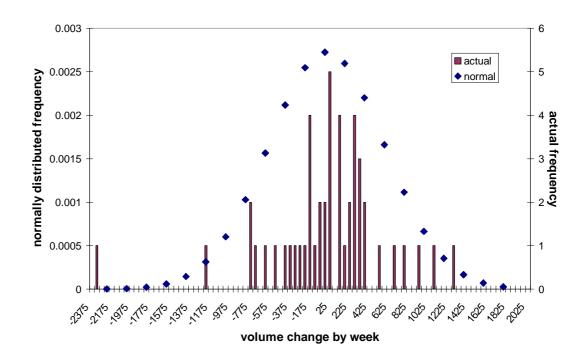
In this section, we first consider changes in the sales volumes of brands in highly competitive retail sectors and then market shares. The changes in sales volumes demonstrate leptokurtosis in time series data and the market share data demonstrate leptokurtosis in cross sectional data.

The first market to be considered is the UK market for beer. Electronic point of sale (EPOS) data was taken from UK supermarkets for 51 brands of beer over 149 consecutive weeks in the early 1990s.<sup>2</sup> Figure 1 shows a typical distribution of the changes in weekly sales volumes for one brand. In this case, the data is taken for one of the three largest selling brands of the time.

The actual distribution of sales volume changes is measured against the right hand axis and is indicated by the bar heights. The normal distribution for data with

<sup>&</sup>lt;sup>2</sup> The data was provided by United Distillers PLC in the course of the Intelligent Marketing Integrated System project funded by the Engineering and Physical Sciences Research Council under contract number IED/4/8022.

the same mean and standard deviation as the sample of actual volume changes is measured against the left hand axis and is indicated by the diamonds. As is evident from the chart, the actual distribution has a peak some three orders of magnitude greater than the normal distribution and very much fatter tails. The actual distribution is classically leptokurtotic.



#### Figure 1: Frequency distribution of sales volume changes - actual and normal

Leptokurtosis characterises cross sectional as well as time series data in systems with dense patterns of interaction among metastable components. In competitive systems, this characteristic is manifest as power-law distributed market shares. Figures 2, 3 and 4 relate market shares to outlets accounting for those shares in three competitive retail trades in the UK: pharmacies, newsagents (confectionery, news and tobacco) and grocers. The data is from the 1993 Neilson handbook on retailing in the UK and Eire [20]. The linear log-log relationship between market share (percentage of turnover) and outlets (percentage of shops accounting for the turnover percentage) is the power law distribution which is classically leptokurtotic.

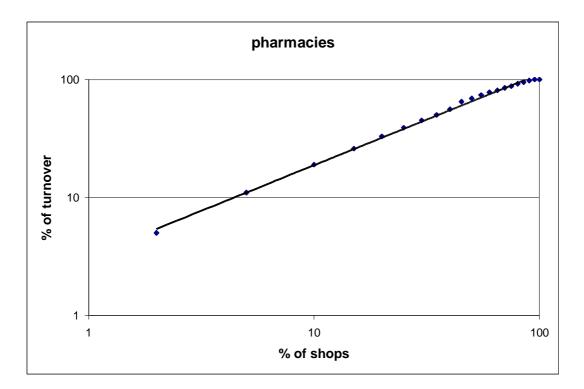


Figure 2: Market share distribution of UK pharmacies (Neilson, 1993)

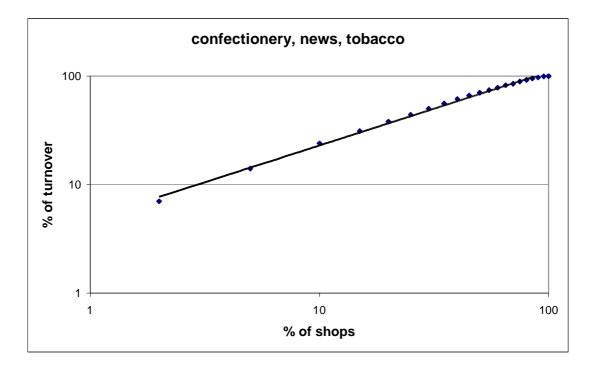


Figure 3: Market share distribution of UK newsagents (Neilson, 1993)

