Emergent Tendencies in Multi-Agent-based Simulations using Constraint-based Methods to Effect Practical Proofs over Finite Subsets of Simulation Outcomes

OSWALDO RAMON TERAN VILLEGAS

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Centre for Policy Modelling
the Manchester Metropolitan University

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Abstract

This thesis suggests a methodology for studying complex systems. This method is intended to be particularly useful for searching and proving tendencies (whether considered emergent or not) in those systems whose dynamics seem to be strongly dependent on the system's components' interaction (such as social systems). These systems are commonly simulated in Multi-Agent Systems (MAS).

It begins by examining the formal notions of simulation, modelling, and theorem-proving. Then it reviews some notions of complexity and proposes a notion of the emergence of tendencies as based on the trade-off between subjective and objective factors of complexity. It next moves on to investigate the dynamics of a system via a platform consisting in a (logical) model constraint-based exploration of the dynamics of a simulation. This platform is suggested for systematically exploring the subspace of simulation trajectories associated with a range of parameters of the model, a range of choices of the processes (e.g., agents' choices), and the logic of the simulation program. Following this, we suggest using this architecture in addition to the higher architectural level given by a MAS and an even lower level, a syntactic constraint-based architecture, as complementary means to investigate aspects of the dynamics of a MAS simulation. The proposed methods are compared with other approaches for exploring the dynamics of a simulation. In particular, differences in terms of the notions of morphism among models, the generality of the conclusions, and the measures of behaviour that each approach allows are emphasised. In addition, enveloping the simulation outputs is proposed as an alternative to statistical summaries. This seems to be especially convenient for studying complex systems and for analysing outputs in case of applying theorem-proving techniques.

This model constraint-based architecture is applied to a MAS-based model exemplifying a typical interaction trader-distributor. A tendency is identified in the MAS-based model and then a constrained proof is performed in the model constraint-based architecture. Afterwards, some implications of this thesis for related areas of science are reviewed. Finally, the appendices include an analysis of the complexity of this model constraint-based exploration of trajectories and two papers particularly relevant to the social simulation and the MAS communities.

Declaration

No portion of the work contained in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institution of learning.

The Author

The author obtained a first degree in Systems Engineering at the University of the Andes in Venezuela in 1992. Then he started lecturing at the Department of Operation Research of the Systems Engineering School and took a Master in Applied Statistics at the same University. He is a Member of the Centre for Simulation and Modelling (CESIMO: Centro de Simulación y Modelado) (`http://cesimo.ing.ula.ve/`) of that university. He has some involvement in the areas of Simulation and Numerical Analysis. His research at CESIMO includes Simulation Methodologies and Simulation of structural change following Carlos Domingo's ideas.

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For my parents, brothers, sister, nephews, and for Jasmina.

1 Chapter 1 - Introduction

1.1 Background

Computer simulation has been widely used for studying and understanding systems. It is of particular importance in the analysis of the dynamics of a simulation. For example, discovering emergent phenomena and demonstrating their existence in social simulations have been recognised as key factors in the understanding of social systems (Nigel *et al.*, 1998; Conte *et al.*, 1997; Edmonds *et al.*, 1998).

Already simulation tools such as Multi-Agent System (MAS)-based Simulation have proved valuable in the analysis of socio-economic domains where the agents have bounded rationality (Edmonds *et al.*, 1998; Edmonds, 1999b, 1999c). Different simulation architectures, methodologies, and techniques have been used in these studies. Some simulation communities have been interested in studying 'complex systems' and 'composite complex systems'. A MAS-based simulation consists of generating in a computer the dynamics of a complex system as resulting from the interaction of simpler agents. The idea is to resemble complex behaviour observed in empirical systems as the product of the interaction of individuals. For example, a social modeller might be interested in generating certain aspects of overall behaviour observed in a society via the interaction of simple models of individuals (e.g., people).

Now, the social simulation community is going beyond bounded rationality to explore how 'social artefacts', such as roles, norms, and organisational design, place bounds on social cognition (Carley *et al.*, 1998). This experimentation is potentially useful, for example: for detecting gaps, inconsistencies, or errors in organisational theories (Moss, 1998; Axtell, 1996); for providing insight into organisational behaviour (Carley *et al.*, 1998); for testing, verifying, and improving models of artificial social agents (Moss, 1998; Moss *et al.*, 1998a); for investigating the interdependence between social factors and agents' cognitive models (Carley *et al.*, 1998), and for aligning models (Mihavics *et al.*, 1996; Axtell, 1996).

Apart from the social simulation community, other examples of research communities for which studying and analysing the dynamics of the simulation are important are: ALIFE (Langton, 1989; http://alife.org/), and microsimulation applications in traffic (Nagel *et al.*, 1998 and 2000).

Usually a MAS-based simulation exhibits very complex dynamics – in this fact lies its promise but it also means that its analysis might also be difficult. Behaviour difficult to capture by a modeller but key for understanding empirical complex systems is resembled

in the simulation in the hope of obtaining hints to improve such an understanding from the simulation experimentation. Nevertheless, commonly it is still little understood how such significant dynamics appear from the agents' interaction. To develop methodologies for helping in understanding the dynamics of a MAS-based simulation becomes a crucial task.

As will be seen in Chapter 3, there are two means by which a modeller can seek to understand the dynamics of computer simulations: through design and through observation of the dynamics using a *post hoc* analysis. In MAS, careful design procedures based on well-understood formal models of agent behaviour can help a modeller to understand the behaviour of individual agents and, in special cases, larger parts of MAS. However, understanding the behaviour of interacting groups of autonomous agents by formal design methods has its limitations, and even the most carefully designed MAS can exhibit behaviour unforeseen by its designers as the simulation dynamics become subtle over the state transitions and large number of data manipulations. Additionally, noted as factors associated with this difficulty are the facts that systems simulated in a MAS are of a high level of complexity and that a modeller (in this case, the observer) has bounded rationality (see Chapter 3). An observer's bounded rationality is associated with limited capabilities for capturing and processing information from the observed system (e.g., the target system or the simulated one).

Because of this the second way in which a simulation can be analysed, namely by inspecting the dynamics in a *post hoc* analysis, becomes the most used approach for studying MAS. Among the existing methodologies for doing such *post hoc* analysis are: scenario analysis (Domingo *et al.*, 1996b), where one single MAS trajectory at a time is examined and analysed, and the Monte Carlo approach (Zeigler, 1976), where the MAS is repeatedly run and statistics are collected about general trends over a sample of trajectories.

However, neither of these traditional ways of analysing computer dynamics is satisfactory for MAS-based Simulations (MABS). On the one hand, it is usually prohibitive in terms of computational resources to explore all simulation trajectories using single scenario analysis. Moreover, its use is restricted to certain 'arbitrarily' and subjectively chosen trajectories. Scenario analysis can help by exposing possibilities for the dynamics of the simulated system. Its value for training the modeller or the policy analyst about 'possible future worlds' has been recognised in simulation applications related to policy analysis (Axtell *et al.*, 1996; Wack, 1985; Domingo, 1996b). Nevertheless

the fact that arbitrarily unexplored trajectories are left aside makes difficult the generalisation of conclusions and the analysis's application in wider theory.

On the other hand, in Monte Carlo techniques the analysis of the simulation is made using a sample of the trajectories. Consequently, an arbitrary (in this case the arbitrariness is random) exploration of trajectories is also carried out. The use of statistics supports only probabilistic generalisations but not mathematically (or logically) valid proofs. Hence the usefulness of these techniques in theoretical applications is limited - they do not give definitive answers but only probabilistic ones.

Consequently there is a need to advance the search for new computational methodologies and techniques that allow a more rigorous exploration of the dynamics of simulations of complex systems.

1.2 Aims of the Thesis

The main aim of this thesis is to help in filling the gap apparent in the need for better computational methods for studying MAS dynamics, as was pointed out in the last section. It is of particular interest to develop a methodology for proving emergent tendencies in Multi-Agent Systems. The following sub-goals of the thesis can be stated as part of an effort for developing methodologies and techniques to help in the exploration and proof of emergent tendencies in simulation of Multi-Agent Systems.

- *To develop a methodological approach for exploring and proving emergent tendencies in computational models*

  The intention is to show that it is possible to prove theorems about social simulation models. The idea is to prove that emergent tendencies are (or are not) general under certain conditions when varying certain factors of the simulation dynamics. This methodology will allow a modeller to elaborate conclusions about the behaviour observed in the dynamics of a simulation theoretically stronger than those that traditional explorations of simulation trajectories are able to support (e.g., Monte Carlo techniques permit one to infer only probabilistically valid conclusions; for more details about traditional approaches and their drawbacks, see Chapter 2 and especially sections 2.6 and 2.10).

- *To develop some techniques for implementing such a methodology*

  The purpose is to apply the methodology in a particular simulation example and to give an indication of how it could be automatically implemented. Appropriate techniques for implementing the methodology in a particular simulation language would be developed.

The language SDML for writing MAS is chosen for such implementation because of the relevance of MAS-based models for studying complex systems in, e.g., social and economic systems, and because this language is suitable for implementing formal proofs (e.g., it is a declarative language with a well-grounded underlying logic). These advantages give SDML advantages over many other existing simulation languages, where usually only traditional explorations of simulation trajectories are allowed (e.g., traditional event-driven simulation languages, e.g, SLAM; see Pritsker, 1995, for more details with reference to this point; this argument is better explained in Chapter 2).

- *To investigate the trade-off between complexity and the usefulness of these techniques*

Factors and criteria for this trade-off will be identified relative to a conception of complexity. This conception will be strongly linked to a notion of what are considered as emergent tendencies in complex systems. In addition, the computational complexity of the task which the approach in this thesis suggests, i.e., proving in a computational model, will be investigated and presented (see Appendix 6 - Complexity of the Search). This investigation allows a discussion of the trade-off between computational complexity and factors related to the usefulness of the technique. On the other hand, difficulties for implementing the technique and modelling approach (MAS-based and constraint-based simulation and proving) in a particular language (SDML) will be examined. Further, how different such difficulties would be in case of modelling in other simulation languages will also be discussed. In particular, for analysing simulation outputs, enveloping of tendencies will be proposed as an alternative to central measures and other statistical summaries. The idea of using an envelope is to present an alternative to statistical summarises and to overcome some of their drawbacks for studying complex systems (for example, that pointed out by Crutchfield, 1992, p. 35; aspects of this trade-off will be discussed in section 8.2).

- *To develop a particular case study*

An abstract simulation based on a 'need problem' will be focused upon. The idea is to apply the developed methodology and techniques in a typical model of an empirical system. The intention is to use this typical example to show that the developed methodology and techniques effectively work. Consequently, it must be possible to arrive at stronger conclusions about behaviour in the dynamics of the model than those possible when using traditional methods; i.e., conclusions must find stronger support in

the theory of the simulation model than in those allowed by existing techniques (this case is presented in Chapter 7).

- *To review the conception of emergent tendencies*

Conceptions of emergent phenomena useful for understanding emergent tendencies in computational models will be reviewed. Of special interest will be criteria for testing whether certain tendencies can be considered as emergent by an observer (e.g., by a modeller). The intention is to achieve a more 'practical' notion of the emergence of tendencies, one more practically useful for modelling complex systems. Existing modelling approaches like those named above as well as modelling in different areas of research (e.g., in physics) traditionally consider the emergence of tendencies only from an objectivist point of view. They are usually linked to objective notions of the complexity of a system. However, it also seems convenient, especially when modelling certain complex systems such as human systems, to examine the relevance of subjective factors for a tendency to be considered as emergent by a modeller. These factors seem to be sourced on a modeller's bounded rationality. The idea is to consider both the objective and subjective factors and to reflect on their trade-off (Chapter 4 is specially dedicated to examining these aspects; a further discussion about the usefulness of this concept is presented in section 8.2.5).

1.3 Limitations of the Study

This thesis is focused on (and limited to) the computational modelling of computer systems. The terminology and concepts are intended with respect to this domain. This bias applies throughout the thesis (except in the discussion in sections 4.2-4.3 and 8.6) and should be held in mind when reading the discussion involving highly polemical terms such as 'complexity' and 'emergence of tendencies'.

This thesis is born out of the following disciplines:

Social simulation. This is one of the areas where a need for better methodologies for analysing and understanding better the dynamics of computational programs has been identified (Conte *et al.*, 1997; Edmonds *et al.*,1999).

Multi-agent systems (MAS), in particular their use for simulation of complex systems conceived as made up from the interaction of sub-systems with certain autonomy (Weiss, 1999).

Automatic reasoning and theorem-proving, specifically their application for analysing the dynamics of computational programs. In the application to be considered in this thesis, it will be useful for studying the dynamics of simulation programs; in particular, the interest is in model-based exploration of simulation programs (Wos, 1988).

Artificial Intelligence. This discipline of research includes modelling using MAS, Automatic Reasoning, and other sub-disciplines of research to be assessed in this thesis (Stuart, 1995).

Computational modelling of complex systems. It includes areas of research relevant to this thesis, such as modelling of social systems and, in general, Multi-Agent-based Simulation (Casti, 1992; Holland, 1998). Methodologies to be developed in this thesis will be useful not only in social systems but also in different sub-areas of computational modelling of complex systems.

Systems theory, more specifically notions of systems from an evolutionary perspective, as is conceived by the group working in Principia Cybernetics at the Center Leo Apostol in Belgium (http://pespmc1.vub.ac.be/CLEA/). Heylighen's ideas will be accorded special importance (Heylighen, 1992 and 2000c; http://pespmc1.vub.ac.be/HEYL.html).

Broadly speaking, the example model will be a representation of a trader-distributor relationship. However, in order to facilitate the development of a technique for exploring and proving in the dynamics of the simulation, a more abstract and limited model will be considered. The case study to be developed will be typical but not necessarily representative of a particular empirical system. This will remove unnecessary time-consuming activities during the research, for example, data collection and validation of the model. traders and distributors in the model will be simplified to agents whose main activities are price- and sales-setting, and price-imitating, in the first case; and order-setting in the second case. It is hoped to apply this technique in future studies using more elaborate examples.

It is foreseen that the methodology and techniques in this thesis will be applied to other areas of MAS-based Simulations, especially when studying complex systems. It is hoped they can be implemented automatically in some MAS. In order for this to be possible, these systems must have certain capabilities for exploring computational program dynamics such as those found in theorem-proving and constraint logic-programming (Moss *et al.,* 1997; Abdennadher *et al.,* 1995; Abdennadher, 1999; Frühwirth *et al.,* 1992; Rainer *et al.,* 1988; Marriott, 1998).

1.4 Outline of the Thesis

First, in Chapter 2, a review of basic concepts and the formal aspects of simulation and modelling are presented. We begin by considering basic concepts in classical systems theory in order to set a first 'world view' for the thesis. Then Zeigler's foundational theory for simulation and modelling will be examined. Afterwards other aspects of simulation (which Zeigler's theory does not consider), such as structural change and modelling in Multi-Agent Systems, will be considered. Following this, concepts related to the formal use of simulation results including validation, verification, and simplification of models are examined. At the end of the chapter, the main drawbacks of traditional *post hoc* analysis of simulation outputs for studying complex systems are pointed out and theorem-proving is suggested as a promising area for alternative approaches.

Following, in Chapter 3, formal aspects of theorem-proving are reviewed, including the most common approaches: the model-based (semantic) approaches and the resolution-based (syntactic) ones. A comparison of these methods is presented. Chapter 3 also includes references to OTTER as an example of a successful theorem-prover. Its more relevant characteristics are described later in the chapter (OTTER is used in the implementation of the case study in Chapter 7).

Chapter 4 offers a review of some conceptions with respect to understanding simulation dynamics. Special attention is given to the concept of emergent tendencies. Some definitions of this, using different ideas of complexity and the notion of bounded rationality, are presented. Special emphasis is placed on a trade-off between the objective aspects (grounded in an observed system) and the subjective factors (grounded in an observer's language).

Chapters 5 and 6 present a methodological approach to the study of complex systems using Multi-Agent Systems. Chapter 5 suggests a model constraint-based exploration of tendencies in a subspace of simulation trajectories. Compared to existing approaches, this method allows a modeller to apply stronger notions of morphisms for comparing models, to reach more general conclusions about the dynamics of a model, and to use other measures of behaviour than statistical summaries. This chapter also discusses aspects of how an analysis of the simulation outputs can help a modeller in better understanding aspects of the simulation and, by extrapolating, of the empirical system. In addition, it proposes enveloping tendencies as an alternative to traditional methods for studying simulation outputs. Then, Chapter 6 proposes a platform consisting of a hierarchy of architectures for understanding MAS-based models. The platform is: at the first level, the

MAS-based model; at the intermediate level, the model constraint-based architecture proposed in Chapter 5; and, at the lowest level, a syntactic constraint-based architecture. This platform is intended to assist a modeller in understanding the simulated complex system by showing complementary aspects of the dynamics of the model.

Chapter 7 offers a case study where the methodology suggested in Chapter 5 is applied. The model consists of a MAS-based model of a typical trader-distributor interrelationship.

Chapter 8 discusses some implications of this research in related areas such of modelling and simulation. It includes a discussion about how the methodology and techniques developed in the thesis may be implemented in other platforms and about the trade-off between the usefulness of such techniques and the complexity of their implementation. Conclusions are offered in Chapter 9.

Finally, the appendices and a bibliographic review are presented. Appendices include a review of the speed-up achieved in the case study when the model is translated from the MAS to the constraint-based architecture (both in SDML), an examination of the computational complexity of the search proposed in this constraint-based architecture, and two papers relevant to the social simulation and MAS research communities.

2 Chapter 2 - Simulation and Modelling

2.1 Introduction

The aim of this thesis is to develop tools and a methodology helpful for understanding simulation of social systems and other complex systems. It is of special interest to apply such a methodology in the simulation of social systems using MAS. Given the difficulties for understanding a MAS-based model from its design, the method commonly used is to inspect the dynamics of the simulation in a *post hoc* analysis.

Understanding of a simulation is closely related to simplification, validation, and alignment of models. They rest in a *post hoc* analysis of simulation trajectories, and in a mathematical notion of homomorphism. Because of the difficulties involved in evaluating the formal notion of homomorphism in simulation, weaker notions are used in the existing techniques. As will be seen, to circumvent these difficulties, the idea of *approximation* is used instead of that of strict morphism. Among the widely used techniques based on this notion of approximation are scenario analysis and Monte Carlo studies. However, as pointed out in the Introduction to this thesis, they both have serious drawbacks.

It is our intention in this chapter to review existing formal simulation approaches and the notion of homomorphism used there, in order to make more explicit the drawbacks of these techniques and then illustrate the advantages of the approach to be proposed in this thesis. In particular, we seek to develop a methodology that allows a modeller to prove emergent tendencies in computer simulation of complex systems. This will lend support to the implementation of the strongest notion of morphism, more so than the weak one commonly used in the named existing techniques.

As a starting point, in this chapter basic notions about simulation and modelling will be presented, to help, as said, in clarifying, for example, drawbacks in the existing techniques and desirable properties in alternative techniques. Then, in the next chapter, concepts related to the proving of theorems in computer models will be reviewed, in order to establish a background helpful for developing a methodology for proving tendencies in a simulation. In Chapter 4, before presenting a methodology for proving a theorem in a simulation in Chapter 5 notions related to the understanding of emergent tendencies in the dynamics of a simulation will be discussed.

In the first part of this chapter, basic aspects of Systems Theory will be described as part of the framework (‘world view’) of this thesis § 2.1). This will be used throughout the whole thesis, but it will be of particular value as a basic reference for discussing notions of understanding and emergence in Chapter 4, where this discussion and the development of a

'world view' will be extended. Then, in the second part of the chapter, the fundamental theoretical aspects of simulation and modelling, starting from Zeigler's formalism and progressing to MAS, will be reviewed. In addition, simplification, integration and alignment of models will be discussed. These are important considerations when modelling complex systems. Finally, in section 2.10 some drawbacks of traditional simulation for modelling complex systems will be discussed.

2.2 Fundamental Notions

The basic notion for the 'world view' taken in this thesis will be presented as understood in classical systems theory (Heylighen, 1992; Ashby, 1964; Checkland, 1981; Checkland *et al.*, 1999; Churchman, 1968; Domingo *et al.*, 2000; Fuenmayor, 1986).

2.2.1 Object

The basic entity identified in reality is called an object. Its basic characteristics or attributes, e.g., colour, size, are called properties.

In this view, objects are one of the basic semantic units. This is one of the more basic notions of object, one widely used in computer science and logic. It is useful for describing more abstract concepts as made up of objects. However, its value is quite limited because processes (e.g., evolution) cannot be passively represented. So, a move towards a higher level of abstraction is necessary in order to identify a basic entity useful for abstracting and modelling processes. Such an entity is the system. More elaborated notions of 'object' will be considered in Chapter 4.