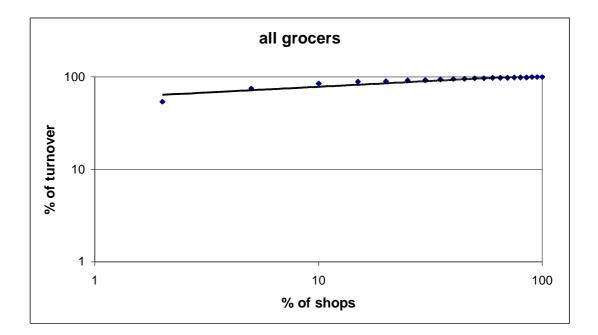
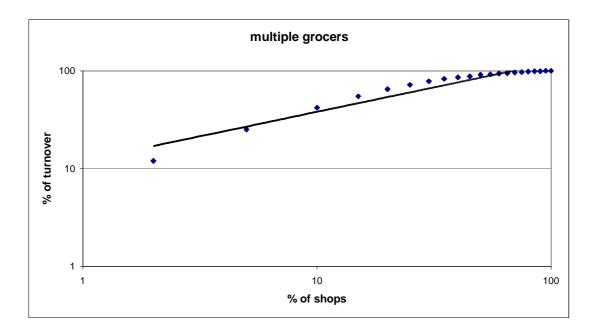


Figure 4: Market share distribution of all UK grocers (Neilson, 1993)



## Figure 5: Market share distribution of UK multiple grocers

Interestingly, the greatest deviation from the power law distribution of market shares, and therefore the least leptokurtotic, is the distribution of market shares for the multiple grocers in the UK which obviously constitute the least competitive segment of that otherwise competitive retail trade.

The evidence here is clear: the competitive retail trades are characterised by approximately power law distributed but certainly leptokurtotic distributions of both sales volume changes and market shares. This result is not surprising since interaction is a necessary condition of the process of competition. Extensive observation as well as simulation data with systems of densely interacting, metastable components suggests that leptokurtosis is the statistical signature of competition in intermediated markets either among brands or among outlets.

# **3** Model based analysis

Intermediaries can function profitably only if they reduce the total transactions costs of their customers and suppliers or they make transactions possible that could not otherwise occur. Moss [16] has used simulation experiments to demonstrate the validity of his earlier [16] arguments that intermediaries must be able to achieve economies of large scale in exchange that are not available to individual producers and users of the traded goods or services. The intermediaries' profit comes out of the difference between total transactions costs in the system without the intermediation and the total transactions costs when exchange in the same goods or services is intermediated. That difference is made possible by the ability of intermediaries to buy and sell both a larger volume of each good and a greater variety of goods than any one producer could sell or user would want to buy. Consequently, storage and transportation costs are spread over a larger number of units than could be achieved by the intermediaries' customers and suppliers. Similarly, communications costs (e.g. advertising and marketing expenditures) can be spread over a larger number of items. Some of the reduced unit costs of exchange are passed on to the intermediaries' customers and suppliers in order to induce them to trade through the intermediary rather than directly with one another. The remainder of the reduction in system transactions costs are available as net revenue to the intermediaries.

The model reported here, unlike the previous analyses, takes explicit account of the inability of each economic actor to know every other actor in the economic system. Specifically, the system is large in the sense that each agent can see only a small part of it and agents are able to communicate directly with one another to exchange information about the existence of other agents.

These conditions suggest an analogy with "word of mouth" communication in social systems. Each agent can "see" or know a small subset of all agents. Typically, people know other people who are geographically close or functionally similar to themselves. Analogously, agents will see other agents in close proximity represented by direct links in a network of agents. Moreover, they will be able to find out about

the existence of agents to which there is at least one path. There is no obvious reason to specify anything like acyclicity of the network.

A standard representation of such a network is agents placed on a grid. If it is relevant that some agents are at the periphery of the network, then the appropriate grid is projected onto a finite plane surface. Those agents towards the edges of the plane will have direct links to fewer agents than will agents towards the centre of the plane. If such "edge effects" are not of interest, then the appropriate grid is projected onto a torus. As a first step in developing statistical signatures for competitive systems, it seems appropriate to implement the conceptually simplest possible model. For this reason, to avoid the complication of edge effects, the agent network is represented by a toroidal grid populated by agents that can "see" a limited number of cells in each of the four cardinal directions.

#### 3.1 Model structure

Cognitive agents in the model buy and/or sell items. These items are represented by the values of digits in an ordered list – a digit string. This could be a bit string (if the allowable digits are 0 and 1) but, in general, the values of the digits in the string can be to an arbitrary base. Only single digits (in whatever base is chosen) are allowable. At each trading cycle, an item generating agent produces a digit string. The values of the digits could represent information or the characteristics of goods and services. The length of the string is constant over each simulation run and is determined user at the start of each run by the model operator.

There is a user-determined number of item sources distributed at random on the grid. Each source holds the current values of digits at specified positions in the digit string. These values change as the system digit string changes.

The intermediaries are cognitive agents that acquire the values of digits from sources. These values can be acquired only as packets of all items held by a source. However, the intermediaries can sell items individually or in any combinations available to them. That is, they can "break bulk" by selling on to other agents only those items from a source that the other agents demand and they can combine the items acquired from several sources. There is a flow of intermediaries chosen at random from the [1, B] interval where B is the maximum number of intermediaries, set by the model operator, that can enter the market at each trading cycle. Each intermediary begins life with no assets and builds asset reserves from profits on the

purchase and sale of items acquired either from sources or from other brokers. An intermediary leaves the market when its asset reserves are exhausted. One consequence of this specification is that an intermediary's sales revenue must exceed the cost of its acquisitions in the first trading cycle of its life.

Each intermediary is initially allocated to an empty cell but can choose to move to some other cell if it is unoccupied and no other agent is seeking to move at the same time to the same cell. The motivation to change cells is the knowledge that there is a profitable intermediary in the neighbourhood of the destination cell.

Customers are cognitive agents that either acquire packets of items from sources in the same way as do the intermediaries or they buy demanded items from the intermediaries or some combination of these. The customer agents each inhabit a cell during the whole of the simulation run. Although the number of customer agents is determined at the start of each run by the model operator, their locations are determined at random.

At the start of each simulation run, customer agents are allocated demands for the values of digits at specified positions in the system digit string. The number of items demanded is determined at random from the [1, C] interval where C is set by the model operator at the start of the simulation run. Intermediaries have no demands of their own but only demands for items for which they have previously received enquiries from customers or other intermediaries.

Intermediaries and customers are synchronous, parallel agents. To enable them to communicate with one another, a series of *communication cycles* is nested within each trading cycle. A limit of eight communication cycles was allowed within each trading cycle though there would have been fewer communication cycles if all demands were filled earlier. In practice, this never happened in the experiments reported here.

## 3.2 Agent cognition

There are two principle aspects to the representation of agent cognition: the problem space architecture taken from Soar [8] and ACT-R [2] and the endorsements mechanism as adapted from Cohen [4].

The goal was to find items in demand. The initial subgoals were to find sources and find intermediaries with the further subgoals to search over the visible cells, to ask suitable agents known to the agent and to listen for their requests or replies to the agent's own requests. Once answers had been heard, if those answers provided suitable information about available items from either sources or intermediaries, then the agent would adopt the transaction subgoal.