2.2.2 System and Process

In classical systems theory, a system can be thought of as a kind of object with additional properties. Among the special attributes a system presents are:

- A *distinction* between the whole, its components, and the interrelations among these components. A component can be a sub-system (a system in itself) or an object. For example, consider a model of a trader-distributor-consumer interaction. The model will represent the whole thing. Among the components are traders, distributors and consumers, and among the interrelations are orders from consumers to distributors and from distributors to traders; sales from traders to distributors and from distributors to consumers; and traders' and distributors' price-settings.
- *Overall properties* of the system are different from components' properties. Such a system's properties enjoy some degree of independence from components'

properties since they are difficult to explain apart from these. For example, among the overall properties of a model such as that described above are the dynamics of prices. Generally, tendencies about prices difficult to understand from the individual behaviour of components (e.g., traders, distributors, and consumers) appear. The term 'properties' of a system will be used instead of 'end' or 'purpose' of a system (as used in some studies) in order to avoid vagueness in the discussion, as that is a less polemical concept.

- *Dynamical structure,* a function given by some laws of change over time, which specifies a system's behaviour. Dynamics introduces the fundamental notion of process. This is significant as it allows a modeller to describe processes as a system's dynamics. For example, the process of evolution as given in nature (a natural system) has been useful in different areas of research, most obviously in biology, but also in complex systems such as social systems. Even more, evolutionary theories have been elaborated in ways that intend them to be useful for approaching systems in general (Heylighen, 1992). Considering the example of a trader-distributor-consumer interaction, each component has a certain behaviour given by its 'goals' in that environment (in such sort of 'market'). They have certain 'rules' for buying, selling, and price-setting. These give the laws of behaviour of each component. Certain laws of behaviour of each component plus some additional laws, such as 'total distributor sales are equal to total consumer' purchases', are useful for generating behaviour of the whole system. In this example, the additional rules might be in accordance with 'marketing theory' or might simply have the purpose of achieving 'consistency' of the whole system.

Other notions of systems found in the literature will be reviewed below.

2.2.3 Subject

A subject is defined as a system with a vicarious mechanism (Heylighen, 1991a). A vicarious mechanism selects among alternative actions in order to satisfy a particular system's goal. This goal might be, e.g., surviving, or perhaps increasing economic benefits. In some subjects, e.g., human beings, this vicarious mechanism rests on genetics and internal models. The genetics provide useful information for surviving as they are passed from individuals to their descendants. This process is actuated for an internal mechanism, which is out of rational control by the subject. Some subjects enjoy a second mechanism at a higher level of complexity, internal models of their environment allowing them to take

choices. Some subjects, e.g., animals, benefit primarily from the first kind of mechanism. The vicarious mechanism is important as it allows the system to take actions in advance of other actions in the environment, in order to adapt in some degree by itself and decrease the likelihood of risky 'corrective' actions from its surroundings (see section 4.3).

2.2.4 Agent

In this thesis, the term 'agent' is used of a kind of system with certain characteristics. Different notions of agents can be found in the literature. For example, in researches in biological and social systems, an agent may be defined as an object with a vicarious mechanism, i.e., a subject; while for people working in physics, an agent may be any entity able to influence others in its surroundings. For instance, when studying the solar system, the earth as a body attracting and rejecting other bodies in the solar system will be an agent. This notion is shared in some modelling computer programs such as the simulation language SDML (Moss *et al.*, 1998a), where an object might be any entity with a rulebase and a database. An agent in SDML is just an object interacting with other objects or an object useful for providing the context (environment) where other objects interact.

It seems useful to check in a dictionary the most common conceptions of agent (Merrier-Webster; http://www.m-w.com/):

Main Entry: agent

Pronunciation: 'A-j&nt

Function: noun

Etymology: Middle English, from Medieval Latin agent-, agens, from Latin, present participle of agere to drive, lead, act, do; akin to Old Norse aka to travel in a vehicle, Greek agein to drive, lead

Date: 15th century

1: one that acts or exerts power

2 a: something that produces or is capable of producing an effect: an active or efficient cause b: a chemically, physically, or biologically active principle

3: a means or instrument by which a guiding intelligence achieves a result

Here, the notion of agent is linked to the notion of influencer. An influencer might be thought of as an object influencing elements in a broader system, but this object does not necessarily have to be considered as a component of the system. While an agent is defined in terms of its actions towards its surroundings, a sub-system is defined as part of the interacting elements defining a system. The conception of agent is more closely linked to the idea of an active entity influencing the environment a subject is studying, but it is not necessarily a key element for helping a subject's understanding. On the other hand, a

system is more closely related to the notion of evolving entity, which is of interest for a subject, as it helps him to model and understand relevant parts of its surroundings.

2.2.5 Model

In general a model is a representation of an object (‘realist position’) or of an idea (‘idealist position’) that a subject has created. In the case of modelling physical systems, a realistic position is taken and there will be a relation of correspondence between the original entity in the environment and the model an agent has elaborated. However, when modelling social systems, which are of particular interest in this thesis, such a physical reference is not clear and the modelling process becomes more subtle as ideas from theoretical and abstract grounds are also appropriate to build the models. Nevertheless, in any case, a model is considered helpful (in practical terms) only if it is simpler or at least easier to manipulate than the corresponding entity it is modelling. So, a model might be an object, for example a picture, or something as elaborate as a system.

People working in social simulation are interested in computational models of the internal representation or models a subject has of its environment in order to reason, make decisions, act, and adapt. Here, models are basically a representation of the subject’s perceiving phenomena, taking decisions, and then acting. Even more, a model might be a representation of good action. A subject with an internal model might have several models of certain aspects of its environment, and will take actions in accordance with a chosen model. It in of interest in this thesis also to consider a subject’s internal model, which may be conceived as a population of sub-models of its surroundings evolving along with perceived phenomena (experience) and reasoning.

2.2.6 Synthesis

So far, a world view where a subject is a particular kind of object has been developed. It is an object that perceives and reasons about its environment. It does this to achieve its goals. It is also a system where ‘evolutionary processes’ are going on. These processes may involve a population of developing models of its environment. A subject’s internal model is useful for abstracting and modelling processes going on in its environment.

On the other hand, the environment is also considered as a particular kind of object. It is and evolving object; e.g., it is an object with behaviour where not only quantitative but also qualitative changes take place.

Finally, an agent is conceived of as a system with special properties. An agent can be seen either as a subject or as a sub-system part of the environment. This is in line with

MAS-based Simulation, where the model of the whole system comes up as the interaction of agents. Each of these agents is a component of the model (system), but each agent also has an internal model of its environment, and consequently, it can be conceived of as a subject.

2.3 Zeigler's Formal Representation of a System

In this section Zeigler's formalism will be recapitulated, though the examples and related discussion are not from him. The idea is to provide in this introductory chapter a formal description of a system as seen in simulation. Zeigler's seems to be one of the first and more successful descriptions of a system in simulation. A description of both a system's structure and a system's dynamics will be given. Originally, its basic formalism is intended to describe simulation models whose structure is fixed, which are common in simulations of systems in industry, e.g., queue systems. However, its ideas can be extended easily to MAS. In fact, each agent in a MAS can be described as a system in Zeigler's formalism and the entire MAS as a composite of a hierarchy of basic and composite agents.

The main parts in his formalism are represented in Figure 2.1. There an input value *x*, and output value *y*, an internal state *q* at two time steps, and the transition from the former $time_1$ to $time_2$, are identified. The output is a function defined by an observer, e.g., something that the observer is interested in. The output function's domain is an internal state of the system. It is supposed there is a function λ generating the output *x* from the state of the system *q*. In addition, there is a change in the system over time (e.g., there is a process): the system's state and the input change over time. Time is introduced as the independent variable.

More formally, Zeigler's (1976) notation for a system *S* is: $S = < T, X, W, Q, Y, d, l >$. Where:

T: Time base (*T* = Reals or *T* = Integers).

X: Input *value* set (each input is a sequence of values)

W: Input *segment* set, subset of *(X, T)*, $W = \{w / w: < 0, t > \rightarrow X, t \in T\}$.

Q: State set.

d State transition function. $d: Q \times W \rightarrow Q$.

l: Output function, $l: Q \rightarrow Y$.

Y: Output *value* set. (there should exist a set of output segments$\{r / r: < 0, t > \rightarrow Y, t \in T\}$)

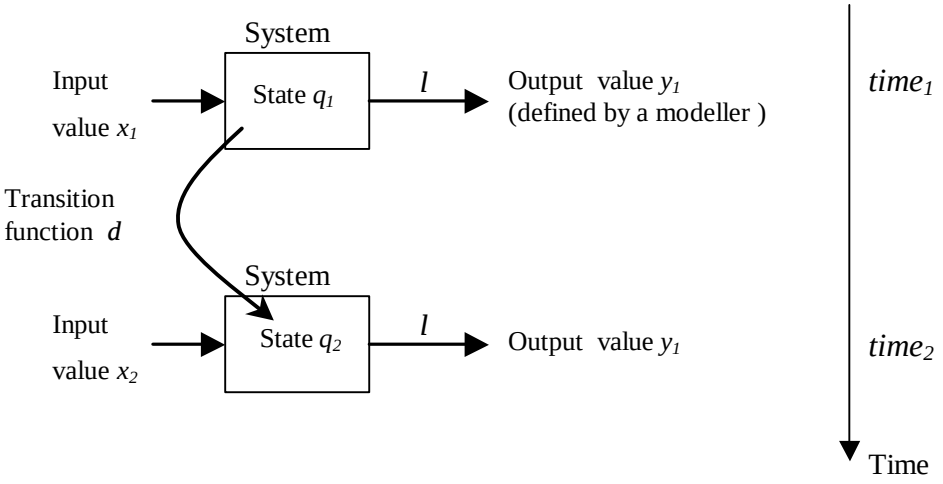

Figure 2.1. Basic notions in Zeigler's formalism: input values x_1, x_2; system states q_1, q_2; output volues y_1, y_2; output function l; and transition function d, changing initial state q_1 into a new state q_2

Example 2.1. Let us consider a linear system

$\bar{q}$: a real vector of size n,

$\bar{x}$: a real vector of size l,

$\bar{y}$: a real vector of size k,

$\bar{q}$: (state) at time t_2 would be given by the application of d at time t_1 ($t_2 > t_1$)

The set of instances for vectors x and y gives a sequence of values over time:

$$d(\bar{q}^t, \bar{w}^t) = \bar{q}_{nx1}^{t+1} = A \times \bar{q}^t + B \times \bar{x}^t$$

$$l(\bar{q}^t) = C \times \bar{q}^t = \bar{y}^t$$

Where t represents t_1, $t+1$ represents t_2, and A, B, C are real matrices.

More explicitly (see also Figure 2.2):

$$\begin{bmatrix} q_1 \\ \dots \\ q_n \end{bmatrix}^{t_2} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} q_1 \\ \dots \\ q_n \end{bmatrix}^{t_1} + \begin{bmatrix} b_{11} & \dots & b_{1l} \\ \dots & \dots & \dots \\ b_{n1} & \dots & b_{nl} \end{bmatrix} \begin{bmatrix} x_1 \\ \dots \\ x_l \end{bmatrix}^{t_1}$$

$$\begin{bmatrix} y_1 \\ \dots \\ y_k \end{bmatrix}^{t_2} = \begin{bmatrix} c_{11} & \dots & c_{1n} \\ \dots & \dots & \dots \\ c_{k1} & \dots & c_{kn} \end{bmatrix} \begin{bmatrix} q_1 \\ \dots \\ q_n \end{bmatrix}^{t_2}$$

Where: n = number of states q_i; l = number of inputs x_j; k = number of observation variables or outputs y_p

The set $<T, X, W, Q, Y, d, l>$ is called the *system structure*. The subset $<X, W, Q, Y>$ gives the *static structure* and the rest of the specification, $d$and l, the *dynamic structure*. The dynamic structure gives the laws of *change of states*. Notice that there are no laws of change either for the static or for the dynamic structure, that is, *this formalism does not consider systems with variable structure*.

Experimental Frame: Zeigler calls the experimental frame 'the limited set of circumstances under which the real system is to be observed or experimented with' (Zeigler, 1976, p. 30). He associates an experimental frame with a subset of the input-output behaviour. More precisely, he formalises an experimental frame E as the subset $< W, Y, l, V>_E$, where the first three components are as given above and V is a subset of Y determining the range of validity of the experimental frame. This subset of Y (i.e., V) contains the control variables of the experiment. The remaining variables in Y are those of interest in the experimentation (Zeigler, 1976, p. 298). *Constraints* are placed in accordance with the modeller's particular interest. In the example, a frame would be given by $<W, \bar{y}, l, \bar{y}>$, where $Y= \bar{y}$ and l are given as in the example, W *is* $(\bar{y}, T)$, where T is defined as the positive integers (e.g., 1, 2, …) and $V= \bar{y}$, e.g., there are not constraints.

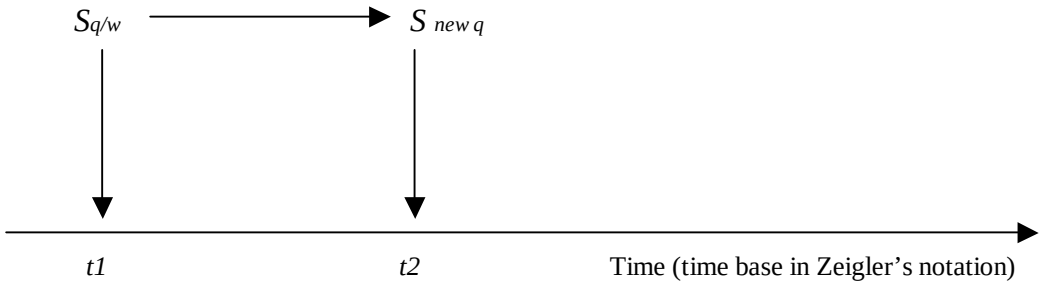

Figure 2.2. Graphic representation of a morphism for example 2.1

2.3.1 Zeigler's Levels of Specification of Systems

The idea is to have a hierarchy of systems' descriptions by increasing levels of elaboration in the sense that the higher the level, the more detail there will be in the specification of the system (e.g., more aspects of his structure will be known). This would be valuable for comparing models' descriptions and to know the degree of specification a model has.

At a *first level* (*level 0*), a collection of inputs and outputs is known over time, but there is no idea about the relationship among them. In a *second level*, an input-output relation is known. Different outputs have been associated with the same input, and, *vice versa*, different outputs have been related to the same input. A *third level* of specification will be useful for associating an input with an output, that is, there is a function relating in a unique manner an input with an output. This level is the most commonly used in science, e.g., in mathematics and physics. Notice that still the notion of process is poor - the weak notion of process is in the output, which can be specified as a function over time. At this level simulation is not an important tool, as the output can be generated only as far as the input and the function input-output are known. This level is not rich enough for helping in those cases where the behaviour of the system is difficult to model via a function, but depends subtly on internal variables (e.g., the state) of the system (e.g., in a social system, or in a queue system). It is in these cases where simulation becomes helpful. In the *fourth level (level three in the specification),* it is supposed that a function for changing the *internal state* of the system is known. The specification of change over time of a system's state brings in clearly the notion of process. Other aspects of a system, such as the output, can be defined in terms of a system's internal state. In the higher level, e.g., at level four, a system is defined as the composite of interacting systems at level two. This level is useful to describe, for example, an agent in a MAS composite of more elemental sub-agents. In fact, it can be used to describe a hierarchy of agents, e.g., a MAS.

More formally, from Zeigler (1976):

Level 0: *Observation Frame*, $S = < T, X, Y >$. The sets of inputs and outputs are distinguished but it is not known how they interrelate.

Level 1: Input/0utput (I/O) Relation Observation (IORO), $S = < T, X, W, Y, R >$, $(R \acute{I} W \times (Y,T)$, *where* $(w,r) \hat{I} R$ ® *(implies)* $dom((w) = dom (r))$. The relation between the input and output sets is distinguished, but it is not possible to differentiate among the different outputs associated with one input and *vice versa*.

Level 2: I/O Function Observation (IOFO), $S = < T, X, W, Y, F >$, $(f \hat{I} F$ ® *(implies)* $f \acute{I} W \times (Y,T)$ is a function, and if $f = (w,r)$ then $dom (w) = dom (r))$. At this level, there is a function between input and output sets which permits one to differentiate between the different outputs associated with an input. It is granted by the *knowledge of the initial state*. Until this level, a move from a lower level to a higher one allows more predictability about the output, given the input, but still there is no knowledge of the system's states.

Level 3: I/O System Specification, S = < T, X, W, Q, Y, d, l >. At this stage, the state set, the transition and output functions are known, but still there is no distinction of the system's components.

Level 4: Coupling of Systems. Here a system is specified as a coupling of several sub-systems or components. Four elements are given: the name of the sub-systems, the specification for each component at level 3, the influencers of each one (other components), and the interface function specifying the input of each component as a function of the outputs of its influencers and the input to the system.

In Zeigler's formalism, it is supposed that the transition function is invariant over time (e.g., a system's dynamic structure is invariant); that is, whenever it is applied over the same initial state and input segment, the same output segment will be generated.

Also, notice that the specification at level zero only gives a pair of the sets of inputs and outputs plus the idea that they are changing over time. There is no specification of structure. This seems to be one of the simplest specifications of a system that can be found in the literature.

Heylighen also formulates specifications of very *simple* (*plain) systems*, for example, systems for which it is not necessary to specify their components or even their states (Heylighen, 1995). He brings in the example of an electron. An electron is considered as a system though no structure has been identified and only the notion of certain behaviour gives its identity. However, 'It has states given by positions, energy or momenta … and their evolution can be expressed by the Schrödinger equation' (Heylighen, 1995, http://pespmc1.vub.ac.be/papers/MST-ConVar.pdf, p. 4). In fact, this model will be at least at level three, as the transition function is known. Heylighen considers that a system is an entity suffering internal change while keeping stability in the sense that it shows differentiation with respect to the environment as an individual unit. He expands this notion, summarising a system as a 'constrained variety or constraint on variety'. This idea can be used for describing systems of order zero: 'The total set of connections (Cartesian product of the set of possible inputs and set of possible outputs) might be interpreted as a maximal possible variety, the subset defining the system as a limited variety, to which actually occurring input-output transitions are constrained' (*idem*, p. 5).

When modelling in traditional economics, the economic system is usually represented at level 4 but the economic agents (e.g., organisations or people) are simplified to level 2. In social simulation this system is modelled at a higher specification level, as the relevant

aspects of an economic subject with bounded rationality, namely, its internal (cognitive) model, are modelled at level 4. One interesting question is whether the simplification made in traditional economics is well justified in the dynamic economic environment of today.

2.3.2 Example: MAS-based Simulation

MAS has been successfully applied to the modelling of complex systems described in terms of components.

A MAS consists basically of a hierarchy of agents. Each agent can be described as a system according to Zeigler's formalism, as was briefly shown in the example given in section 2.6.1. The lower level of agents in a MAS, that of agents without sub-agents, will be described at level 3 in Zeigler's formalism (see paragraph 2.3.1). The other, higher levels (the containers) could be described at level 4.

Agents change as their rulebase changes over time. A relevant example is the evolution of their cognitive models given via a set of rules. The change of rules over time allows more creative and subtle processes than those happening when only elimination and introduction of agents are implemented.

In declarative programming, laws of behaviour are given as a set of rules, which are 'atomic' components admitting to be modified, eliminated, created, and replaced. This is appropriate for evolving the dynamic structure of agents.

2.3.3 Morphism between Systems. Simplifying

Now a criterion for comparing models will be reviewed. This comparison is based on the concept of *experimental frame*: 'equivalence' among models is always relative to an experimental frame. This conception of equivalence is based on the mathematical concept of morphism. A weaker notion of model equivalence will be presented in section 2.6.1. This will be useful for simplification, validation, and alignment of models.

Evaluating morphism consists in checking if the same output-input set a modeller is interested in (e.g., the experimental frame) is observed in two models. It might consist also in checking if a subset of the output a model offers under certain conditions (this gives an experimental frame) coincides with the correspondent outputs other models generate under the same conditions; in such a case it is said that there is a morphism between these two models. As a system's output depends on the system's state, also 'equivalence' between the compared system's states has to be checked. In fact, a system *S* will be called 'equivalent' to a second system *S'* (that is, a morphism exists from the second to the first system) if the state and output of system *S* can be accessed via a mathematical

transformation (the morphism function) from the state and output, respectively, of system *S'* at any time and under certain conditions of experimentation (e.g., under an experimental frame).

Notions discussed in this section can be used to classify and assess homorphisms appropriately when using the methodology to be developed in Chapter 5 for comparing models. Notions of homomorphism associated with this methodology will be discussed in section 5.10.