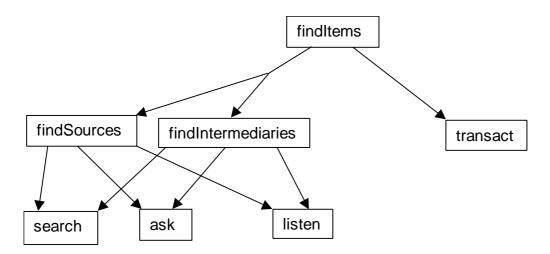


Figure 6: Cognitive agents' problem space architecture

The urgency of acquiring any particular item – the value of the digit at any particular position in the system digit string – was determined by endorsements. Items that changed frequently in value were valued more highly than items that changed infrequently. The frequency of change was learned by experience and was in fact determined by a mutation probability. The maximum mutation probability and the distribution of probabilities among positions of the digit were set for the duration of the simulation run by the model operator.

The choice of intermediary, when their was a choice, was also determined by endorsements. Intermediaries that were known to the agent and had been reliable in the past or relatively inexpensive and that provided most or all of the items demanded by the purchasing agent were preferred to agents that lacked any of those endorsements. A fuller description of the endorsements mechanism as used in the model reported here is to be found in several papers by Moss (*e.g.*, [17], [16]).

The goal setting and actions taken in each goal and subgoal took place within communication cycles. There were two alternative specifications. In one, all of the rules were fired within the communication cycle making use of the assumptions mechanism of SDML [19]. The SDML assumptions mechanism ensures that the rules firing for any agent at any time step are sound and consistent with respect to strongly grounded autoepistemic logic. However, the rules implemented in the present model required considerable backtracking by the assumptions mechanism to ensure that no contradictions emerged. Once the logical issues were resolved, a further level of time steps – *the elaboration cycle* -- was specified within the communication cycle so that one layer of problem space was handled in each of these lower level time steps. This procedure effectively makes the problem space architecture more procedural and, by avoiding backtracking, considerably increases the speed of each simulation run.

#### 3.3 Communication among agents

Although the broadcasting of information is common in the retail trades either as advertising of brands or advertising of individual and multiple outlets, agents in this model were not programmed to broadcast information. All communication took place directly between individual agents.

Direct communication among agents in SDML is implemented by having the sending agent place the desired clause on the database of the receiving agent. Because agents are parallel, synchronous agents, it is not feasible for one agent to change the state of another agent while that other agent is changing its own state. Consequently, SDML allows parallel agents to access clauses placed on their databases by other agents only at the following common time step – in the case of the present model, at the subsequent communication cycle.

Being able to follow links from one agent to another to get or give information is entirely analogous to word-of-mouth communication. One agent can communicate with another agent within its horizon. If the second agent informs the first agent of the existence and address of a third agent beyond the horizon of the first agent, then the first agent will be able to communicate directly with that third agent. If the third agent informs the first agent of the address of a fourth agent, then communication from the first to the fourth agent becomes possible. Precisely this procedure was implemented in the model reported here. It was assumed that consumer agents would engage in word of mouth communication concerning the locations of both intermediaries and sources but that intermediaries would not pass on that information since it was commercially valuable to them.

#### 3.4 Parameter values

All of the simulation runs employed, with one exception, parameter settings that were taken exactly from runs of Moss' [16] unit-square model. The system digit string

contained 40 digits; there were 15 sources and 100 customers. Each customer could demand up to 12 items and each source could hold up to 15 items.

The exception was the permitted number of entrants as intermediaries in the market. This parameter was shown in the unit-square model to have no effect on the efficiency of intermediated exchange. The maximum number of broker agents that could enter the market in any trading cycle was therefore set at 15 which is rather higher than in the runs with the unit-square model. The choice of a larger number of entrants was motivated by the intention to investigate the effect of word of mouth communication among agents: with more intermediaries in the market there was more information for customers to communicate by word of mouth.

In all runs, agents could identify the existence of sources or other agents within eight cells of their own position in the cardinal directions (up, down, right and left). The only parameter setting that was changed for the different simulation runs was the size of the grid. Three grid sizes were used:  $50 \times 50$  (2500 cells),  $30 \times 30$  (900 cells) and  $25 \times 25$  (625 cells). A larger grid size implies a lower density of agents.