Consider now the notion of morphism as given by Zeigler. There is a morphism between $S = < T, X, W, Q, Y, \delta, \lambda >$ and $S' = < T', X', W', Q', Y', \delta', \lambda' >$ if there exist functions *g, h, k* such that (Zeigler, 1976):

$1. g : \Omega' \rightarrow \Omega$

$2. h : \overline{Q} \xrightarrow{onto} Q', (\overline{Q} \subseteq Q)$

$3. k : Y \xrightarrow{onto} Y'$

$\forall q \in \overline{Q}$ and $\forall w' \in \Omega$

$4. h(\delta(q, g(w'))) = \delta'(h(q), w')$, transition function preservation.

$5. k(\lambda(q)) = \lambda'(h(q))$ output function preservation.

Each evaluation of δ takes the state *q* and the input *x* at time *t* and generates the new state *q* for the next time step *t+1*. The output function λ takes the state at time *t* to generate the output, also, at time *t*. This is represented in Figure 2.3.

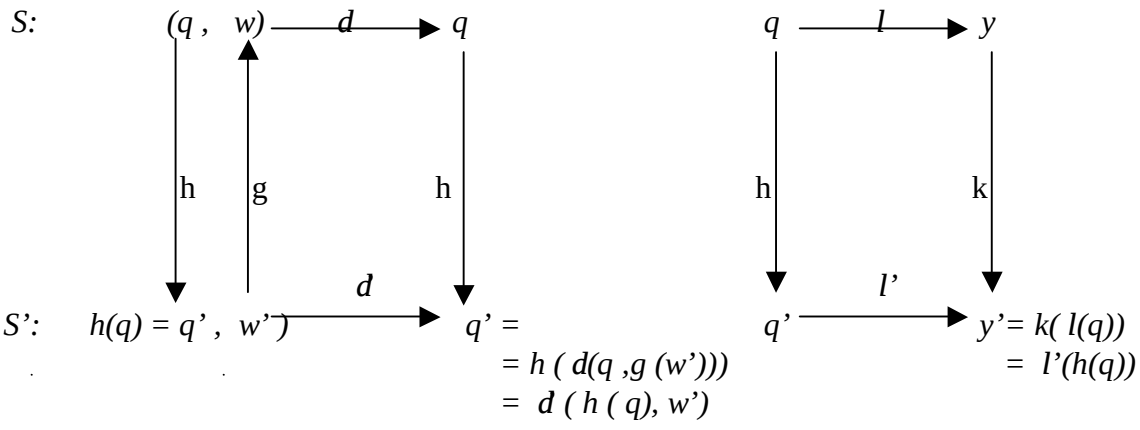

Figure 2.3. Graphic representation of a homomorphism

If *S* and *S'* have the same observation frame, that is: *T = T', X = X', Y= Y', W = W'*, and $\overline{Q}$ is equal to *Q*, then *g* and *k* are the identity functions. The graph in Figure 2.2 becomes as shown in Figure 2.4, and equations 4 and 5 turn out to be:

$$4a.h(d(q,w)) = d'(h(q),w),$$

$$5a.\ l(q) = l'(h(q))$$

S' is an *image morphism* of *S*, and the function *h* is called a *homomorphism*. If in addition *h* is one to one, it is said that there is *an isomorphism* between *S* and *S'* (or that *S* and *S'* are isomorphic).

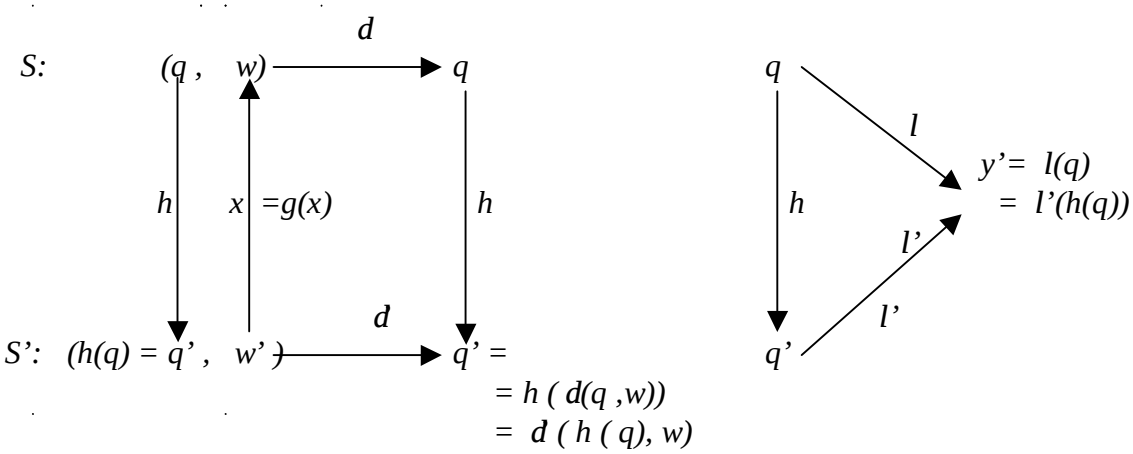

Figure 2.4. Morphism between systems *S* and *S'* under a similar observation frame

Zeigler (1984, p. 251): explains a valid simplification in these terms: 'We say that a lumped model [for us, *S'*] is a *valid simplification* of a base model [for us, *S*] in frame *E* [for us, an observation function *l*] if the two models are equivalent in *E* [e.g., a morphism from *S* to *S'* exists] and the lumped model is simpler than the base model with respect to some measure of complexity.'

Example 2.3

Continuing with the example of the linear system, we define *H* as:

$$H = \begin{bmatrix} h_{11} & \dots & h_{1n} \\ \dots & \dots & \dots \\ h_{m1} & \dots & h_{mn} \end{bmatrix}$$

$\overline{q}$ ' is the state of the system *S'* given as a vector of size *m*. It is supposed that the number of

states in *S'* is smaller than the number of states in S (that is $m < n$).

So, at any *t*:

$$\bar{q}' = H \times \bar{q} = \begin{bmatrix} h_{11} & \dots & h_{1n} \\ \dots & \dots & \dots \\ h_{m1} & \dots & h_{mn} \end{bmatrix} \begin{bmatrix} q_1 \\ \dots \\ q_n \end{bmatrix}$$

Or, more explicitly:

$$\begin{bmatrix} q_1 \\ \dots \\ q_n \end{bmatrix}^{t+1} = \begin{bmatrix} a_1 & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} q_1 \\ \dots \\ q_n \end{bmatrix}^t + \begin{bmatrix} b_{11} & \dots & b_{1l} \\ \dots & \dots & \dots \\ b_{n1} & \dots & b_{nl} \end{bmatrix} \begin{bmatrix} x_1 \\ \dots \\ x_l \end{bmatrix}^t$$

$$\begin{bmatrix} y_1 \\ \dots \\ y_n \end{bmatrix}^t = \begin{bmatrix} c_{11} & \dots & c_{1n} \\ \dots & \dots & \dots \\ c_{k1} & \dots & c_{kn} \end{bmatrix} \begin{bmatrix} q_1 \\ \dots \\ q_n \end{bmatrix}^t$$

And for *S'*:

$$d'(\bar{q}'^t, \bar{w}'^t) = \bar{q}'^{t+1} = A' \times \bar{q}'^t + B' \times \bar{x}'^t$$

$$l'(\bar{q}'^t) = C' \times \bar{q}'^t = \bar{y}'^t$$

The two systems have the same number of outputs and inputs. However, system S' has a smaller number of states.

Now, we can look for conditions to satisfy transition and output function preservation (*4a.* and *5a.*; see also Figure 2.5)(here it is assumed time variables $\bar{q}$, $\bar{w}$, and $\bar{x}$ are evaluated at time *t*):

4a $h(d(\bar{q}, \bar{w}')) = d'(h(\bar{q}), \bar{w})$, becomes

$H \times (A \times \bar{q} + B \times \bar{x}) = d'(H \times \bar{q}, \bar{w})$

$H \times (A \times \bar{q} + B \times \bar{x}) = A' \times H \times \bar{q} + B' \times \bar{x}$

$H \times A \times \bar{q} + H \times B \times \bar{x}) = A' \times H \times \bar{q} + B' \times \bar{x}$

5a $?(\bar{q}) = l'(h(\bar{q}))$, becomes

$C \times \bar{q} = C' \times H \times \bar{q}$

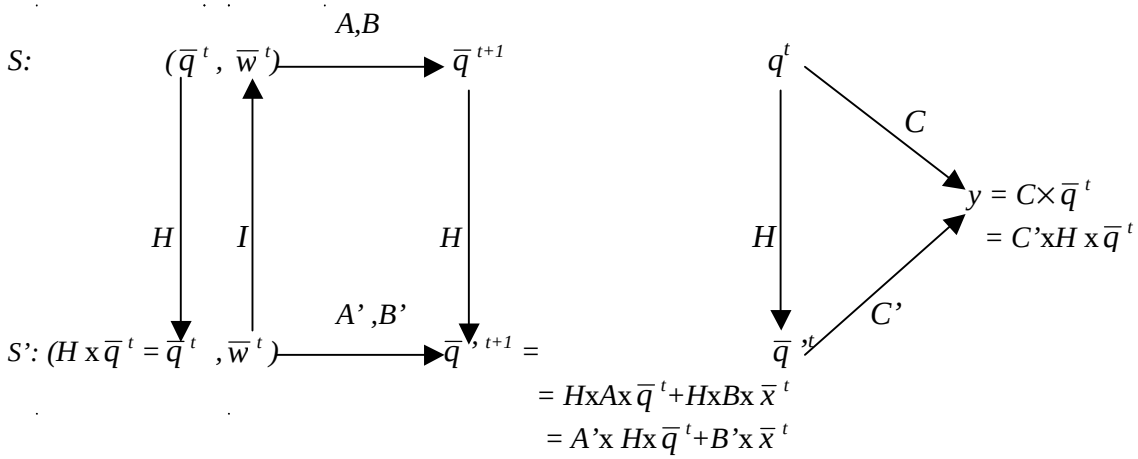

Figure 2.5. Homorphism between systems *S* and *S'* (likewise *A*, *B*, *A'*, and *B'*; also *H* is a real matrix)

As is known from previous equations:

$$d(\bar{q}^t, \bar{w}^t) = \bar{q}_{nx1}^{t+1} = A \times \bar{q}^t + B \times \bar{x}^t$$

$$l(\bar{q}^t) = C \times \bar{q}^t = \bar{y}^t$$

Vectors $\bar{q}, \bar{x}$, and $\bar{w}$ are evaluated at time *t*.

For any *t* and arbitrary vectors *x* and *q*, *4a*, and *5a*, are true if:

6. $H \times A = A \times H$
7. $H \times B = B'$
8. $C = C' \times H$

Example 2.1. A case where *n = 3, m = 2, k = 3,* and *l = 2* is:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}; B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}; B' = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}; C' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Example 2.2. Again a case where *n = 3, m = 2, k = 3,* and *l = 2*:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}; B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}; C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}; B' = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}; C' = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

It is easy to verify conditions *4* and *5*. Systems *S* and *S'* have a similar number of inputs and outputs. The size of the space of states is different, three for *S* and two for *S'*. We notice the exhaustive partition that function H makes in the space of states of *S*. The first component of $\overline{q}$' depends on the first component of $\overline{q}$, and the second component of $\overline{q}$' depends on the remaining two and three components of $\overline{q}$. As the relation among states is not one to one, this is not an isomorphism (see Figure 2.5).

Assume a simplified system for *S: S' = < T', X', W', Q', Y', đ, l' >*, is given. *S'* would be smaller in the sense that *Q'* could be inferred from *Q*, but the opposite would not necessarily follow.

Zeigler identifies at least 4 ways to simplify a model:

1. Dropping of components, descriptive variables, or relations.
2. Replacing one or more deterministically controlled variables by random ones.
3. Coarsening the range set of one or more descriptive variables.
4. Grouping components into blocks and aggregating the descriptive variables also into blocks.

2.4 Homomorphism and the Idea of Metarules

If S' is a homomorphic image of *S*, the behaviour of *(S')'* is a subset of the behaviour of *(S)'*. In fact, it is possible to generate any state and any output of *S'* from *S*, using the homomorphism function *h*. Should *S'* be a valid simplification in a given experimental frame, either of the models generates the behaviour of interest, but it would be cheaper in computational terms to simulate system *S'*. Moreover, in this experimental frame, the behaviour of *(S')*' is the behaviour of *(S)' constrained*. In declarative programming, where functions are given in terms of rules, *S'* might be seen as *S* plus some constraining 'metarules' (as behaviour of *S'* is accessible form *S*). This notion of metarules is used by

Holland (1998). In the example, those metarules will constrain the space of states of *S*, eliminating some sort of redundancy.

In such an example a redundancy occurs when given both values q_2 and q_3 because only one value, the total $q_2 + q_3$, is sufficient for calculating the required output, as the simplified system *S'* shows. The metarule that would have to be added to *S* in order to get *S'* will imply this condition. Note that columns two and three of *C'* are identical. Such a condition would be expressed mathematically as:

$$(c_{i2} = c_{i3}, \forall i) \Leftrightarrow ({y^t}_i = c_{i1} {q_1}^t + c_{i2}({q_2}^t + {q_3}^t), \forall i) \Leftrightarrow ({y^t}_i = c_{i1} {q_1}^t + c_{i2} {q_{total}}^t, \forall i)$$

The resulting output and the transition from two states differing only in the two components (q_2, q_3) will be similar (under the experimental frame defined above) if the sum of q_2 and q_3 gives the same result.

In this particular example *S'* is simpler than *S*, as the systems are of a similar nature (linear systems) but less memory is necessary to keep information about the states of *S'* and less calculations are necessary to generate a simulation state transition - the size of the matrices and vectors involved are smaller than those in *S*. If *S'* and *S* had different natures, such a comparison might not be so straightforward. Here the measure of complexity is defined in terms of computational resources, namely in terms of the amount of memory necessary to keep a system's state and the number of manipulations required for a simulation step transition.

Given an experimental frame (e.g., the sort of input-output behaviour of interest) and a transition function, the homomorphism might be defined in several ways corresponding to alternative manners of simplifying. However, given a system and an experimental frame, it might not be easy or even possible to find the ideal simplest system, *S'*, of *S* under such a experimental frame (assuming it exists). A notion of satisfying seems to be convenient. In addition to the experimental frame and modeller's goal, available theories, methodologies, and modelling tools (e.g., for simulation) seem to be other factors involved in simplifying.

This formal definition of system homomorphism has been useful in simulation. However, generally in practice this formal (mathematical) notion of homomorphism cannot be applied, as no homomorphism as seen in Zeigler's formalism exists and alternative weaker notions of homomorphism have to be used for comparing systems. This point will be addressed in more detail in section 2.6, where an illustrative example will be given.

2.5 Systems Exhibiting Structural Change

Call *S(t)* and *S'(t)* the structures of systems *S* and *S'* respectively at time *t*. Extrapolating our notion of morphism for a system with static structure, *S'* is a simplification of *S* if *S'(t)* is an image of *S(t)* under a suitable morphism. However, to require morphism equivalence at any time instant seems to be too strong, and even unrealistic in practical applications. A weaker notion of 'similar' evolution among structures might be more convenient where evolution of the structures at different time instants is allowed. To account for this, more relaxed conditions for a valid simplification when the systems are under structural evolution would have to be looked for.

The homomorphism function might change over time, e.g., $h = h(t)$. If the function *h(t)* is given in terms of rules, its change over time can be implemented by modifying only some rules rather than by redefining the whole function *h*(t), as would be done in a procedural program. This gives a sort of modularity and flexibility. Consequently, to follow the structural evolution of two systems and express homomorphism between them seems to be easier in a declarative program than in a procedural one.

MAS built in languages like SDML are an example. Structural change such as elimination and/or introduction of components of the system specified at level 4 (as seen above) or modification of components' interrelations can be made in modularly. In addition, processes, e.g., dynamic laws of systems, can be modified by changing, aggregating, and/or eliminating rules. Also rules can be shared among agents and processes. This gives a sort of modularity at the level of process implementation useful for considering changes in the dynamical properties of a system. Rules could be seen as an object that might be created, shared, inherited, aggregated, and destroyed. Rules can also be grouped conveniently and reused in different modelling specifications (e.g., a module hierarchy in SDML).

This sort of structural change, though far beyond what is allowed by known simulation languages in industry (e.g., SLAM II, Pritsker, 1995), is still very limited compared with structural change in empirical systems. In empirical systems structural change occurs at 'infinite' levels. Even more, in highly evolved systems (or systems at a high level of complexity; see Chapter 4), this process might be creative (Domingo *et al.*, 1996a).

2.6 Approximation: Loosening the Morphism Criterion for Validation, Aggregation, and Alignment of Models

Validation is a particular case of weak homomorphism between two systems. A sort of equivalence under a weaker criterion than the mathematical morphism defined above is applied to check agreement between two systems. A weak notion of homomorphism will be described in sub-section 2.6.1. It is supposed there is a bigger system *S*, e.g., reality, and the model or system to be validated *S'*. *S'* will be a valid model if it is a homomorphic image of S under the chosen weak homomorphism criterion.

Aggregation of models consists in integrating several models representing complementary aspects of a system rather than each component. Simulating each of these components usually requires a lot of computational resources, and so it is convenient to find a way to simplify them. Again, the idea of a weak homomorphism is applied, first to simplify the components, and then to validate the new, 'bigger' model resulting from that aggregation. Some researchers in social simulation have proposed creating a cascade of integrated models (Moss *et al.*, 1998b). They describe it as a 'bottom up' modelling procedure in contrast to the 'top down' modelling procedure used in classical models of economics and climate change. They also argue that validation might be done against collected data in reality or 'domain experts'.

Finally, two models are *aligned* if there is a weak homomorphism among them, the idea being to compare models in different platforms, although one model is not necessarily simpler than the other. This notion is used in Axtel *et al.* (1996) for comparing two programs built independently for modelling a similar case. The criterion used in this work for checking homomorphism is weaker than the one given by Zeigler and discussed above (this criterion will be discussed in sub-section 2.6.2).

2.6.1 In Traditional Simulation

Several factors are involved in the difficulties associated with proving homomorphism, among them are: lack of precision in measuring the phenomena, error propagation in the computer, and factors little understood in the empirical system an commonly modelled as random processes. Usually, it is not possible to cancel these restrictions, and the criterion of perfect agreement in a relation of homomorphism has to be replaced for one of *approximation* in practical applications.

In this section the relaxation of the validation procedure for the case of simulations of models with fixed structure will be reviewed (Zeigler, 1976).

'The validation criterion becomes one of '*REASONABLE*' agreement between the systems' outputs The notion of reasonable agreement is formalised in a concept of tolerance. Basically, it is defined as a metric (|| . ||), over the difference between the outputs (behaviour of interest) of the two systems, *S* and *S'*. This difference is required to be smaller than a predefined tolerance. One of the most popular metrics is the square mean.

To calculate this measure of agreement exactly, it is usually necessary to sum over all outputs of interest. Consequently, this requires a *post hoc* analysis of all trajectories. However, it is not usually possible to generate all the outputs of a system via examination of the simulation trajectories. Alternative looser simplification procedures are needed. Techniques where only a limited number of trajectories are analysed are associated with *post hoc* inspection of trajectories where *weak notions* of morphism are applied. Among the existing techniques for doing this looser *post hoc* analysis are scenario analysis and Monte Carlo techniques. Given certain drawbacks of these existing techniques, additional methods are an open area of research (see section 2.10). It is the main aim of this thesis to propose alternative methods, but yet practical, for analysing simulation dynamics, which rest in stronger notions of morphism than the weak ones to be described in this section (see Chapters 5 to 7).

Different *weak homomorphism criteria* can be defined. Taking a phrase from Zeigler (1976, 330) (he uses *f* for || . ||): '*f picks out those aspects of the output CONSIDERED TO BE IMPORTANT FOR DETERMINING ACCEPTABLE AGREEMENT*'. As pointed out before, in Zeigler's simulation theory only quantitative change in the system's outputs is taken into account, this is also true when checking a weak homomorphism.

Example 2.4. A simplification using a weaker homomorphism criterion

Suppose there are two MAS-based models of a trader-distributor interaction. There will be two principal types of agents: Trader and Distributor. Each model is defined as the interaction of several traders and distributors. A distributor's main task is: to place orders to a trader. A trader's main tasks are price- and sales-setting, and price-imitating. The experimental frame will be to observe changes in prices, sales, and orders over time.

Assume now that in a first model the agent trader contains sub-agents warehouses and trucks, where trucks are used for delivering goods to distributors and warehouses are employed to keep control of the input-output of goods.

Now suppose that in the second model warehouses and trucks do not exist as agents. Data and rules kept in warehouses in the first model are now kept in the trader's database

and the rulebase respectively (e.g., now the level of goods can be represented as: *goodLevel(Warehouse, Amount)*). Trucks are just obviated and delivering is modelled using a delay function.