## **4** Simulation results

Experimentation confirmed that agent density is a critical factor in the viability of agent trading, more surprisingly that a high proportion of demands are satisfied only when virtually all trading is via intermediary agents and leptokurtosis characterises market shares among trading agents when intermediation is viable. The results presented in this section bring out the relationship between agent density and, in turn, market effectiveness, pricing, the extent of intermediation and the nature and role of the statistical signature.

# 4.1 Market effectiveness

One natural measure of the effectiveness of markets is the proportion of total customer demands that are satisfied through transactions. The time series of these proportions for three scales of grids are shown in figure 7. The population density of customers and sources increases from figure 7a down to 7c. With a density of one customer in every 25 cells, as in the run reported in figure 7a, on average 3.2 per cent of demands were filled. With one customer for every nine cells, the percentage of filled demands rose to 14.6 per cent but the supplies were very erratic. The reason for the erratic nature of the supplies was that brokers typically found sources for items

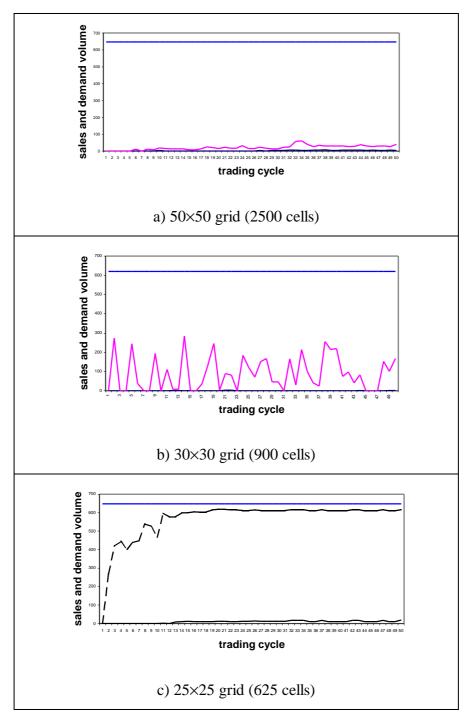
that potential customers wanted but the number of such items was sufficiently small that the revenue typically did not cover the transportation and storage costs. The survival of brokers in the environment modelled here, as in the unit-square environment [16], requires each broker to be able to sell on to several customers the same items obtained from a small number of sources. In that way, the transportation charges to a point close to the customer agents as well as the storage or processing charges are incurred once for a relatively large number of sales. This enables the intermediary to undercut the cost to the customer of acquiring the items directly from sources because the intermediary is able to share out the same costs among several customers.

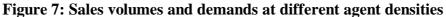
In figure 7c, it is apparent that the density of one customer for every 6.25 cells results in a high and relatively constant percentage of satisfied demands. The increase in the proportion of demands satisfied over the first 13 trading cycles is due to the appearance and survival of new broker agents and the spreading knowledge of their existence by word of mouth among customer agents.

These results extend those of Moss [16] who found in the unit-square model that intermediated exchange could take place only if the number of customer agents was large in relation to the number of sources. In this case, the same number of customers and sources were active in all simulations as were active in the best functioning markets in the unit square model. The difference of course is that in the unit square model every agent knew of the existence and location of every other agent and every source. Consequently, we infer that a second condition for intermediation to be viable is that the density of agents in the market exceeds some critical level where density is defined on the number of agents known on average to each individual agent.

## 4.2 Prices

When agents are distributed so densely that every agent knows every other agent and also knows every source, then the customer agents can compare the cost of items from intermediaries with the cost of acquiring them directly from sources. Provided that customer agents always choose the cheaper source, intermediaries will have to keep their prices sufficiently low that the total costs of exchange in the system are less than they would be in the absence of intermediation.