The first model of trader needs to be specified at level four (as it has components), while the second model can be specified at level three (see the next diagram for an overview of the first model of a trader).

First Model of a Trader

Specification:

$< T, X, W, Q, Y, \delta, \lambda >_{Trader}$; *where: < T: Time base, X: input value set, W: input segment set, Q: state set, Y: output value set, δ transition function, λ : output function >*

T: simulation time (iterations), given by a sub-set of positive integers (e.g., 1, 2, 3, …, total number of iterations).

X: inputs at iteration *i* are, e.g., orders placed by Distributors at this agent at the last iteration (e.g., i-1), as well as prices and sales of other Traders at the last iteration.

W: (X, T).

Q: among its states we have price, accumulated value of orders Distributors have made at this agent, its purchases, state of its sub-agents, e.g., level in warehouses (from now it is left implicit that this value is given at iteration *i*).

Y: price, sales.

δ among them purchases and sales estimates, and price-setting.

λ: according to the given experimental frame, prices and orders and probably some statistics or graphs about their behaviour.

Sub-agent Warehouse

Specification: $< T, X, W, Q, Y, \delta, \lambda >_{Warehouse}$

T: shared with Trader.

X: Trader's purchase and amount the truck takes for delivering (this part of the trucks-warehouses interaction).

W: (X, T).

Q: level of goods.

Y: it has been assumed no aspect of Warehouse's state is of interest for a modeller, but level of goods is accessed by the Trader.

δ new level of goods has to be calculated as a function of level, purchases, and sales at last iteration.

λ: null function, as there is no interest in observing behaviour of the level of goods (Warehouse's state).

Sub-agent Truck

Specification: $< T, X, W, Q, Y, \delta, \lambda >_{Truck}$

T: as given for Trader.

X: data about delivering assigned by Traders, e.g., amount of goods to be carried, destination (buyer) and level of goods at Warehouse (part of the trucks-warehouses interaction).

W: (X, T)

Q: a truck might be delivering, waiting for an order, or out of service. If delivering, the amount of goods carried and the state of the trip are specified.

Y: there is no aspect of interest for the modeller.

δ as in the other agents, its states depend on previous states. For example, if the truck is in service and a delivering has been assigned by the Trader in the last iteration, then its new state will be delivering.

λ: null function.

Other facts about this first model are:

The input of a sub-agent (truck or warehouse) depends on the output of some of the other sub-agents (the influencers). In this example, the interaction between a warehouse and trucks serving it is simple: a truck takes produce from a warehouse in accordance with the level of this product, and a warehouse updates its level of product in accordance with the amount taken by the trucks. The states of warehouses and trucks are considered accessible by their container (trader).

Now, let us see how a trader is modelled in the simplified model trader-distributor:

Second Model of a Trader

Specification: $< T, X, W, Q, Y, d, l >_{Trader}$;

T, X, W, Y, l are defined as given in the previous model of Trader.

Q: among the variables given the state of a Trader we have good's price, accumulated value of orders placed at it by distributors, the Trader's purchases, and variables representing states of its sub-agents in the first model, e.g., level of goods (this was previously kept in its sub-agents warehouses).

d this function not only has to generate the transition for data that was in Trader's database in the previous model but also for updating data that previously was updated by its sub-agents, e.g., now delivering is calculated by using a delay function. That is, delivering is modelled as if a portion of the total amount to be delivered were sent each day. This might be implemented using an equation in finite-differences as those employed for numerically approximating and solving differential equations (e.g., Euler's method). As previously noted the data structure has to be modified in order to identify explicitly the missing gents (e.g., an field for identifying each warehouse has to be introduced in the predicates of warehouse 'level'of good).

A simplification between these two models cannot be based on a homomorphism as it was originally defined, but only on a weaker criterion. Proofs of strict homomorphism might be more difficult than the simulation itself and quite unlikely to be achieved successfully. As previously stated, no strict homomorphism exists in practice. Let us assume that Monte Carlo techniques are chosen as the weak method for comparing the two models.

Our experimental frame (see above) specifies interest in observing the behaviour of prices, sales, and orders. Consequently, the evaluation criterion has to be based on these outputs.

The comparison may be made following these steps:

1. A random set of simulation trajectories for each model is generated. It can be implemented using different random seeds for calculating random variables for different trajectories. Random variables are used for modelling certain little-known aspects in the simulation model. For example, assume that for price-setting a trader

imitates other traders' prices. Then, a random variable can be introduced for modelling a trader's choice of another trader.

2. Data of interest are collected from the generated trajectories. In this case data about sales, orders, and prices are gathered.
3. Then a statistical test (here the weak criterion for comparing the two models) is applied to check the hypothesis that the two samples (e.g., each pair of samples of orders, sales, and prices) are coming from a similar statistical distribution and so are statistically similar. Among the methods for making this comparison is the non-parametric Rank Sum Test (Hoel, 1966). Should the test prove successfully, it would be said that the models are equivalent in accordance with this weak notion of homomorphism. As the second model is simpler than the first one (notice that it has less components and less variables), there would also be a valid simplification.

Notice the weakness of this procedure in that the conclusions valid for the explored subset of trajectories are extrapolated by using probabilities to the whole space of trajectories, even though unexpected things can happen at the randomly left trajectories. As previously stated, it is the purpose of this thesis to propose alternative ways for doing the searching for trajectories, and then, based on this, to define alternative notions of morphism.

2.6.2 In Simulation of Structural Change

In simulations where the structure of the model undergoes qualitative change, the experimental frame is generally defined not only in terms of quantitative outputs but also in terms of the structural change of the system; that is, qualitative changes are an aspect of interest. For example, in a MAS-based simulation where agents are eliminated and introduced, the behaviour of interest might be the number of agents of a certain type appearing at the end of a simulation. Notice that in this case quantitative measures can be defined over the qualitative change.

So, a way for doing the qualitative comparison is by defining quantitative measures over qualitative behaviour. Illustrations already exist in the literature, for instance, that given by Axtell *et al.* (1996) for the alignment of models. They compare the average number of stable regions observed in the simulation of a model of cultural transmission. In this experiment, the factors are the number of cultural attributes, the attribute's levels, and the size of the territory where the interaction happens. The statistical comparison is made only at one time instant of the simulation (at the end of it). Axtell *et al.* also mention an

alternative procedure using statistics: to observe if certain internal relationships among variables hold in both models. They call this comparison ‘relational’ equivalence, and the statistical one ‘distributional’ equivalence.

2.7 Verification and Validation of a Simulation

In a simulation processes of three types can be identified, (1) those programmed or explicit in the simulation design (which can be used for verification); (2) those not clearly given in the design but desirable in the model as well as being well known in the modelled system (which can be used for validation); and (3) those called open aspects, which are little understood in the target system and about which a simulation might give hints and help a modeller hypothesise about the behaviour of the model and, extrapolating, about the behaviour of the target system.

For example, in an event-driven simulation of a queue system, the process of arrival of entities into the queue might have been programmed to follow a probabilistic distribution. This first sort of behaviour can be foreseen and so is not of interest for validation. This is an aspect to be verified. Verification can be achieved by comparing the design of the model with some formal specification. In fact, verifying a model consists in comparing the structure of the model with some specification given by a theory.

On the other hand, in the event-driven simulation a modeller might notice certain behaviour about the size of the queues not directly given in the simulation design. Checking if the corresponding behaviour in the target system - which may supposedly be measured in some way (e.g., via a statistical sample of the behaviour of interest) - coincides with behaviour in the simulation is what is called validation of the model.

Finally, there should exist many other features a subject (a modeller) does not comprehend well in the dynamics of the modelled system and which he wishes to understand better by observing the simulation. For example, some special features called emergent tendencies are worthy of special attention in social simulations.

2.8 Event-Driven Simulation

Strategies in event-driven simulations are based on the use of a future event list (FEL). Each event is associated with: the time when it will be activated (activation time), possibly additional conditions for the event to be activated once the time-condition is satisfied, and the consequences of the event (namely changes to be made in the system’s state and the scheduling of further events). Events in the list are ordered by their activation time. For example, in a queue system, the event ‘arrival of an entity into the system’ will be

associated with the arrival time (activation time of this event), other conditions for such an arrival to happen, changes to be implemented in the state of the system (for example, to place the entity in the queue if the queue is smaller than a certain size), and, finally, the scheduling in the FEL of a new arrival.

As can be seen from these examples, the FEL is used for guiding the state transitions over time. Size of time steps is taken as the difference between the time of activation of each pair of consecutive events in the FEL. Two of the most popular strategies in event-driven simulation are simulation by events and simulation by processes. In event-driven simulation each event is programmed in a separate module and a main program executes that module corresponding to the next event in the FEL. In simulation by processes, the process an entity carries out is the key aspect. The program consists in a sort of net for the 'life-cycle' of the entity. For example, in a queue system, the entities are those units queuing and their cycle of life is given by a succession of events: to arrive, to queue, to be served, to leave. The program consists of a net of nodes following the temporal order given by the entity's life-cycle, where the consequences of each event (as well as possible decisions the entity has to take or decisions of the system, e.g., gates) are programmed. An event fires as the entity reaches the associated node(s). So nodes are used to represent: entrance to the system, queue, service, output of the queue, and exit (destruction of the entities). Examples of languages where these strategies have been implemented are SIMAN, SLAM, GLIDER, and SPSS. This sort of simulation is common in industry.

2.9 Simulation of Systems with Variable Structure

The basic aspects of a system with variable structure and its simulation will be reviewed in conformity with the ideas of Domingo *et al.* (1996, 2000); Testa *et al.* (1999), and Heylighen (Heylighen, 1989, 1991, and 1995). Then speculation about simulating structural change according to Domingo's ideas and a MAS will be presented.

Testa *et al.* consider it fundamental to recognise change at both levels: at the level of the static structure (what he calls form) and at the level of the dynamic structure (function). They call this change 'fluctuation'. More precisely, they talk about 'fluctuation within a probabilistic range'. They argue that there is a mutual dependence among the two changes and note that such an interdependence cannot be ordered either hierarchically or causally. They describe the processes of dissolvence and emergence as the two facets of the same coin where a whole (system) arises from its components (sub-systems). The first is the rise of new properties of the whole and the second the constraint of its components 'in a

synergic process of integration and re-organisation'. Examples of biological systems are given.

In Domingo *et al.* (2000), another concise idea of structural change is found, for here they affirm that 'in structural change systems change the behaviour, the components and the relations among them.' They identify other processes leading structural change in addition to that of aggregation described by Testa *et al.* Among them are bottom-up and top-down changes. In the former case the process of structural change is driven from above - the whole system drives the constraint in, e.g., properties of the components and elimination or introduction of new components. In the latter case, the parts are responsible for directing that process. Moreover, they argue that the two processes can be present simultaneously. For example, a revolution can be a bottom-up change led by members (components or actors) of a state (e.g., groups, associations, people) creating changes in the whole system (the state). The opposite happens when there are economic associations among countries. Then the new rules for the economic interchange among the involved countries are elaborated at the top level and then 'imposed' upon the lower levels of the system (the economic actors). However, the systems these examples refer to can present both kinds of changes over a time period of enough length. For example, in the case of a revolution, after the new political force takes power, it starts promoting changes from above (top-down) as it implements, for instance, its economic and educational policies.

Domingo identifies two main problems for the simulation of structural change (SC):

- First, those related to the laws of change, that is, aspects related concerning when to change the structure and how the variation selection of structures will be given.
- Second, that of the level of aggregation. In nature, the aggregation of sub-systems seems to happen at many levels (perhaps at infinite levels) starting from small atomic levels possibly still unknown in physics. Obviously, its modelling in a computer cannot be attempted. In this device only certain levels of aggregation of structural change can be modelled (the relevant ones for certain purposes and in accordance with the available modelling tools).

A graphical representation of several levels of control, where structural change is allowed, is given in Figure 2.6.

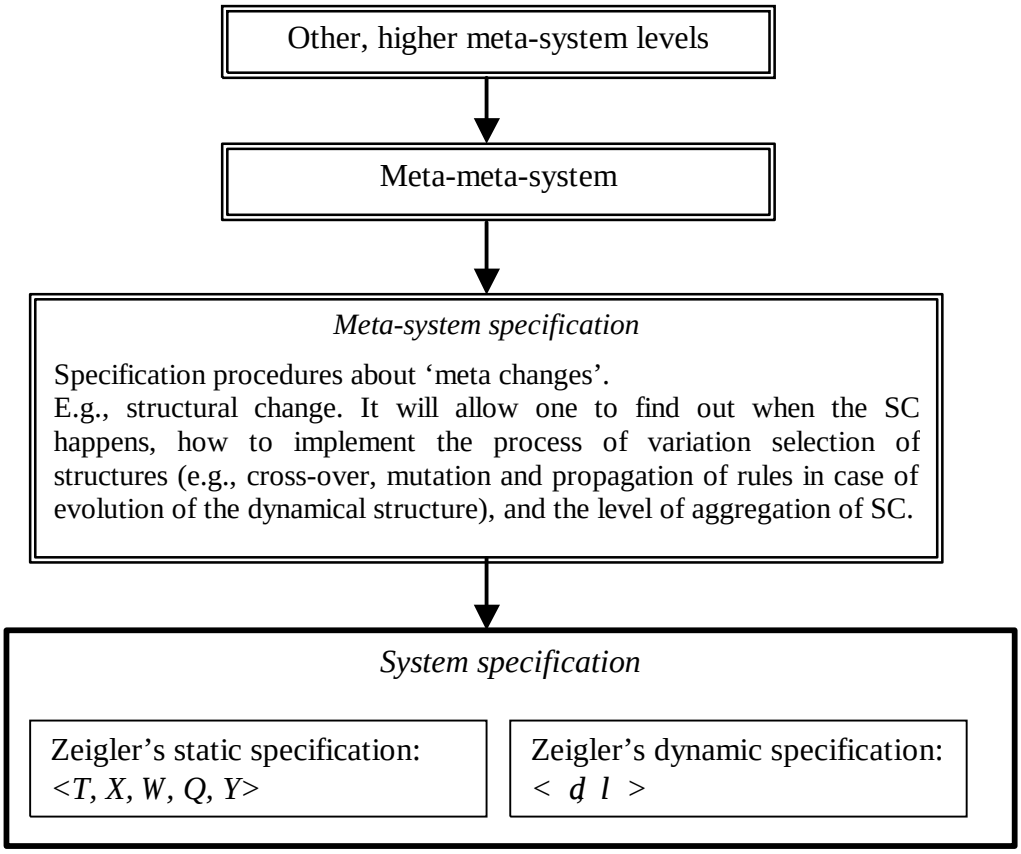

Figure 2.6. Graphical representation of several control levels where structural change takes place

Example 2.5. MAS-based Simulation: A Weak Case of Simulation of structural change.
In MAS some levels of aggregation are at the level of agents. Agents can be created, destroyed, and their rulebase modified. There are several levels of aggregation but the number of levels is finite. Moreover, these structures (agents) and levels of aggregation are fixed. Another level of aggregation is that of the cognitive model of the agent. Examples of agents' internal models are presented in Moss *et al.* (1998a). Here aggregation occurs at the level of rules. In the following pages we will present examples of how the problems averted to by Domingo have been handled in MAS.

At the level of aggregation of agents, since structures are fixed, the first problem, (namely the process of variation selection) is solved in an easy way – it is simply obviated. At the level of cognitive models more elaborate procedures have been used: evolutionary programming. Here rules are evolved using genetic algorithms or genetic programming (Edmonds, 1998). These two examples of simulation of SC are part of two strategies Domingo *et al.* propose to solve the named problems. Structural change in the case of an agent can be a decision either of the environment or of the agent itself. For instance, a trader in a market might be eliminated by the economic environment (e.g., if his profits are

too low) or he may choose to leave the market. A cognitive model can suffer structural change when the agent undergoes a new experience[1].

Both Heylighen and Domingo recognise the importance of a meta-model when modelling structural change. The meta-model, or controller, would be responsible for solving the problems Domingo has indicated: namely, to decide when the change happens, to implement the process of variation selection of structures, and to choose the aggregation level (see also Heylighen, 1991b; Domingo *et al.*, 1996a). Several aspects of their proposal coincide with the strategy followed in a technique to be presented in Chapter 5 and implemented in the example contained in Chapter 7.

2.10 Towards Alternative Methods for Analysing the Dynamics of Simulations of Complex Systems

A *post hoc* analysis of a simulation consists in examining outputs obtained from the simulation (as has already been described in this chapter). This is the strategy used in many simulation studies to understand a simulation after it has occurred owing to difficulties in studying it from its design. Conclusions of this analysis can be extrapolated to the modelled 'real system' (if it exists) under certain conditions (e.g., the morphism criterion discussed in previous sections). In this thesis a *post hoc* analysis is of special interest for assisting in explaining emergent tendencies in a simulation.

Usually it is not possible to include all possible simulation trajectories in a *post hoc* analysis of a model-based simulation, as might be suggested by a strong notion of morphism like that of Zeigler introduced in section 2.3.3. An exhaustive generation of simulation trajectories in a simulation is generally impracticable owing to the limited computational resources available to a modeller. It might also produce too much information and make the analysis of the outputs difficult. Because of this, alternative approaches have been developed along the weak idea of morphism offered by the notion of approximation introduced in section 2.6. Among these alternative methods are scenario analysis and Monte Carlo techniques.

Scenario analysis consists in analysing a modeller's selected subset of trajectories. On the other hand a Monte Carlo Analysis consists in analysing a randomly chosen subset of trajectories. In a Monte Carlo study, conclusions from a 'big enough' sample of trajectories are probabilistically extrapolated to unexplored trajectories (see section 2.6.1).

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[1] For more about a typology of SC, see Domingo *et al.* (2000).

Both of these methods leave unexplored trajectories. Neither of these explorations is exhaustive. In the first method the omitted trajectories depend on the 'subjective criteria' a modeller applies to choose the trajectories to be explored and, in the second one, they are left randomly. Because of this the analysis is limited, as the conclusions cannot be generalised and applied in wider theory – only probabilistic generalisations are allowed (in the second method).

On the other hand, probabilistic extrapolation is valid only if small changes in the settings of the simulation produce small changes in the outputs of the simulation – e.g., if outputs are continuous with the settings of the model. As this assumption is not valid in complex systems (e.g., in systems with chaotic behaviour), a Monte Carlo analysis of a simulation of a complex system might omit trajectories likely to happen in the 'real system' where the behaviour of interest might be quite different to that observed in the explored ones. This arbitrariness in the chosen trajectories engenders difficulties for generalising conclusions when simulating complex systems.

The dynamics of complex systems are usually very sensitive to changes in the properties of the components and, in general, to the evolution of the structure of the system itself, since small changes in the properties of the system might result in big changes in the outcomes of the system. For example, the behaviour of a chaotic system is very sensitive to changes in the initial conditions of the simulation; a slight change in the initial conditions may produce big alterations in the processes this system is undergoing (e.g., that system (in weather forecasting) defined as the surrounding of a geographical area determining the weather conditions of this area). Another example is a social system. In a social system chaotic behaviour is present because of its high level of complexity. It is sourced mainly in the high number of entities (humans) interacting in a society and in the complexity of each of these entities (see the discussion concerning modelling systems at high levels of complexity in Chapter 4, especially section 4.3.3).

Similar susceptibility is expected to be present in the dynamics of a simulation of a complex system. For example, phenomena in a MAS-based simulation of a social system might be unexpectedly contingent on changes in the parameters of the model and the choices of the agents, so that small changes in parameter-settings or in agents' decisions might result in significant changes in the simulation outputs. This inappropriateness of modelling complex systems by using classical modelling approaches based, e.g., on averages, has already been discussed by many thinkers. One of them, Crutchfield (see, e.g., Crutchfield, 1992), argues that in these systems, 'fluctuations dominate behaviour and

averages need not be centred in around the most likely value of its behaviour. This occurs for high complexity processes … since they have the requisite internal computational capacity to cause the convergence of observable statistics to deviate from the Law of Large Numbers' (Crutchfield, 1992, p. 35, in the version of the paper available at http://www.santafe.edu/projects/CompMech/papers/SATTitlePage.html). In this statement, Crutchfield seems to be considering systems at a high level of complexity, e.g., those with a vicarious mechanism (whose ability and activity he refers to as 'internal computational capacity'), such as human systems. Aspects of these systems and their simulation are examined in section 4.3.3.

All this creates doubts about the appropriateness of techniques that arbitrarily leave simulation trajectories unexplored as well as about the use of statistical summaries for studying simulation outputs. This thesis is aimed at developing alternatives to such existing methods, for studying complex systems: on the one hand, stronger notions than that of approximation for overcoming drawbacks in existing techniques but more practical than Zeigler's original notion of morphism (for this, 'constraint-based exploration of simulation trajectories' and a 'platform of simulation architectures' will be suggested), and, on the other hand, alternatives to the use of 'statistical summaries' for analysing simulation outputs are needed (in this sense, the 'envelope of simulation trajectories' will be proposed).

Ideas not commonly used in simulation such as those implemented in theorem-proving and Automatic Reasoning (described in Chapter 3) will be introduced for exploring the dynamics of a system. A proof of a theorem in a simulation rests in a more exhaustive exploration of the dynamics of a simulation than that carried out in the traditional methods named above (see Chapter 5 and, in particular, section 5.5). A proof will allow a modeller, on the one hand, to explore a fragment of a simulation theory, and, on the other hand, it will permit the implementation of measures of behaviour other than averages in the explored simulation theory. As we are interested in studying emergent tendencies, we propose, as an alternative measure of behaviour, the envelope of the tendency in the explored fragment of the simulation theory. Hence, the methodology to be proposed should be a valid alternative to existing methods for analysing the simulation of a complex system.

In addition, this methodology will be in line with Heylighen's call for computational mechanisms for helping a social agent (in this case a modeller) to cope with the complexity of the social system it inhabits (see section 4.3.4).

3 Chapter 3 - Proving Theorems in Computational Models

3.1 Introduction

One of the main purposes of this thesis is to develop a methodology to analyse and understand the dynamics of simulation in MAS. It is of interest to analyse regularities in the dynamics of a simulation not explicitly given in the simulation design and which are relevant for understanding the modelled system, despite being poorly comprehended by the modeller (these regularities will be called 'emergent'). As observed in section 2.10, existing methods for analysing the dynamics of simulations of complex systems have serious drawbacks and this thesis aims at looking for alternative approaches.

As was shown in the previous chapter, those existing methods for investigating simulation behaviour allow one to draw only limited conclusions about the dynamics of the simulation in only very specific spaces of the simulation theory. Those methods explore only a single simulation trajectory at a time and use the notion of approximation for proving morphisms. It is the intention in this thesis to investigate about more powerful methods for analysing such dynamics, that will permit one to reach more general conclusions and to define stronger notions of morphism than those allowed when using the notion of approximation. Concretely, this thesis aims at developing methods for proving theorems about the dynamics of a simulation. As it is usually difficult to explore the whole dynamics of a simulation (e.g., about the whole simulation theory), this thesis seeks to find methods to prove theorems with respect to a sub-space of the space of the simulation dynamics and, correspondingly with respect to a fragment of the simulation theory.

Though in this thesis a methodology for proving theorems in a simulation will be developed, this work is also exploratory in the application of theorem-proving strategies for proving tendencies in the dynamics of a simulation. There have been no previous developments in this line, as far as is know. Because of this, a review of a wide variety of existing methods, not only those directly related to the strategy to be proposed in this thesis, will be considered in this chapter. This review might be used for classifying: the theorem-proving strategy followed in the theorem-prover to be used to implement an example (e.g., OTTER), existing methods for simulation, and the theorem-proving strategy for proving theorems about the dynamics of a MAS to be presented in the next two chapters.

The introduction to simulation given in the previous chapter, together with the review of formal aspects of logic-programming and theorem-proving to be presented in this chapter,

will provide the theoretical background for a move towards the methodological developments to be given in the next chapters of this thesis. The main intention of this chapter is to provide a review of the different strategies for theorem-proving.

First, in section 3.2, basic conceptual aspects of proving theorems, using computational algorithms, will be shortly addressed. Then, the difficulties that have been found for finding a good and general computational algorithm to prove satisfiability/unsatisfiability of a formula will be briefly touched upon. Also, in this section, a basic conceptual framework related to proving theorems in a theory useful for the rest of the presentation will be offered.

Then (in section 3.3), similarities/differences between exploring dynamics of a computational program in theorem-proving and in a simulation will be reviewed. The intention is to preview the most straightforward similarities between simulation runs and explorations in theorem-proving. Direct parallels between the search strategy called 'model exploration' in theorem-proving and generation of trajectories in simulation will be found.

Section 3.4 will discuss in brief seminal work for proving in computational models. The initial interpretation-based exploration of 'logical models' in a theory, namely Herbrand's universe and Herbrand's theorem, will be addressed. Herbrand's theorem is the basis for the more efficient methods for checking falsity of a set of clauses in first-order logic. This theorem became the fundamental reference for developing resolution, the most widely used procedure for proving. This section provides the starting point for a more comprehensive review of theorem-proving strategies in section 3.5, whose methods are based on these two achievements of Herbrand.

Having presented Herbrand's foundational work for theorem-proving in section 3.4, the main strategies for theorem-proving will be reviewed in section 3.5. It will include a consideration of the most popular search procedures and strategies in theorem-proving. Two main branches of theorem-proving approaches will be presented in accordance with the orientation of their inference procedure, which might be either predominantly semantic- or predominantly syntactic-oriented. Simultaneously, examples of theorem-provers using these procedures and strategies will be offered. First, the syntactic methods will be presented. These have been the most successful ones. In this group is the theorem-prover OTTER. OTTER will be used to program the case study of this thesis and so will be an important reference as a source of ideas for developing the methodology to be presented in Chapters 5 and 6. This review may be used to describe the main aspects of OTTER and other theorem-provers found in the literature. However, neither the simulation method to

be proposed in the next chapter nor the common simulation methods correspond well to a syntactic-driven proof. Then, the other group of strategies, that consisting of the predominantly semantic-oriented, will be reviewed (in sub-section 3.5.2). In this group will be found the so-called model-based exploring approaches and, among them, tableaux. This sort of search has similarities with the method to be proposed in the next chapters, but again this method does not fit completely well in the sort of search described in this sub-section.

However, the given taxonomy, as also happens with any other given classification, is somewhat arbitrary, as most of the theorem-provers are in part semantically and in part syntactically oriented. Hence, it will be worth going further and analysing strategies for theorem-proving using additional criteria.

To this purpose section 3.6 will be offered. In this sense, in sub-section 3.6.1 additional criteria for classifying theorem-provers are given. Then, using the criteria provided so far, the main aspects of Bonacina's (1998, 1999) classification are recapitulated in section 3.6.2. As, once again, neither the proving strategy to be developed in this thesis nor other simulation methods fit well in the given (Bonacina's) taxonomy, a further discussion will be presented in sub-section 3.6.3.

Having opened the panorama of theorem-proving strategies, the main aspects of a new tendency in logic-programming - constraint logic-programming and its variant, rule-based constraint logic-programming - will be considered (see, e.g., Frühwirth *et al.*, 1992). This new area of research seems to be relevant to achieving efficient methods for exploring dynamics in simulations. There, inference procedures using a constraint-driven search as an alternative to unification are implemented. It appears that there, novel and promising ideas for efficient implementations of exploration of behaviour in a simulation can be found.

Finally, the usefulness of meta-reasoning for an efficient exploration of the dynamics in a computational program and, in particular, for *implementing context-driven inference procedures* will be highlighted. Context-oriented search seems to be convenient for exploring and proving theorems in the dynamics of a simulation.

3.2 Proving: an Ancient Problem

The problem consists in finding a proof procedure to verify the validity or inconsistency of a formula or a theorem with respect to a theory. In simulation, a theorem might represent a tendency in the dynamics of the simulation.

According to some authors (e.g., Chang and Lee, 1973), such a problem was first tried by Leibniz, then by Peano, and, in this century by Herbrand's school. Nevertheless, a general procedure for proving theorems in a theory was proved impossible by Church and Turing in 1936 (Chang and Lee, 1973). In any sufficiently expressive formal system, there will always be invalid formulas for which these procedures will never decide. For example, 'in certain cases the generation of clauses never stops, because no finite database satisfies all clauses, but a contradiction does not arise either. This is because of the 'undecidability of satisfiability' (Stickel, 1988). However, there are procedures that are useful for attempting to prove certain valid formulas.

In logic-programming, a method or program able to provide an answer (in this case, to the dichotomic question of whether a formula is satisfiable or not) within a finite delay is called an algorithm, while those that can provide and answer when such an answer is positive but there is no guarantee of getting an answer when it is negative, are called semi-algorithms or procedures (Gochet *et al.*, 1988). Similarly, it is said that a theory is decidable if it admits a decision algorithm and semi-decidable, or partially decidable, if it only admits a decision procedure. Methods for proving the validity of a theorem (e.g., resolution) for first-order logic are procedures (Gochet *et al.*, 1988).

In consequence, in this chapter the aim is to discuss appropriate procedures for proving in particular situations. As a preliminary step towards this goal, the main ways to check the validity of a theorem with respect to a computational theory will now be reviewed.

According to Chang and Lee (1973, p. 45) there is no guarantee of finding any procedure to check the invalidity of an invalid formula. On the other hand, no general procedure exists for verifying the validity of a valid formula. However, it is always possible to find a specific procedure for checking the validity of a particular valid formula.

The most important approach for proving validity of formulas in theorem-proving was given by Herbrand in the 1930s. The first algorithm for implementing Herbrand's method was given by Gilmore in 1960. Then, this algorithm was modified, the most important improvement being that given by Robinson. These algorithms verify the validity of a formula by detecting inconsistency of the negation of the formula. The important point here is that: if the formula is valid – and obviously such detection is not possible - the program will halt after a finite number of steps (Chang and Lee, 1973).

3.2.1 Basic Concepts and (Logical) Model Search/Construction

In this thesis the phrase '*logical model'* will be used to refer to the concept of model in the context of logic. This will help to prevent confusion, as the notion of model in logic and in logic-programming is different from the concept of model in modelling and simulation, which has been used until now in this thesis.

First, some basic concepts will be defined in order to provide a context for the notion of model in logic-programming (Gochet *et al.*, 1988). However, only some notions will be covered and other more basic concepts, such as predicate, clause, literal, function, term, constant, proposition, unification, and predicate, will be assumed as known.

Seven terms that will be frequently used below and have a special significance for understanding the notion of theory (e.g., that implicit in a simulation) and proofs in a theory are now defined:

Signature: collection of predicate and functional constants.

Language: set of terms and formulas generated by the signature through the syntactical rules of predicate calculus.

Structure: relative to a language is an interpretation of this language.

Theory: *relative* to a language is a set of formulas of this language.

Axioms of a theory: that set of formulas in a language defining the theory.

Theorem in a theory: logical consequence of the axioms defining the theory.

Logical model of a theory: a structure for which all formulas of the theory are valid.

A logical model of a theory in predicate logic might be given with reference to ground terms of the language, that is, specifying whether each ground term is true or false. Then, the definition of interpretation given in the semantic specification of the language for variables and formulas can be used to determine whether a variable or a formula is valid under a certain logical model or under some interpretation. Alternative techniques for defining a logical model or a set of logical models are: specifying only the set of valid ground terms (the remaining ones are assumed to be false by default), or by a formula. Figure 3.1 illustrates the interrelationship among these concepts

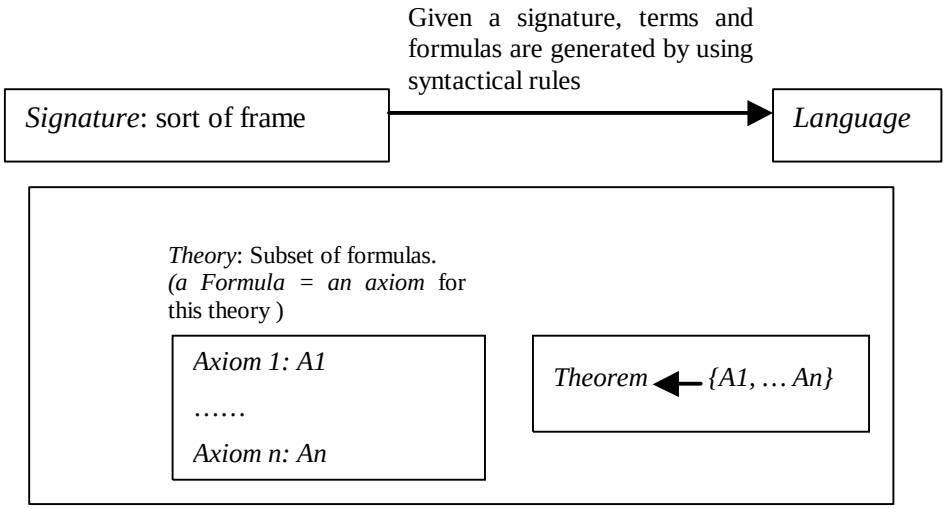

Figure 3.1. Graphic representation of the relation among the concepts: *Language, Signature, Theory, Axioms of a theory, and Theorems in a theory*

Example 3-1: Signature: collection of predicate and functional constants.

Signature: *P(,), Q(,), R( )*

Language: any formula *A ¦ B,* where *A* can be *P, Q, or R* valued on certain variables, and ¦ can be any of the operators commonly used in the literature for defining a propositional first-order logic *{v, ?, …}*

An example of a theory is *S = {(1)~ P(x, y) v ~ Q(y,z) v R(x), (2) P(x,y) v R(y,z), (3) Q(x.y) v R(x)}*

A theorem for this theory is *R(x).*

Assuming a propositional language, S = {(1)~ P v ~ Q v R, (2) P v R, (3) Q v R}:

An interpretation (signature) would be {R = Q= true, P = false}

The formula R can be used to represent the set of logical models M = {R = true, Q = V_1, R = V_2} where V_1 and V_2 are in the domain {false, true}.

In general, a logical model search procedure for proving is an interpretation-guided exploration of logical models. An example of a logical model search using backward-chaining is tableaux. Also, forward-chaining logical model procedures have been implemented, for example event-driven simulation and partitioning of rules in some declarative MAS, as will be seen below.

3.3 Simulation, Logic-programming, and Theorem-proving

Logic-programming and theorem-proving can be applied to many research subjects. In this thesis its particular interest is its use for proving tendencies in the dynamics of a

simulation. To make clear the relation between simulation on the one hand and, logic-programming (and especially theorem-proving) on the other, the relevant aspects of the dynamics of a simulation and inference procedures in logic-programming and theorem-proving will be reviewed. The main similarity is found between what in simulation is called a 'trajectory' and what in logics is called the logical model of a theory.

A trajectory of a simulation corresponds to a logical model or valid interpretation of a theory in logics. In fact, the generation of trajectories in a simulation can be seen as a particular case of logical model generation. It is a generation of logical models for the theory represented by the simulation program. However, it is not always required to know all valid facts in a logical model, as is the case in tableaux, in order to check theorems in a trajectory. A simulation implemented using declarative programming will be expressed in terms of clauses and there may exist alternative ways of generating logical models corresponding to the different possible simulation trajectories. A formula will be a theorem of the simulation program if it is true in all simulation trajectories. A semantic proof of certain regularity in a simulation can be implemented by generating all trajectories and observing whether the regularity holds in all of them (this is a 'logical model checking'). This assumes that, as is common in simulation, a finite-state structure representing all logical models exists.

Simulation trajectories are usually generated following an event-driven or at discrete, equally spaced intervals of the independent variables. This is the general case, for example, in MAS and finite difference methods for solving differential equations. Its generation usually consists in a time- and/or space-driven generation of states starting from a certain initial state. This is a sort of forward-chaining inference. For instance, in the simulation of a differential equation, the simulation might follow a temporal and/or a spatial order. Consider, for instance, the case of a two-space heated plaque whose temperature at the internal points changes over time and is different for different points. The differential equation might be like this:

$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial^2 x} + \frac{\partial^2 T}{\partial^2 y}\right)$; where: T represents the temperature, x and y are the spatial variables, and t represents time. These sorts of equations are solved using finite differences: the partial derivatives are replaced by finite difference approximations, then a set of finite values in the range of interest is chosen for the independent variables *x*, *y*, and *t*, and, finally, T is obtained for the chosen points of the independent variables step-by-step,

starting from the known values of *T* in the boundaries of the plaque. Should this flux be null, time is no longer a variable of interest, although it may still be interesting to know the plaque's temperature at each point; then the simulation steps would only be over the spatial edges. In this case the simulation steps follow only a spatial sequence. In the case of the simulation of processes involving humans (for example, simulation of a queue system in industry, in a road, or in a bank, or simulation of social interaction in a market), the dynamics is usually carried out following a temporal sequence.

Similarly to what happens in simulation, many logic-based computational systems follow a forward-chaining strategy. Consequences are generated as the antecedent of inference rules match facts already existing in the database. This is a strategy also used in theorem-proving. Generations of trajectories over time or in temporal logical model exploration have been formalised in Temporal Logics. See, for example, Fisher *et al.*'s proposal of a clausal resolution procedure for discrete temporal logic (Fisher *et al.*, 2000). Obviously, theorem-provers that use forward-chaining inference have similarities with simulation.

A common strategy for proving in computational models, which is extendible to simulation, is to add the negation of the theorem into the database and then show that this causes a contradiction. A drawback of this sort of search is that many irrelevant facts (for the proof) may be generated before the contradiction appears. To handle this problem, heuristic strategies have been developed. More details about these procedures will be given below.

In contrast to this, searches in logic-programming (e.g., in Prolog) are not usually forward-chaining but backward-chaining or goal-driven. A goal is managed as a question to be answered with respect to new goals or facts. Assume this is a goal at level 1. If this goal matches the head of a rule (consequent), the body (antecedent) of the rule (with the respective update of the variables according to the variable unifications) becomes the new goal in a next level (level 2). This new goal might consist in a conjunction of predicates. In this case, each of those predicates would be a sub-goal to be satisfied at the new level – thus, a new list of sub-goals is constructed. The next task is to check if each of these goals is satisfied. Then, one of them is taken as the goal to be treated. Once the head of a rule matches this sub-goal, then a new list of sub-goals at another level (level 3, given by the antecedent of the matching rule) is set. The search stops as soon as all resulting goals become true, that is, as soon as all the goals are shown to be facts.

If during that search new sub-goals have priority over previous target-goals to be tested (e.g., goals are listed in order LIFO: Last-In First-Out), the search is called depth-first. On the other hand, if new sub-goals are considered only after previous target goals (e.g., goals are arranged in order FIFO: First-In First-Out), the search is called breadth-first. Generally a depth-first search generates less data than a breadth-first-search. It is the search commonly implemented in logic-programming (e.g., in Prolog).

Despite these differences in how the inference is driven, the inference procedures in both logic-programming and resolution-based theorem-provers are variants of the semantic and syntactical methods to be explored below.

To summarise, in some aspects simulation is closer to logical model generation in logic-programming and theorem-provers than to logic-programming languages such as Prolog, as those procedures and simulation usually follow forward-chaining inference procedures while Prolog's inference procedure works in backward-chaining. On the other hand, however, simulation also has similarities with certain backward-chaining search procedures like tableaux, as they both follow a logical model generation strategy. Accordingly, in principle it seems convenient to try proving in simulation using procedures already implemented in forward-chaining theorem-provers and in logical model generation. Nevertheless, syntactic backward-chaining methods appear attractive to lessen certain problems (e.g., the huge amount of accumulated data) appearing when implementing logical model generation and forward-chaining methods. A recently developed approach is constraint logic-programming, which seems promising because of its flexibility for driving the search by exploiting semantic information in the data rather than using only traditional unification. Aspects of this approach will be utilised in this thesis as the methodology to be proposed in Chapter 5 consists in certain constraint-based explorations of trajectories.

3.4 Seminal Work: Towards a Procedure for Checking Unsatisfiability of a Set of Clauses in First-Order Logic

The first attempts for checking the unsatisfiability of a set of clauses before, in the next section, going on to those commonly used in theorem-proving will be presented in this section. It is common to attempt to prove the unsatisfiability of clauses. A theorem is proved by showing that its (classical) negation is unsatisfiable.

3.4.1 Interpretations

The barest procedure for proving the falsity of a set of clauses S is to check if they are false under all possible interpretations of the theory. Evidently, there might be a huge number of possible interpretations making this task impossible. This is only realistic in a few cases and not in those of interest in this thesis.

Notice that this procedure is not syntactic but semantic. Each interpretation for which a theory is valid is a logical model of the theory. Other approaches for proving based on interpretations will be reviewed in section 3.5.

3.4.2 Herbrand Universe and Herbrand's Theorem

In order to improve the previous procedure, it seems convenient to find a sub-domain SD of all possible domains D of clause interpretations, so that, to know if a set of clauses S is unsatisfiable, it is necessary and sufficient to check the interpretations of S in SD. The first answer came from Herbrand and the sub-domain is called *Herbrand universe* of S (for more details, see Chang and Lee, 1973; Wos, 1988; Gochet *et al.*, 1988). Chang and Lee (*idem*, p. 52) define it in this way:

'Let H_0 be the set of constants appearing in S. If no constant appears in S, then H_0 is to consist of a single constant, say $H_0 = \{a\}$. For $i = 0, 1, 2, \ldots$ let H_{i+1} be the union of H_i and the set of all terms of the form $f^n(t_1, \ldots, t_n)$ for all n-place functions f^n occurring in S, where t_j, $j = 1, \ldots, n$, are members of the set H_i. Then each H_i is called *i-level n constant set* of S, and H_∞, or $\lim_{i \to \infty} H_i$, is called the *Herbrand universe* of S.'