Once the density is attenuated in any way, the choice for customers is no longer whether they engage in direct exchange with the sources or intermediated exchange with brokers. The choice becomes one of trading or not since, at less than the highest densities, not all sources will be known to all customer agents. In this case, unless some expenditure constraint is imposed on the customer agents, there is nothing to limit price. Increasing rates of price inflation were indeed encountered in the 625-cell simulation but these obviously had no effect on the volume or stability of trade. No attempt was made in these models to introduce price competition among intermediaries although the customer agents did positively endorse intermediaries they knew to be cheaper than others and, so, other things being equal, would choose the intermediaries offering lower prices. At the same time, they valued reliability – orders translating into deliveries – more highly the cheapness.

We conclude that, while price competition and low prices generally are doubtless important features of some systems (for example, real societies), price competition is not a core consideration for the functioning of exchange processes in large systems.

#### 4.3 The extent of intermediation

Demand satisfaction in all of the modelled markets was very largely a result of intermediated transactions. In Figure 7, the time series in each case represents, from the bottom up, acquisitions of items by customers directly from sources and the total of satisfied demands. The horizontal topmost line is total demand. Evidently, in all cases direct acquisition from sources was negligible.

In the most successful (most densely populated) market, intermediary agents were not on average very long lived and, as indicated in Figure 8, there were always a large number of broker agents.

Because there was a stream of broker agents entering the market, each of them would attract demand enquires from and make supply offers first to agents within their visibility horizon and would communicate with increasingly distant customer agents as knowledge of their existence spread by word of mouth. Consequently, their customers would tend to be relatively close to them. This gives scope for a larger number of brokers to be active in a large system than in a small system (*i.e.* a system where every agent knows every other). As is seen from figure 8, once the market became established, the effective system was marked by a gaggle of brokers. The size distribution of these brokers (by sales volume) is the subject of the next section.

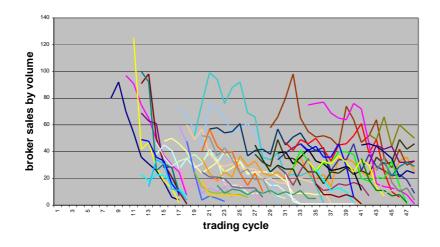


Figure 8: Intermediaries' sales volumes in a 25 '25 (625 cell) grid

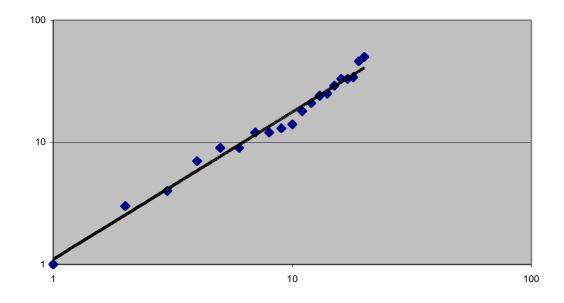


Figure 9: Intermediaries' market share distribution at the 50<sup>th</sup> trading cycle

# 4.4 Statistical signatures

It is clear from figure 9 that the power law holds for cumulative sales volume against the rank of the broker (from lowest to highest sales) at the  $50^{\text{th}}$  trading cycle of the simulation of the 625-cell market. That the power law was obeyed consistently during the trading cycles is shown by figure 5 and table 1. Figure 10 shows the trend lines of the power law data for every ten trading cycles from the  $20^{\text{th}}$  cycle. The intercepts and slopes of the trend lines for the  $20^{\text{th}}$  and  $30^{\text{th}}$  cycles are not distinguishable at the 95 per cent confidence level. The trend lines for the  $40^{\text{th}}$  and  $50^{\text{th}}$  cycles are significantly different from each other and the other two. In all cases,

of course, the slopes are significant and positive indicating that, however many intermediaries are active in the market and despite the finding that none of them are long lived, there is always a substantial monopolistic element.

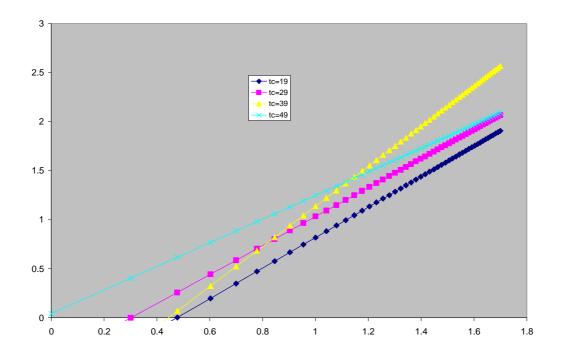


Table 1: Power law regressions on market shares at the 50<sup>th</sup> trading cycle trading  $\mathbf{R}^2$ cycle b а 19 -0.73751 1.555198 0.953736 29 -0.44535 1.477288 0.988578 39 -0.905152.042664 0.942295

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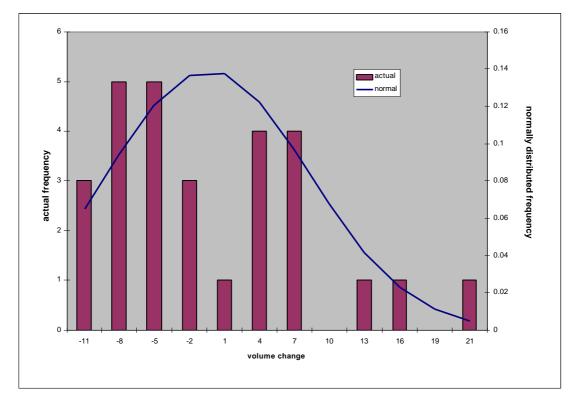
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0.04263

Figure 10: Intermediaries' market share distributions regression lines

Table 1 reports the regression estimates of the power law:  $\log y = \log a + b \log x$ where y is percentage of total sales volume and x is the percentage of intermediaries accounting for that sales volume. The table demonstrates that the power law holds but the particular distribution changes unsystematically over time. The exponent b is not significantly different in trading cycle 29 from the value in trading cycle 19, but they are significantly different at 90% confidence from either of the other two which are themselves significantly different. This result, depicted graphically in Figure 10, is

what Mandlebrot's (1963) work leads us to expect from a leptokurtotic distribution of sales volumes.



# Figure 11: Frequency distribution of simulated sales volume changes – actual and normal

The leptokurtosis of each intermediary's sales volume changes mirrors that of brand data as reported in section 2 above. As seen in Figure 11, the peak of the distribution of sales volume changes for the most successful of intermediaries is some three orders of magnitude greater than the peak of the normal distribution with the same sample mean and standard deviation. The observed tail is considerably fatter than in the normal distribution both absolutely and relative to the peak.

# 5 Conclusion

Previous work on leptokurtotic distributions in market data has concentrated on real or simulated organised financial markets. The present paper extends those results to the more general class of intermediated markets.

The obvious conjecture from these results and the literature on self organised criticality is that, for intermediated markets, leptokurtosis is a measure of the density of interaction among intermediaries and between intermediaries and their customers and suppliers. If the distributions of market shares and sales volume changes are leptokurtotic, then the critical density necessary for competition to generate a high degree of demand satisfaction has been achieved. The simulation model reported here simply replicates the results in statistical physics on leptokurtosis in conditions of dense interaction among metastable entities. The model is itself highly abstract and is not bound by assumptions describing any particular type of market such as a stock exchange.

If the frequency distributions observed in real data and in the data from simulation experiments are stable, then they are stable Paretian distributions with infinite (or, equivalently, undefined) variances. As Fama [6] pointed out nearly forty years ago, the sample variance in these circumstances is "probably a meaningless measure of dispersion [and] other statistical tools (*e.g.* least-squares regression) which are based on the assumption of finite variance will, at best, be considerably weakened and may in fact give very misleading answers." The same conclusions follow if the observed and simulated data are not from a *stable* distribution. The sample variances will not converge to any population variance. Moreover, there is no guarantee with a stable Paretian distribution that even the mean can be defined. If the distribution is not stable, then the absence of a defined population mean follows *a fortiori*. At best, it cannot be assumed that statistical techniques requiring defined means and finite variance are applicable to such competitive intermediated markets as the retail trades, the wholesale trades or the financial markets.