Examples:

a) if $S = \{P(a)\}$, then $H_0 = \{a\}$;

b) if $S = \{P(a), P(f(x)), Q(g(y))\}$, then

$H_0 = \{a\}$;

$H_1 = \{a, f(a), g(a)\}$;

$H_2 = \{a, f(a), g(a), f(f(a)), f(g(a)), g(f(a), g(g(a))\}$;

$H_2 = \{a, f(a), g(a), f(f(a)), f(g(a)), g(f(a), g(g(a)), f(f(f(a))), \ldots\}$.

Instances of a clause C obtained by replacing variables in the clause C by members of the Herbrand universe are called *ground instances.* In the second example above, $P(g(a))$ and $P(f(g(f(a))))$ are examples of ground instances of clause $P(f(x))$.

However useful Herbrand universe is, it is usually hard to check all interpretations over it because it is still too big. Further steps toward more useful procedures were encouraged by Herbrand's theorem. Chang and Lee offer a version of it:

Herbrand's Theorem. A set of clauses S is unsatisfiable if and only if there is a finite unsatisfiable set S' of ground instances of clauses of S.

The importance of the theorem is that it will be sufficient to have a finite set *S'* of unsatisfiable ground instances of *S* to know that the whole set S is unsatisfiable. This theorem has become fundamental for developing resolution, the most widely used procedure for proving. The main difficulty for applying this theorem is the requirement of ground clauses. A resolution goes forwards; it has to be applied not only over grounded clauses but also over ungrounded ones (Chang and Lee, 1973).

3.5 Approaches to Theorem-proving: Syntactic (clausal)- and Semantic (interpretation)-based Searches

In this section some of the main approaches in theorem-proving will be considered. First, those implementing syntactic-driven procedures based on clause manipulation will be outlined, followed by those that are semantic-driven since they are founded in logical model generation. Among the resolution methods and heuristics to be listed below, there were used in the examples presented in this thesis (see Chapter 7): hyperresolution, the set of support strategy (a particular case of semantic resolution), subsumption, fewest-literal preference strategy, and demodulators. Other, alternative methods currently used in theorem-proving or in declarative programming will be presented.

3.5.1 Syntactic (clausal)-based Inference Procedures

The great advantage of resolution is that it can be applied over clauses having variables. Therefore, the size of the universe of clauses over which it works is usually smaller than when using Herbrand's universe while the number of interpretations to be considered is still the same. Actually, as resolution can be applied over clauses having variables, many or even an infinite number of interpretations can be considered at a time.

Resolution-based theorem-proving has become a very important area of research since 1965, after the introduction of resolution by Robinson (Chang and Lee). Further refinements and search strategies have been developed, e.g.: hyperresolution (Robinson himself), semantic resolution (Slagle), the set of support strategy (Wos, Robinson, and Carlson), lock resolution (Boyer), and linear resolution (Loveland, 1970; Luckham, 1970) (more details will be given below).

Many successful resolution-based theorem-provers have been developed. One to be used in this thesis is OTTER (see McCune, 1995; also see www-unix.mcs.anl.gov/AR/otter/).

3.5.1.1 Robinson's Principle for First-Order Logic (Chang and Lee)

The important point in Robinson's resolution principal is that it generates a new clause from two existing ones. The generated clause will replace the two existing clauses. Then, Robinson's resolution, also called binary resolution because it uses two clauses to generate the new one, can be seen as an inference rule (Wos *et al.*, 1988). In fact, binary resolution can be applied in different forms; for example, it can be seen as a generalisation of the inference rule that gives *(P or R)* from: *(P v Q) Ù (~Q v R)*. Moreover, it can be considered as a generalisation of 'modus ponens' and 'syllogism' (Wos, 1988). Applying it iteratively, a chain of clauses can be generated. If the original set of clauses is inconsistent, then a contradiction occurs in the chain - this is called the completeness of the method. Binary resolution is complete. Obviously, completeness is a desirable attribute for resolution methods. If the set of clauses is satisfiable (there is no proof), binary resolution will halt once no more clauses can be generated. Nevertheless, complete proofs methods are only available for a restricted range of relatively inexpressive logics.

The main advantage of binary resolution is that it is complete and that it does not put restrictions on either of the resolvents. So it is one of the most generally applicable resolution variants. However, it might be inefficient in the sense that other resolution methods might do certain proofs in shorter manner in terms of time or generated data. Moreover, other resolution procedures might be more recommendable in certain situations, for instance, in keeping with the characteristics of the clauses over which the proof is going to be executed. For example, when translating a simulation program into a theorem-prover, it may be convenient to see the original data of the simulation program (e.g., that given the initial state) as unit clauses and the rules of the simulation program as multi-unit clauses and then apply hyperresolution (see below) in the search so that the multi-unit clauses operate as the inference rules (see below and Wos, 1988). Sometimes, theorem-provers allow the user to choose the resolution method(s) to be applied.

In the following some alternative resolution methods will be informally reviewed as some of them are used in the theorem-provers of interest in this thesis, e.g., in OTTER. Then, the different strategies used together with resolution for improving the search in a theorem-prover will be considered. These strategies are heuristics aiming to make the search shorter and computationally less expensive.

3.5.1.2 Semantic Resolution

Semantic resolution is one of the first improvements over binary resolution in the search for methods to decrease the number of redundant clauses.

The main idea is to divide the set of clauses S into two subsets S1 and S2, preventing resolution within clauses in a subset. This can be explained by using an example from Chang and Lee:

Example 3-3.

S = {(1)~ P v ~ Q v R, (2) P v R, (3) Q v R, (4) ~R}

Applying binary resolution to (1) and (2), it is found that *R1 = ~ Q v R*, from *R1* and (3) is obtained *R2 = R*, and from *R2* and (4) it is concluded that the set is unsatisfiable. Binary resolution has been applied using clauses in the following order: (1), (2), (3), (4). The same result is achieved if binary resolution is applied in the order (1), (3), (2), (4). Though this new inference is clearly redundant, an automatic program might produce it if not prevented. Note that binary resolution might give even more redundant information; for example, from (1) and (4) a useless resolvent is obtained in the direction to prove the unsatisfiaility of *S*. Some of these redundant inferences are prevented by refinements of resolution procedures, as will be seen in this and in the coming sections.

Using the interpretation *I = {~ P, ~ Q, ~ R},* the set of clauses can be separated in two subsets: S1 the subset of satisfied clauses and S2 the subset of unsatisfied clauses. That is, *S1 = {(1) ~ P v ~ Q v R, (4) ~ R}, S2 = {(2) P v R, (3) Q v R}*. This prevents (1) being resolved against (4). However, (1) can still be resolved with (2) and (3), taking clauses in *S* in a different order. To prevent resolving clauses in different orders when the result is the same resolvent, an additional tactic is used: ordering of predicate symbols. Assume the order *P > Q > R* is used. Given priority to predicates of higher order, binary resolution will be applied in the sequence (1), (2), (3), (4). In fact, this ordering is arbitrary and semantic resolution can be considered as an inference rule applied simultaneously over several clauses, in this case over (1), (2), and (3). As (1) is used to match both (2) and (3), it is called the *nucleus*, while (2) and (3) are called *satellites.* Semantic resolution is complete (Chang and Lee, 1973).

Chang and Lee give the following formal definition for semantic resolution. It is reproduced to add clarity in the exposition and because of its usefulness for defining other resolution variants to be set in the next sections. Note that Chang and Lee name the resolvent *semantic clash* (Chang and Lee, 1973):

'Let I be an interpretation. Let *P* be an ordering of predicate symbols. A finite set of clauses $\{E_1, \ldots, E_q, N\}$, $q = 1$, is called a *semantic clash* with respect to *P* and *I* (or *PI-clash* for short), if and only if $E_1, \ldots, E_q$ (called the *electrons*) and *N* (called the *nucleus*) satisfy the following conditions:

1. $E_1, \ldots, E_q$ are false in *I*.
2. Let $R_1 = N$. For each $i = 1, \ldots, q$, there is a resolvent R_{i+1} *of* R_i *and* E_i.
3. The literal in E_i, which is resolved upon, contains the largest predicate symbol in E_i, $I = 1, 2, \ldots, q$.
4. R_{i+1} is false in *I*.
5. R_{q+1} is called a *PI-resolvent* (of the *PI-clash* $\{E_1, \ldots, E_q, N\}$).'

Observe that if the nucleus is valid over the chosen interpretation, then the satellites are false, and *vice versa*. Also, it is important to notice that an application of semantic resolution as an inference rule does not need to have intermediate steps; that is, it can be thought of as applying simultaneously over the nucleus and the satellites. Here its usefulness is in reducing several inference steps into one step. This might also turn out to be a reduction in redundant data.

Additional refinements have been proposed over semantic resolution, for example, ordering of clauses. Some of them have turned into other variants of resolution, such as lock resolution. Nevertheless, some of those procedures are not complete (e.g., semantic resolution with ordering of clauses) or have disadvantages when implemented automatically. In the coming sections, some of the more successful variants of semantic resolution will be examined.

3.5.1.3 Hyperresolution

Positive (negative) hyperresolution is a special case of semantic resolution where the interpretation is chosen as that with all literals negative (positive). As a consequence, all satellites will contain only positive (negative) literals and the nucleus will contain at least one negative (positive) literal, and yield (if successful) a clause with only positive (negative) literals. Hyperresolution will be the resolution method chosen in OTTER for carrying out the exploration of the dynamics of the model.

Example 3-4

Considers S as given in example 3-3, i.e., $S = \{(1) \sim P \vee \sim Q \vee R,\ (2)\ P \vee R,\ (3)\ Q \vee R,\ (4)\ \sim R\}$, (1) is the nucleus, (2) and (3) are the satellites, and $I = \{\sim P,\ \sim Q,\ \sim R\}$. It is a case of positive hyperresolution: literals in the interpretation are all negative, the nucleus has at

least a positive literal (R), all literals in the satellites are positive, and the resolvent is positive (*R*).

3.5.1.4 The Set of Support Strategy

Though this is understood as a strategy, it is presented here because it can be considered as a special case of semantic resolution. The set of support strategy was proposed by Wos, Robinson, and Carson (Wos *et al.*, 1965). It is a very successful strategy and has been adopted as the main strategy in many theorem-provers (e.g., OTTER).

It consists in a division of the set of clauses to two groups, as in any other variant of semantic resolution. The first of these will be a set of valid clauses and the second a set of unsatisfiable clauses. There are several ways to make this division. For example, if we wanted to prove that the theorem *T* is valid in the theory given by the set for clauses $\{A_1, A_2, \ldots, A_n\}$, the aim would be to prove that $A_1 ? A_1 ? A_2 ? \ldots ? A_n$ **?** $\sim T$ is unsatisfiable. The first and second sets to be $S_1 = \{A_1, A_2, \ldots, A_n\}$ and $S_2 = \{ \sim T \}$ might have been chosen.

In theorem-provers such as OTTER, the set of valid clauses, called *set of support ('sos'!),* is chosen as the set of facts the user can easily recognise as valid and the other subset, called *usable set,* is chosen as the rest of the clauses including the negation of the theorem. For example, the *sos* might contain those facts given the initial state in a simulation and the *usable set* might be given by the remaining clauses in the original set of clauses S and the negation of the theorem. Clauses in the *set of usable* may be used as the nucleus for hyperresolution or/and as inference rules. In OTTER no inference is allowed using only clauses from the list usable –at least one clause should come from the list *sos*.

The set of support strategy is *complete* (Chang and Lee, 1973). However, when combined with hyperresolution, 'completeness is not guaranteed' (Wos *et al.*, 1988, p. 436).

In the example to be given in Chapter 7, the rules to run the simulation will be translated as clauses and placed, along with the negation of the theorem to be proved, in the *set of usable*. Data in the simulation language will be placed as ground instances in the *sos*.

3.5.1.5 Linear Resolution

Linear resolution was proposed by Loveland (1970) and Luchham (1970). It later underwent several improvements (according to Chang and Lee, 1973): 'that given by

Kowalski and Kueher (1971) and that by Loveland (1972) which is more important for computational implementations.'

Linear resolution starts with two clauses (C_0 and B_0) given the resolvent (C_1). Then C_1 is resolved against another clause B_1 to get C_2. Afterwards, the new clause is resolved against clause B_2 to get another resolvent C_3. This procedure is continued until a contradiction or the empty clause appears (if the proof is successful). The basic characteristic of the 'chain' of clauses generated by the linear resolution procedure is that the resolvent in one step is used as one of the clauses to be resolved upon in the subsequent step. Linear resolution is complete (Chang and Lee, 1973).

The already described *depth-first* and *breath-first* searches are particular cases of linear resolution (Chang and Lee, 1973). Linear resolution style search has been popular in logic-programming (e.g., depth-first search in Prolog) and in theorem-proving when using tableaux. It seems more closely related to backward-chaining search than to forward-chaining search.

3.5.1.6 Strategies or Heuristics

In this subsection some of the more popular strategies for driving the inference procedure in (syntactic) theorem-provers and, in particular, implemented in OTTER will be discussed. The main role of these strategies is to choose, among the commonly huge amount of possible deductions and 'path searches', those that hopefully will lead to the proof with less data generation, or at least preventing as far as possible the generation of useless data.

- **Deletion Strategy or Subsumption**

The idea is to eliminate copies of the same clause or clauses that are instances of other clauses. 'The clause A subsumes the clause B when there exists an instance A that is a subclause [an instance] of B' (Wos, 1988).

Subsumption can be trivial or as complicated as may be wanted. A trivial case is *P(x)* subsuming *P(a),* where *a* is a constant and *x* is a variable. A more complicated case is *P(x)* subsumption of *P(a) v Q(b),* where *a* and *b* are constants and *x* is a variable. In this case, if *P(x)* is true for any *x, P(a)* is also true and then *P(a) v Q(b)* is true, whatever happens with *Q(b).*

- **Fewest-literal Preference Strategy**

When using resolution procedures such as hyperresolution and UR-resolution, for different reasons it is convenient to have clauses with few literals. The main reason is that it is easier

to resolve the clause with others successfully. Another motive is that usually the theorem to be proved has few literals (generally one).

Its advantages are clear, for instance, when using it along with hyperresolution and the set of support strategy. For example, in OTTER, when applying hyperresolution, clauses in the *sos* are used as satellites while those in the *set of usable* are used as the nucleus. The shorter the number of literals of those chosen as satellites, the easier it will be to apply hyperresolution.

Some theorem-provers (e.g., OTTER) implement this strategy by giving each clause a weight proportional to the number of literals, after which clauses are ordered and then selected according to this weight.

- **Heuristic-Evaluation Function**

This consists in giving each clause a weight as a function of certain clausal features such as a number of literals and a number of distinct variables. Usually the function is a weighted linear combination of these features. The difficulty with this procedure is in calculating appropriated weights. Because of this, it is not implemented in theorem-provers. As can be seen, fewest-literal preference strategy is a particular case of heuristic-evaluation function.

Usually, theorem-provers implement a *weighting heuristic*, giving the user the option to specify the criterion for clause weighting.

- **Demodulators**

Demodulators are procedures 'to simplify and canonize information'. (Wos *et al.*, 1988). Clauses giving semantically redundant, though not necessarily syntactically redundant, information are discarded. The main idea is to rewrite terms in a clause. For example, *SUM(1,2)* and *3* are terms giving the same though syntactically different information. As *3* is 'simpler', it would be a candidate to canonise this and other numerical expressions semantically equivalent (e.g., MULTIPLY(3,1)). It would be used to rewrite semantically equivalent expressions.

Demodulators are very useful for doing numerical manipulations. For example, a demodulator in automated reasoning languages and theorem-provers (e.g., OTTER) might be: SUM*(x,y) = x + y*. Here '+' has the meaning of the numerical operation addition, while SUM is a predicate symbol. Once the predicate SUM is used in a term, the demodulator is called, then a sub-program, doing the numerical operation is called, and, finally, the result of the addition is sent back to rewrite the term. The usefulness of demodulators in simulation is obvious, as numerical operations are common. However, there are few

theorem-provers offering demodulators, among them OTTER. In theorem-provers implemented over Prolog, such as SETHEO, numerical manipulations can be achieved using Prolog facilities. Demodulators will be used in the example (given in Chapter 7) for implementing functions originally written as backward-chaining rules in SDML.

Consider a more complicated example of demodulation taken from Wos *et al.* (1988). It can be checked that the following two predicates have the same information:

EQUAL(sum(sum(x,v),sum(y,z)),sum(sum(x,y),z))

EQUAL(sum(x,sum(y,z)),sum(sum(x,y),z))).

EQUAL is a literal interpreted as meaning equal.

In general, a demodulator has the form *D: EQUAL(r,s)*, where *r* and *s* are terms. If a clause *C* has a term *t* matching *r* and the result of the matching is *r'*(*r' = t*), then applying the demodulator *D* to *C* will result in replacing *C* by *C'* given by the substitution of *t* for *s'*, where *s'* results from the appropriated instantiation of the demodulator *D'*: EQUAL(r',s'). The demodulator given above for *SUM* can be thought as *EQUAL(SUM(x,y), x + y).*

- **Paramodulators**

Paramodulation has some similarities with demodulation, though it is very different. Paramodulation generates a new clause *C* from two clauses *A* and *B*, while demodulation replaces and *subsumes* an old term for a new one.

Concretely, should we have the clause *A* with an equality literal, lets say *EQUAL(r,s)*, and clause *B* with a term *t* matching, for example, the left side of this literal, *r*, then the matching, where *r' = t*, gives the instances *A', B'*, and *EQUAL(r',s')* (for the clauses *A* and *B* and the *EQUAL* literal, respectively). Call K' the literal *EQUAL(r',s')* and B'' the clause *B'*, where *r'* has been replaced by *s'*. Then the resulting clause *C* of applying paramodulation to *A* and *B* using the *EQUAL* literal *(EQUAL(r,s))* of *A* and the term *t* in *B*, will be the disjunction of *A'-K'* and *B''*.

Example 3-5

Consider $A = P(x) \vee EQUAL(PROD(x,0), 0))$, and $B = Q(x) \vee R(PROD(2,0))$,

Where:

$r = PROD(x,0), s = 0; t = PROD(2,0)$

Applying the explained procedure it will give:

$r' = PROD(2,0), A' = P(2) \vee EQUAL(PROD(2,0),0), B' = Q(2) \vee R(PROD(2,0)),$ $K' = EQUAL(PROD(2,0),0), B'' = Q(2) \vee R(0).$

Finally, the disjunction of *A' – K'* and *B''* gives:

P(2) v Q(2) v R(0).

If *A* were *A = EQUAL(PROD(x,0), 0))*, the result would have been *Q(2) v R(0).*

3.5.1.7 Forward-chaining Search

As was stated above, forward-chaining search consists in applying resolution rules over the clauses in the database, generating consequences that are added to the database if they are not subsumed by clauses already existing in the database. This process is repeated iteratively until a certain stopping condition is found (for example, a contradiction or an empty clause) or it is not possible to generate more consequences. Forward-chaining is the 'counterpart' of backward-chaining.

- **Example: OTTER, a first-order logic theorem-prover**

OTTER is the theorem-prover used in this thesis for implementing a MAS simulation case (see Chapter 7), and so it will be a source of ideas for the methodology to be presented in the next chapters. Two reasons for using OTTER in this thesis are: first, because it is one of the most successful theorem-provers, and second, because it offers facilities for data manipulation (e.g., demodulators). This is convenient for implementing cases like the example from social simulation given in Chapter 7. This example is about a trader-distributor interaction, where numerical manipulation is made, e.g., for calculating sales, orders, prices. Other theorem-provers are very restricted to symbolic manipulation and do not provide these sorts of facilities.

OTTER is a forward-chaining theorem-prover that uses a set of support strategy. That is to say, the original set of clauses is separated in to two groups, a first one containing only valid clauses called set (list) of support and a second group of (assumed) unsatisfiable clauses called usable list (this set of clauses is unsatisfiable if the theorem is valid). The usable list contains those clauses to be used as inference rules and the negation of the theorem to be proved.

OTTER offers a wide range of inference rules and strategies, which can be turned on and off by using commands. Among the inference rules OTTER offers are: binary resolution, hyperresolution, unit resolution, and UR-resolution. The strategies it presents include: weighting and subsumption.