The consequences for theoretical analyses of exchange are no less far reaching. The justification for assuming a defined and constant population mean and finite variance over substantial intervals of time must be predicated on the existence of a stable equilibrium in exchange. Otherwise, there is nothing to justify the absence of the extreme events that contradict the assumption of defined population variances and means. The simulation evidence and real data combine to ensure that the assumption that equilibrium exists and is stable has neither intellectual nor empirical justification where intermediated markets are concerned.

Where social systems are concerned, it is not entirely likely that a stable Pareto distribution prevails. This history of the development of the rules of organised financial markets suggests that institutional change is a common and natural response to extreme events. In living memory, the introduction of automated trading on the world stock exchanges in the 1980s was blamed for the global price crash of 1987. The consequent changes in rules were intended to ensure that significant price changes would be less likely to result from any similarity of rules in several expert

Another example followed the introduction of limited liability by systems. registration in the UK in the Companies Act of 1856. The common practice was to issue shares that were not paid up – allowing the limited companies to call in the event of need on shareholders for additional funds equal to the difference between the par value of a share and the paid up portion. This was intended to provide individual companies with an additional source of liquidity, thereby to make the shares less risky. In fact, the first financial panic after 1856 led to widespread calls on the unpaid portions of shares and the consequent exacerbation of the panic because those who were making the calls were also being called upon for payments in others' shares. There was not sufficient liquidity in the system to meet the calls. Consequently, it became normal practice to issue shares fully paid up or, in the United States, with no par value in order to avoid that source of extreme events. It is not possible to state with any certainty that these changes in rules, custom and practice changed the distribution of price changes but it is equally impossible to state a priori that such changes do not influence the distributions.

Understandably, conventional and even many heterodox economists will find these results and the conjectures they support to be wholly unpalatable. The habits of thought and the training that are essential to both theoretical analysis of stable equilibrium and empirical analysis based on the assumption of well defined population means and variances are rendered otiose by the results and the correctness of the conjectures (if correct they be). There is a long and ignoble history in the economics profession of ignoring results that undermine fundamental tenets of conventional theory. Some of these have been rehearsed by Moss [17] and his cvolleagues [14] in relation to the modelling of climate change. They include the consequences of the Lipsey-Lancaster theorem of the second-best which undermines the empirical value of the social welfare function and the results of the 1960s capital theory controversy on the empirical value of the aggregate production function and related measures of technological change. Mirowski [13] ascribes the same fate to the work of Mandelbrot demonstrating, on the grounds reported here, the inapplicability of classical statistical, econometric and equilibrium analysis to the financial markets. To ignore such results is, of course, to practice bad science.

Good science grabs hold of results that demonstrate an unbridgeable gulf between observation and conventional analysis. In the natural sciences, the difference between observation and theory and the analytical techniques corresponding to the theory, have led not only to new, different or more general theory but also to new techniques of observation and empirical analysis.<sup>3</sup>

If it is right that leptokurtotic distributions in cross sectional and time series data imply dense patterns of interaction and metastable individual behaviour, both of which are ignored or denied by conventional economics, and that leptokurtosis is commonly observed, then it follows that habits of mind must be changed in the search for new theory and the development of new techniques of empirical analysis that are appropriate to the data we observe. Experience in physics suggests that simulation studies will be a key technique though it is of course possible that some analytical results will emerge that explain observation more simply and more generally than simulation studies. But the first step must be to determine how widely we find leptokurtotic distributions in economic data and how well dense interaction patterns and individual metastability explain leptokurtosis and, further, whether such behaviour can be validated empirically.

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<sup>&</sup>lt;sup>3</sup> For a clearly stated and crucially important example of this attribute of good science, see [5].

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