OTTER also offers demodulators and paramodulators. These are very helpful in certain applications. In simulation, numerical manipulations and rewriting is not only a factor of efficiency but it also becomes essential in practice. In OTTER demodulators are applied as rules in backward-chaining whose intermediate or partial consequences are not written in the rulebase. They work as functions in imperative programming, in the sense that they are

called with a set of parameters and then they return the corresponding result. For example, a demodulator could simply be a function to multiply two integers: *MULT(a, b) = a * b* (for more about demodulation and paramodulation, see above).

OTTER main inference procedure works as follows (McCune, 1995):

'The main loop for inferring and processing clauses and searching for a refutation operates mainly on the lists usable and sos:

While (sos is not empty and no refutation has been found)

1. Let given_clause be the `lightest' clause in sos;

2. Move given_clause from sos to usable;

3. Infer and process new clauses using the inference rules in effect; each new clause must have the given_clause as one of its parents and members of usable as its other parents; new clauses that pass the retention tests are appended to sos;

End of while loop.7

... Otter's main loop implements the set of support strategy, because no inferences are made in which all of the parents are from the initial usable list.'

In step 3., the *retention test* checks that the *new clause* is not subsummed for existing clauses.

A difficulty when using most of the existing theorem-provers is their poor interaction with the user, and OTTER is not an exception. In McCune's (1995) words: 'Although Otter has a primitive interactive feature … it is essentially a noninteractive program. On unix-like systems it reads from the standard input and writes to the standard output:

otter < input-file > output-file

No command-line options are accepted; all options are given in the input file.'

As has been said, for implementing an example in OTTER in Chapter 7, hyperresolution, the sets of support and usable, subsumption, fewest-literal preference strategy, and demodulators will be used. However, when re-implementing the example in SDML, the search strategy will be model-driven by using automatic partitioning of the set of clauses and other strategies SDML offers, which are more common in simulation and in logic-programming languages than in theorem-provers (see next section, especially sub-section 3.6.3).

3.5.2 Semantic (interpretation)-based Theorem-provers

The inference procedures in this section are based on a search over interpretations as opposed to clausal manipulation studied in the previous paragraph. Aspects of semantic-

driven search were already considered in brief when considering semantic resolution, but there the use of interpretations was restricted to separate clauses in groups. The procedure is still predominantly syntactic as it is based on clausal manipulation after the separation is made. In the methods to be reviewed in this section, interpretations are more intensively used to drive the search. In fact, each inference step is guided by the validity of certain terms.

Recent theorem-provers have been developed using the notion of semantic search, particularly using tableaux. Comparing with interpretations where all possible interpretation have to be checked, tableaux are more efficient as it considers only those interpretations that might make valid (literals in) certain clause(s) - in this sense it is also guided by clauses.

3.5.2.1 Tableaux

Tableaux generate indirect proofs as 'the premises of an argument form together with the negation of the conclusion are tested for joint satisfiability' (Gordon *et al.*, 1992). ' … In essence, a tableau is a survey of the possible interpretations of S with each branch representing an interpretation' (Bonacina, 1999). Tableaux are an 'attempt to systematically construct a structure from which a logical model can be extracted for the negation' of the theorem to be proved (Fisher *et al.*, 2000).

The idea is to generate a branch for each possible valid interpretation. Once a contradiction appears in a branch, this branch is *closed*. If all children of a node are closed, the node is also closed. If a branch is not closed (there is no contradiction in it), it is said to be *open*. An open branch, where no more inferences can be made, offers a *logical model* of the theory (remember, the theory in this case is given by the conjunction of the premises and the negation of the theorem). In this case the whole set of clauses defining the theory is satisfiable, the theorem is valid, and the negation of the theorem is false. On the other hand, if all branches of a tableau are closed, then there is no valid interpretation and the theorem is valid.

The set of braches gives the tableau. If at least one branch is open, it is said that the tableau is open. If all branches are closed, the tableau is closed. As an (tableaux) inference rule (an example of this sort of inference rule will be given below) is applied, there is a transition from a tableau (state of the search) to another tableau (next state in the search). The proof will be successful if a final state with a closed tableau is reached. As soon as the procedure generates an open branch where no more inferences can be made, the proof fails. Tableaux are complete.

Now *logical model elimination tableaux* will be explained as an example of inference procedures used in tableaux. Starting from the set *S*, defined as the union of the premises and the negation of the theorem to be proved, roughly the inference procedure works as follows (Bonacina, 1999):

Assume the starting clause is $C_i = L_1 \vee L_2 \ldots \vee L_n$. This gives a first state in the search represented by a tableau with C_i as its unique node. A branch named L_m is created for each literal L_m of C_i representing the different ways this clause can be satisfied. Then suppose there is another clause in *S:* $C_j = F_1 \vee F_2 \vee \ldots F_k$, and that for the substitution s: $-L_1 s = F_1 s$. Then this clause C_j will be added at the node corresponding to branch L_1, closing the branch for F_1 and applying s to literals in branch L_1. (As is known, it is possible to apply binary resolution to these two clauses, giving as resolvent the disjunction: $(L_2 \ldots \vee L_n \vee F_2 \vee \ldots F_k)s$). This gives a new state in the search process: the original tableau with node C_i expanded by this second node C_j which has branch F_1 closed, and the application of the substitution s. Repeating this procedure to the remaining literals in nodes for C_i and C_j, the inference proceeds. A final state is achieved either because an open branch where no more inferences can be made is found (a logical model for S given by this branch has been found, the negation of the theorem is valid in this logical model, and the proof fails) or the tableau is closed (the proof succeeds).

Example 3-6.

Assume, $S = \{(1) \sim P \vee \sim Q \vee R,\ (2)\ P \vee R,\ (3)\ Q \vee R,\ (4)\ \sim R\}$, where the premises and the theorem are clauses (1)-(3) and (4) respectively, a possible tableau is shown in Figure 3.2.

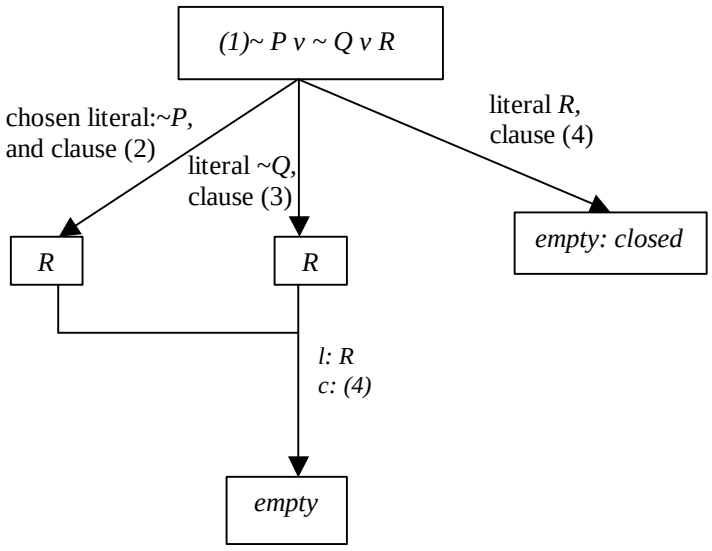

Figure 3.2. A tableau for example 3-6

3.5.2.2 Prolog-based Logical Models

Two theorem-provers written as extensions of Prolog will be considered: SATCHMO: SATisfiability CHecking by [logical] MOdel generation; and PTTP, a Prolog Technology theorem-prover. Both of them are based on logical model search. Their most relevant aspects will be considered. First, common characteristics will be enumerated and then particular aspects of SACHTMO will be reviewed, as it presents more similarities with OTTER and the way an exploration of simulation trajectories using SDML's facilities will be implemented.

In both of them result from attempts to extend Prolog with a complete procedure to manage non-Horn clauses (i.e., not only Horn clauses as is usual in Prolog). A difference between them is that while SACHTMO is written on top of Prolog, PTTP is an extension compiled along Prolog's source code. To be implemented on top of Prolog gives SATCHMO flexibility and portability. Both are tableaux-style implementations. They introduce additional inference procedures from those Prolog has, are semantic oriented, and use splitting and backtracking to generate alternative paths in the search.

SATCHMO is based on the introduction of a *new set of rules* in order to be able to manage clauses not allowed in Prolog. These rules are fired in a forward-chaining manner. In fact, the primary idea for writing SATCHMO comes from the observation that range-restricted clauses can be exploited by the inference procedure. A range-restricted clause is a clause such that every variable occurs in at least one negative literal. If a clause is written in the form of a rule with the negative literals in the antecedent, it is said that the clause is range-restricted if every variable is in at least one literal in the antecedent. Then, the idea is that if in the rule *a implies b (or in the clause ~a | b), a* is valid but *b* is not, then b can be taken as deducted via this rule and added to the database.

SATCHMO is based on hyperresolution. Splitting and backtracking are introduced in SATCHMO by allowing disjunctions in the consequent of rules. Other problems Prolog's search faces are also solved: the difficulty of infinite generation of clauses due to recursive clauses as a consequence of unbounded search is worked out by using subsumption (Rainer *et al.*, 1988).

A drawback of SATCHMO is that range-unrestricted clauses might need full instantiation of some variables over the whole Herbrand, universe which might lead to an 'explosion' in the number of ground instances. This is prevented by an appropriate transformation of range-unrestricted clauses into range-restricted clauses, which is always

possible. See, for example, CPUHR tableaux, which are an adaptation of PUHR: Positive Unit Hyperresolution tableaux (Bry *et al.,* 1996), the tableaux used to formalise SATCHMO, for managing constraints and existentially quantified variables (Abdennadher *et al.*, 1997).

A logical model in SATCHMO is given by a set of ground terms, more concretely by those ground terms that are satisfied in the logical model (the remaining ones are assumed violated by default). This notion of determining all facts in a logical model is slightly different in simulation and especially so when proving tendencies due to practical reasons. For example, not all facts of a trajectory are generated when searching for a tendency; once the tendency is found in the trajectory, the program backtracks leaving the remaining facts (irrelevant for the proof) as unknown.

3.6 Classification of Theorem-provers

3.6.1 Criteria

The classification will be based on the character of the search, which might be:

1. *Forward- vs. backward-chaining (i.e., assumption expansion vs. sub-goal reduction)*

As is already known, in forward-chaining the precedent of a rule is checked and, if it matches the database, then the rule fires and the consequent with the corresponding instantiations of variables is added to the database. In case of disjunctions in the consequent, different branches will appear, each one labelled with an assumption. On the other hand, if the search is backward-chaining, the consequent of a rule is checked and, if the goal is a conjunction of several sub-goals, a branch is open to check each of them. In this case, the sub-goals label the generated branches.

2. Semantic vs. syntactic focused: interpretation- vs. clausal-driven search

Resolution is basically syntactic or clause oriented; it generates a clause from several other clauses considering basically the form of the clause and the meaning of terms (see above). Alternatively, tableaux are in essence semantic oriented, as the generated branches in a node are precisely those alternative interpretations that make certain clauses valid.

3. Implicit vs. explicit splitting of the search space

Optional solutions in the search may appear for different reasons, for example, owing to a disjunction in the antecedent and/or in the consequent of a clause, or owing to existentially quantified variables in the consequent (see, for example, Abdennadher *et al.*, 1997). In a splitting search, one branch is investigated at a time. This sort of search will be called explicit search. In this case, if attempting to prove unsatisfiability, the program backtracks

once a contradiction appears in a branch. Alternatively, in an implicit search the inference is carried out over all paths simultaneously. Branches in an implicit search will be called implicit branches.

To illustrate this, compare OTTER and tableaux methods. In the new versions of OTTER both kinds of searches can be implemented (the older versions accept only implicit search), while in tableaux only explicit search can be followed. OTTER does not give explicit information about the closed branches in an implicit search. That is different when applying an explicit search. In this case OTTER provides a summary for a closed branch at the moment the branch is closed and OTTER backtracks to explore another branch. One of the main advantages of an implicit search when compared with splitting is that it is easier to prune the search tree when similar paths are found. On the other hand, one of its main disadvantages is that it needs more memory as all paths are kept in memory simultaneously.

Nevertheless, a classification based on these criteria is somewhat arbitrary, as almost any computational logic program is a combination of syntactic, semantic, forward-, and backward-chaining strategies. For illustration consider OTTER. It is predominantly syntactic, as based on clause manipulation, and forward-chaining oriented, but it also implements semantic ideas, such as the separation of clauses into two groups (the sets of support and usable) and by the use of demodulators and paramodulators (see above).

3.6.2 Bonacina's Taxonomy of Theorem-prover Strategies

Bonacina's classification (Bonacina, 1998 and 1999) is based on the third criterion above. She considers an implicit branch search as clausal oriented, and a splitting one as goal oriented. The two main categories in her classification are, on the one hand, those theorem-provers with a 'sub-goal reduction strategy ... if one considers the single object they work on as the goal, each step consists in reducing the goal to sub-goals' (Bonacina, 1998, p. 257) (e.g., tableaux). On the other hand, she considers those theorem-provers using strategies she calls 'ordering-based strategies'. She characterises these strategies as working 'with a set of objects [e.g., clauses], they use a well founded ordering to order the objects, and possibly delete [e.g., using subsumption] objects that are greater than and entailed by others' (*idem,* p. 256). In this group is OTTER.

However, it is confusing to consider clausal orientation as an implicit branch search and splitting as a goal-orientated search. For example, as noted above, in the new versions of OTTER splitting has been implemented even though OTTER is clausal oriented.

Bonacina's depiction of the search process seems similar to the description of a simulation. She illustrates the search process as a chain of transition states, where a transition is generated by an application of an inference rule. Each state is a step towards a desired final state, where the proof is successful.

In the case of a logical model search using tableaux, each state is a tableau. Given one state, a new state is generated by the application of a transition rule over this tableau and the generation of a new branch. The new state will be the tableau given by the addition of the new branch to the old tableau. The desired final state is a closed tableau.

In case of a clausal search, a state is given by the set of clauses in a search step. This set might consist in subsets of clauses, for example, the sets 'usable' and 'support' in OTTER. Additional subsets might be given by the implicit branching of the search. A transition step consists in the application of an inference rule giving a new set of clauses defining the new state. The proof is successful once a desired final state is reached; that is, any subset of clauses representing a search path contains a contradiction.

3.6.3 Additional Considerations about Bonacina's Taxonomy: Classifying Generation of Trajectories in a Simulation

The criteria given in section 3.6.1 will be used to add details about the search process in the theorem-provers SATCHMO and OTTER, and to include in this classification logical model generation in a simulation, namely an event-driven simulation, and the MAS-based simulation scheme adopted in SDML.

1. SATCHMO. This uses the sub-goal (backward-chaining) strategy of Prolog combined with a logical model (semantic) search with splitting.
2. OTTER. Originally OTTER was purely an assumption-based forward-chaining procedure using clausal (syntactic) orientation with implicit path generation. New versions of OTTER also allow the explicit use of splitting.
3. Event-driven simulation. Usually simulation is forward-chaining and logical model-oriented. It is rare to find simulation languages implementing splitting either implicitly or explicitly. When experimenting with the dynamics of a simulation, individual trials have to be generated by explicitly choosing the different combinations of the factors for each trial. As it is a logical model search, it can be interpreted as a tableau, but note that in a simulation not all facts have to be deduced and some are left as unknown. Although it is similar to tableaux (since it is logical model-oriented), it is different, as inference is forward-chaining rather than

backward-chaining. In theorem-provers like OTTER, the order transition rules as given by certain clause weighting, while in event-driven simulation this order is given by the future event list.

4. SDML. Like most simulation languages, this is a forward-chaining logical model-oriented language, but it enjoys facilities for splitting and backtracking and it is not necessarily committed to generate all valid clauses in a logical model. State transition rules follow an order given by the hierarchy of agents, the hierarchy of time levels, and, finally, a partitioning of the space of rules according to their dependencies.

3.7 Constraint-based Search

The idea is to develop search and inference procedures that allow a more flexible manipulation of the semantics than traditional logic-programming and forward-chaining systems do. The call has been for techniques that allow a semantic-driven search more adaptable to the search process than unification (Frühwirth *et al.*, 1992).

A first answer came from logic-programming, in the form of constraint logic-programming, commonly using Prolog, both as a platform and as the programming style (Frühwirth *et al.*, 1992). It is basically a backward-chaining inference system. A second answer came from rule-based forward-chaining systems. Examples are SATCHMO and CPUHR-tableaux calculus (Addennadher, 1995; *idem*, 1997; see also Frühwirth, 1994; Abdennadher *et al.*, 1999). Among the advantages of these forward-chaining systems over constraint logic-programming are introduction of user defined constraints and meta and higher-order reasoning via rewriting of rules.

Constraint logic-programming can be characterised by the interaction of the logic-programming inference engine and a constraint-solver algorithm. constraint-solvers specialise in managing constraints in certain domains (e.g., integers, booleans). The logic-programming engine calls the constraint-solver with a set of constraints, getting as the answer a normalised solution for it and then pruning the search space in accordance with this solution. A disadvantage constraint-solvers face is that they are not open to the user, making difficult their modification and application to new domains. Some rule-based forward-chaining systems allow user manipulation of the constraint-solver. There will exist both user-defined and built-in constraints. The first group of constraints will be managed by rules the user gives, while the second one will be managed by constraint-solvers. With illustrative purposes, in the next paragraphs aspects of CHR will be considered in more detail.

‘CHRs define determinate conditional rewrite systems that express how conjunctions of constraints propagate and simplify … CHRs define *simplification* and *propagation* over user-defined constraints. Simplification replaces constraints by simpler constraints … Propagation adds new constraints which are logically redundant but might cause useful further simplification’ (Frühwirth, 1994, pp. 90-91). In what follows a CHR operational semantics will be considered to some extent in order to bring in an idea about how it works.

In a CHR system, the inference engine is characterised by the interaction of a logic-programming engine, a constraint-solver (working on built-in constraints), and the rules working on user-defined constraints.

Using Frühwirth’s notation, a *state* in the search is given by the tuple of sets:
$< G_S, C_U, C_B>$,
where G_S is a the set of goals, and C_U and C_B are constraint stores for user-defined and built-in constraints. A constraint store is a set of constraints.

An *initial state* might consist of $< G_S,\{\},\{\}>$, i.e., constraints are all embedded in the set of clauses defining the problem and in the goal.

In case of logical model generation, a final state in a branch of the search is given by $< \{\}, C_U, C_B>$, if the search succeeds or by $< G_S, C_U, \{\mathsf{false}\}>$, in case it fails (i.e., the branch is closed).

Consider how a *state transition* would be. Suppose the state is:

$< \{C\} \cup G_S, C_U, C_B>$: that is, a subset of constraints {C} has appeared as a new goal during the search process. Calling constraint-solver, if $(C \wedge C_B)$ is logically equivalent to C_B', in the next step it would produce: $< G_S, C_U, C_B'>$.

Similar modifications are generated by CHR rules. In fact, CHRs will either generate simplifications in C_U while adding constraints to G_S, or add constraints into C_U logically equivalent to some constraints in G_S but leaving these constraints’ store unchanged (constraints propagation) (Frühwirth, 1994).

Example 3-7

The sort of reasoning exposed above will be illustrated by using a simplification of an example borrowed from Abdennadher *et al.* (1997).

Problem:

- John has enrolled in courses either before 1994 or after 1995

- John was enrolled in cs30 before 1997.

In clausal form, it is expressed as:

enrolled(john, A, B) $\rightarrow$ (B < 1994) v (B > = 1996)

(A, C) = ({enrolled(john, cs30, T)}, { T <= 1996})

Using notation for operative semantics given above, this will be produce:

goal: enrolled(John, A, B), constraints: (B < 1994) v (B > = 1996)

Applying logic-programming two sub-goals or branches are found:

(A, {T <= 1996, T <1994}) and (A, {T<= 1996, T >= 1996})

Then applying constraint-solver at each branch, they are extended into:

(A, {T <1994}) and (A, {T = 1996}), respectively.

For the second branch the final state is achieved:

{enrolled(John, cs30, 1996), {}).

In this example it is assumed there is only one set of constraints to be solved by the constraint-solver. For more details about CHR, see Frühwirth (1994) and Abdennadher S (1999), or look at http://www.pst.informatik.uni-muenchen.de/~fruehwir/.

3.8 Meta-Level Reasoning and Proving Tendencies

The sort of meta-reasoning certain reasoning languages enjoy is quite convenient when proving tendencies. Examples of languages that have explicit mechanisms for presenting meta-reasoning are SATCHMO and SDML. SATCHMO allows the use of writing and eliminating of clauses not only in the database, but also in the rulebase using ‘assert’ and ‘retract’ (Prolog’s built-in functions). Additionally, since it is written on top of Prolog, SATCHMO itself can be easily modified. On the other hand, SDML enjoys similar benefits from a meta-agent being allowed to read and to write rules in other agents.

These supplementary facilities permit reasoning about the inference system itself, e.g., writing rules depending on how the search develops. An example would be: given only hints about a theorem related to an overall tendency defined in terms of certain output, the theorem might be elaborated by continuously updating an envelope for the output in accordance to the simulation results. Also, after all logical models have been found, relevant tendencies might be searched either over the whole space of logical models or over a subspace of it.

4 Chapter 4 - Understanding Phenomena and Simulation Dynamics

4.1 Introduction

In this chapter the development of the 'world view' introduced in Chapter 2 will be continued. The main aim of this is to set up a framework for a conception of emergence of tendencies, which is a central notion in this thesis. A conception of the emergence of tendencies seems to be strongly linked to a subject's knowledge and to an object's complexity. A notion of the object-subject dichotomy will be emphasised as a useful distinction once the subject's bounded rationality has been recognised.

The interest is not only in tendencies observed in an empirical system but also in a simulation. For instance, MAS-based simulation seems to be very helpful for understanding some complex systems, for example, social systems. There, aspects of the complexity of the 'real system', from the point of view of the scientific community studying such a system, are recreated in the simulation. The importance of observing and explaining tendencies in social simulation has been pointed out for researchers in this community (see, for example, Edmonds *et al.*, 1998; Nigel, 1995; Conte *et al.*, 1997; Carley, 1998). The re-creation of properties of target complex systems in the computer lab is what makes the simulation interesting in these cases, as it is not possible either to implement this sort of experiment with the target system or to get satisfactory knowledge of these tendencies via models of a different nature (mathematical models or historical studies, for example).

In applications such as social simulation, where the number of agents is large and/or the interrelations are subtle and hardly contingent to the dynamics of each simulation trajectory, analysis of the design seems to be of limited use and a *post hoc* study becomes necessary. It is common to find structural change or fluctuation in these systems, making their analysis difficult (the simulation of systems undergoing structural change was discussed in sections 2.5 and 2.9). Some of the difficulties involved in studying structural change, and then complex systems, are: structural change develops differently in different simulation trajectories, it is difficult to foresee how it will unfold (e.g., what the new structure of the system will be after it happens), and it is not easy to identify the factors triggering it.

The notion of emergent tendencies will be seen in this thesis as relative to a modeller's difficulties in understanding an observed system, and linked to, on the one hand, a modeller's cognitive model and bounded rationality, and on the other hand, by the observed system's 'objective complexity'. The discussion will start by considering the

usefulness of the distinction between subject (observer) and object (observed phenomena) for understanding emergent tendencies. Then, the central discussion in this chapter will be addressed. First, a picture of a hierarchy of systems at different levels of phenomena in accordance with their potential for complexity (which will be called objective complexity) will be drawn. This is based on Heylighen's ideas. Second, a subjective notion of complexity in accordance with Edmonds's conception will be introduced. Third, using these two notions, a notion of what an emergent tendency might be and what are its causes will be discussed. Finally, aspects of a simulation related to emergent tendencies for a modeller observing a simulation will be examined.

4.2 Subject's Bounded Rationality and the Subject-Object Dichotomy

A basic notion in the view taken in this thesis is the *object-subject dichotomy*. How this dichotomy is understood and its relation to the idea of a subject's bounded rationality will be discussed. Bounded rationality is a central concept in social simulation – it is understood as the existence of significant bounds in an agent's cognitive model for gathering information from their environment and for processing it (Edmonds, 1999b). A pragmatic position will be taken for defining the emergence of tendencies, a stance useful for the purposes of this thesis – in particular, a position from which a subject's understanding will be conceived as conditioned by the interrelation between the subject's language and the observed object's complexity. It is not claimed that this notion has general philosophical or theological validity. Instead, it is based on the assumption that the agents and subjects of interest have bounded rationality and on the notion of object-subject dichotomy to be discussed below.

It seems that the less bounded is the rationality of an agent with respect to relevant aspects of the environment (for its purposes), the more deterministic the world (surroundings) will be for this agent. Here 'relevant' means those macro regularities that the agent perceives in its environment that are useful for reasoning and taking decisions in terms of some goals. Bounded rationality means that many aspects of the world will be effectively indeterministic. Different agents with perfect rationality (assumed deterministic in itself) and the same goals will share the same perfect knowledge and will take the same decisions under similar 'environmental' conditions.

In general, the behaviour of a population of agents seems to be non-deterministic, even if they have perfect rationality. This is because of their differences in goals and experience (or the particular environmental circumstances they have become involved into). If a group

of agents with perfect rationality collaborate to achieve a common goal and the environment has no intention against them (e.g., there is no other kind of rational agent in such an environment whose goals conflict with theirs), then the 'resulting world' might be deterministic for these agents. Behaviour of one of these agents will be predictable from another agent (or from a modeller) having the same information. Examples of agents with perfect rationality include the agent in a market as it is conceived in classical economics (Edmonds *et al.*, 1998). These economic agents do not necessarily have the same goals and collaborate. Economists use certain 'game theory' rules according to which, under certain conditions, behaviour of their (classical economic) agents is predictable. Unless the opposite is explicitly said, henceforth by 'agent' is meant agent with bounded rationality.

There seem to exist even more reasons for the behaviour of agents with bounded rationality to be indeterministic. Bounded rationality means that other aspects apart from goals and experience might make agents behave differently. These differences are rooted in the limited information an agent can capture from its environment and in the limited capabilities the agent has to process the collected information. For each agent, there will be a different 'world view'. Different agents will behave differently under the same circumstances. Agents with bounded rationality use different languages, though the divergences might be only in the beliefs used as data to feed to their 'inference procedure'. This might happen even if they have similar experiences (it is assumed agents are able to learn from experience).

Each agent (a subject) has models of its surrounding environment – almost certainly different from other agents' models. As a consequence, there is a subjective 'world' for each one. There will be shared and unshared beliefs among agents. As more agents share a belief over time and space, it can be said that this belief is less subjective. Similarities among agents' beliefs introduce the notion of knowledge with different degrees of subjectivity. Indirect justifications of subjects' knowledge such as experimentation, theoretical developments, and modelling might influence subjects' beliefs. For example, it is possible for human beings to decrease the degree of subjectivism of their knowledge by improving understanding of phenomena via modelling (e.g., simulation) theory development and historical discussion (see Figure 4.1). An increase in the degree of subjectivism of an agent's beliefs is understood in this thesis as an increase in the degree of objectivism of these beliefs. Decrease in the degree of subjectivism of a belief will hopefully bring that belief closer to the 'real' (hypothetical) characteristic of the object the subject refers to with that belief.

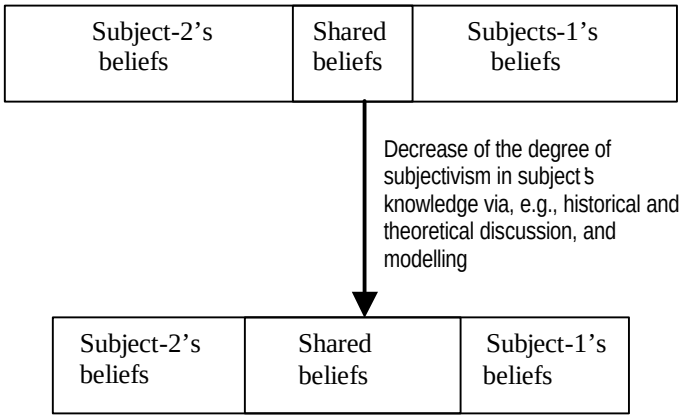

Figure 4.1. Increasing objectivism in subjects' beliefs

In this discussion, it has been assumed implicitly that an agent reasons about its internal model of the 'world or surroundings' rather than about the world itself. On the one hand, there is a sort of subjective world of the agent given by its mental models, goals, perceived phenomena, beliefs, language, and every other 'mental' construct it has. On the other hand, there is the 'real world' or empirical world – the source of phenomena; i.e., there is a 'real interaction' subject-world, where the actions of this and other subjects, on the one hand, and the new circumstances arising in the world as the result of those actions, on the other hand, turn up as, in part, the consequences of this subject's actions; and there is a perception by this subject of this interaction and of the consequences of its actions. Such a pragmatic 'division' between a subject's perception of the world, mental models and language, and the empirical world itself is what in this thesis is called the subject-object dichotomy. The notion of subject, then, is linked to this mental model, language and perception, while that of object is linked to phenomena where it is supposed the 'real' interaction subject-world takes place. These cognitive agents (called subjects) will focus on those parts of their surrounding with relevant behaviour for their intentions or goals, as it is supposed to be useful for understanding phenomena, for modelling their surroundings, and for taking better decisions. Hence, a subject-object dichotomy, where the subject uses mental models to understand the object (which is a system) has been identified.

In our later discussion about emergent tendencies and complexity, this notion of an object-subject dichotomy will be exploited and recognised as originating from both subjective and objective factors. For example, a sort of trade-off between these subjective and objective factors is associated with the likelihood that a subject finds emergent tendencies in an object. It will have an effect on a subject's recognition of emergence and

complexity either in its environment or in a simulation. Then, in the following section, the factors related to an objective view of complexity in an object, namely the levels of complexity of systems, will be reviewed. Afterwards, a conception of subjective complexity will be considered. Finally, the two notions of complexity will be used to make up some conceptions of emergent tendencies relevant for studying complex systems.

4.3 Objective Causes of Complexity: Levels of Complex Systems

There are in the literature several categorisations of systems in accordance with levels of complexity, as the levels can be defined in many ways (see, e.g., Heylighen, 1997; 1991a; Simon, 1984). The main goal of this section is to present a relevant categorisation of levels of systems, as this will be useful as a reference when settling notions of objective complexity and emergence of tendencies.

The purpose of this classification is to elaborate the fundamental distinction among systems in different levels of the hierarchy. Thus, the higher the level of a system in the hierarchy, the higher the level of control the system exhibits (following Heylighen, 1991a). The evolution of a system from a lower level to a higher one is understood as the appearance of variety at the higher level of control of the system, plus the rise of a new control mechanism to select among these new options. This selector works as a sort of constrainer over the new variety. A level subsumes a lower one in the sense that systems in the higher level present the same characteristics and properties that systems in the lower level have, plus some additional ones. The higher the level of a system, the higher its potential for adaptation - it has more options about actions and mechanisms to discern among these actions, increasing the likelihood of taking good decisions. This new variety involves new properties and adds unpredictability not present in the lower levels, as well as an increase in the autonomy of the system. Unpredictability in a system's behaviour increases as there are more choices given by the increase in variety. The higher autonomy and unpredictability introduces a new potential for complexity (Heylighen, 1991a, 1995).

Heylighen (1991a) and Simon (1984) are particularly interested in complexity of living organisms (systems with a *vicarious mechanism*) and generally include in their descriptions only these kinds of systems. They see such a hierarchy as the result of organisms' ability to cope with complexity in order to survive or to behave more satisfactorily. The first level in the hierarchy is that of the simplest systems - inanimate systems. A brief review of the complexity and the state of human knowledge at this level, such as is seen in physics, is offered by Gell-Mann (1995; 1995/96).

An example of a *hierarchy of systems* is that given by Heylighen (1991a). The more relevant characteristics of this description are summarised in Table 4.1.

This table gives a hierarchy of phenomena for living organisms. In the next sub-section, an alternative categorisation (including inanimate matter) will be given. There, the intention is neither to dig much into different levels of systems at each level, nor to have a level for each level of control that has appeared in organisms during evolution (Heylighen, 1991a), but to capture their basic characteristics in relation to sources of complexity and unpredictability in their behaviour – i.e., a useful classification identifying the factors making difficult a subject's understanding of systems at each level.

Four levels of systems will be defined. In the first one inanimate matter will be situated. In a second level all living beings without reasoning will be placed, namely, those of biological systems, including those at the level of 'origin of life' and those at the level of 'learning' in Heylighen's hierarchy. Then, in a third level, organisms able to reason (or self-aware organisms in more philosophical terms) such as 'human beings' will be described. Finally, in a fourth level a meta-being, an individual able to deal better than a common human being with the variety a human being faces in a modern society, will be considered. This fourth level is intended to provide a human being with a mechanism helping him to process the huge amount of information he has to cope with daily in a dynamic modern society and in dealing better with the choices he has in such an environment.

Level	*Novel characteristic*
Origin of life (first pre-rational level)	Systems at this level have a vicarious selector given by DNA. 'The living cell is characterized by a self-producing (autopoietic) organization, where the DNA controls the production of proteins and enzymes, and the enzymes control the production of DNA' (Maturana & Varela, 1980).
control of position (simple reflexes)	New variety: movement. Selector-controller: control of position. 'The movement will now be a function of particular features of the environment sensed by the system. Sensory organs here act as media translating features of the environment into an internal representation (vicarious selector, see Campbell, 1974) which allows informed decision-making.'
complex reflexes	*Needed for controlling simplex reflexes.* Not discussed by Heylighen.
Learning	Variety: mental model with non-deterministic decisions. Control: Hebb rule. 'The variety is here due to the fact that the synaptic strengths, which determine the probability that a stimulus or excitation would travel from one neuron to another one, are variable, so that the same pattern of excitation may lead to different results. The variability of synaptic connections accounts for what Turchin calls the capability to associate, i.e., to create variable associations between representations. Part of the control is realized through the Hebb rule, stating that a synaptic connection that is used often will increase its strengths so that the probability that it will be used later on increases... The Hebb rule makes it possible for the system to learn by experience. ... The associations that are formed through learning are limited to phenomena experienced in spatial or temporal contiguity'.
The rational level	Variety: concepts from a context can be used in other contexts. Controller: experience (e.g., via social mechanisms). 'The fact that the human can imagine a dog producing musical sounds, while the rabbit cannot, signifies that the human has a larger variety of possible actions. ... concepts are separable from their context: they retain (part of) their meaning when brought into contact with radically different contexts. This can be understood by noticing that the concept can be distinguished from the concepts it is associated with, and that this distinction is stable or invariant: it does not change with the context (Heylighen, 1990)... This means that we shall have to postulate a specific mechanism that explains how any conceptual separation, however partial, is possible... such independent external phenomena that can be associated with a concept may be called symbols. A symbol is a stable, easily recognizable phenomenon that can be combined with other symbols'.
Meta-beings	Variety: high quality information from the environment is filtered and provided to a human. Controller: the named filter itself and human rationality. 'In the present view, the newly emerging control would be situated on the level of the individual rather than on the level of society. Each human individual would dispose of a metarepresentational framework, implemented through an advanced man-machine interface, that would help him or her in manipulating knowledge, in creating new concepts and theories, and in efficiently gathering and organizing all the existing facts and values (s)he needs to solve his/her problems. Practically, what was considered to be a privilege that only a few persons of genius might achieve during their lifetime, namely the creation of a completely new theory modelling important parts of reality, would now become an automatic, everyday activity. This construction of a model would come as natural to meta-humans as the formation of a sentence in verbal language comes to us'.

Table 4.1. A summary of part of Heylighen's hierarchy of systems' complexity

4.3.1 Level 1: Matter, Inanimate Systems

This is the most basic level; here are the simplest systems. Different instances of similar systems (e.g., planets with similar physical constitution) under the same external conditions will behave in exactly the same way. Behaviour of the system is basically determined by external circumstances (environment).

Among the most important cause-effect laws in this level there are: gravitation, electromagnetism, and quantum mechanics. Note that these laws depend on the environment or on the interrelation among different systems and objects. Their effect cannot be changed arbitrarily by a particular system.

Uncertainty is pure in the sense that there is no 'intention' in the entities involved in phenomena. Autonomy is null. Complexity is due to randomness and basic laws such as those previously mentioned.

Example 4.1

Matter in universe. Takes as an example planets or components of an atom as understood in physics. Entities show attraction and/or repulsion forces from where regularities such as planets' orbits around a central entity (a star) can be noticed by certain subjects at the level of rational systems, for instance by humans.

The main science studying systems at this level is physics.

Objectivism (as understood in this thesis) and even absolutism (here absolute knowledge is understood as knowledge about reality theoretically perfect and complete) at this level seem to be reachable via a theory of complex inanimate systems, which may be based upon quantum field theory. One of the greatest defenders of absolutism has been Kant (Solomon, 1996).

4.3.2 Level 2: Adaptive Systems: Living Organisms

New particular phenomena: adaptation and self-reproduction of systems.

At this level will be that subset of systems achieving the level of the simplest subjects. i.e., those that have a vicarious mechanism but are not aware of it (see Chapter 2). The simplest level of vicarious mechanisms is found in the pre-rational level in Heylighen's description. The higher level in this hierarchy is given by those systems with a cognitive model but unable to reason. In Heylighen's view, at this level agents can learn but cannot reason.

The mechanism for surviving, in the pre-rational level given by Heylighen, is expressed in nature via the processes of reproduction, variation, and selection of individuals in a

population of systems by using, for example, DNA. Entities in a generation reproduce and yield descendants for a next generation. Via some selection mechanism only the 'well adapted' will participate in this reproduction. Randomness is present in the variation process (mutation). Generally, there is adaptation of the species over its evolution to new circumstances by passing on features that allow well-adapted behaviour from generation to generation. Characteristics of those well adapted are transferred to their descendants. Useful information about both system structure and behaviour is passed via genes. This gives, in part, the additional variety a system at this level has with respect to a system at the previous level. The new level of control (internal variation-selection) is based on a system's genes.

The new control (new respect to the lower level of systems described above) of a system at this level consists of the mechanism used to actuate appropriate information content in the genes for 'good behaviour' in the particular circumstances the system faces, and to pass useful information to descendants (DNA). The activation mechanism involves basically an organism's enzymes. New unpredictability in behaviour of the system appears, as there are no deterministic decisions about which information in the genes will be actuated and which will be passed to descendants. Systems have more choices at this level, as there is an internal vicarious mechanism where by a selection about behaviour is made. In this sense, as there is a new variety and an internal selector, the system has won certain autonomy.

This level of systems includes not only pre-rational organisms, discussed until this point, but also organisms able to learn. However, learning at this level is somewhat primitive; this corresponds to Heylighen's learning level. At the learning level given by Heylighen, selection of models in the mental level of a system is not in control of the system itself but of laws such as 'Hebb rule'. From the system's 'point of view', this is a sort of blind learning. A system's decision in a particular situation might depend on its 'feelings' about a similar situation it had found before and the evaluation of the consequences of its actions in that past situation in accordance with its goals. Very primitive feelings such as fear or confidence, which are associated with pain or satisfaction, can be recognized in some systems at this level, for example in animals. In simulation, this sort of unconscious learning can be modelled by weighting of rules of behaviour (Edmonds *et al.*, 1998).

Example 4.2

Typical example: living organisms like animals or organisations made up of these entities (e.g., colonies of ants). Another example might be a population of evolving computational agents representing those agents at this level observed in reality.

Main science: biology. Computational models using MAS have been widely used for helping biologists.

This level might be redefined using other sources of knowledge than biology, modelling, and applications in computer science. For example, for some philosophers, the additional source of complexity at this level would be what Arthur Schopenhauer called will (Solomon, 1996). At this level some philosophers include the notion of 'hidden unconscious wills'. Among these wills Solomon mentions the 'will to live' and the 'will of love'.

4.3.3 Level 3: Self-aware Systems or Systems able to Reason

New phenomena at this level: a systems' self-awareness and 'rational' control over its actions. The subject's vicarious mechanism becomes more complex; a control at the cognitive (mental) level appears. The new phenomenon at this level is described as a certain degree of freedom at the rational level by computer scientists and modellers (e.g., as defined by Heylighen; see a summary of his view in Table 4.1), and, as free will by many philosophers (Salomon, 1996). This idea of freedom at the mental level means a certain ability 'to play' with mental constructs some systems at the previous level already have, e.g., concepts and beliefs in Heylighen's view, but for which these systems in a previous level do not have variety.

Some philosophers consider that phenomena at this level, such as a human's intentions and goals, have a more abstract and subtle origin. They suggest that 'free will', is the final source of phenomena at this level. Several definitions of free will going from strong notions, which are of interest in, e.g., philosophy and theology, to weak notions, which are of interest in, e.g., computer science and modelling, might be displayed. Philosophical views of free will seem to bring philosophers into endless discussions, which are not of interest in this thesis. Likewise, notions of free will in theological terms might be explained in very 'soft terms', e.g., as linked to 'virtues' some of 'god' or divinity (e.g., someone in a higher level than human beings) give humans. In a less strong way, such notions might be linked to emotions and other human virtues and probably related to some conceptions of autonomy. For instance, they can be seen as linked to very 'special capabilities' of a

human such as initiative and the ability to change his goals. Finally, weaker notions of 'free will' than those previously considered could be seen as associated to special abilities in an agent's reasoning.

For example, Edmonds (2000) has presented some guidelines for modelling a weak notion of 'free will'. He describes a notion of free will in these terms: 'It means that an agent is free to choose according to its will, that is to say that sometimes it is its deliberations on how to achieve its goals that determine its actions and not just its circumstances (including past circumstances)' (*idem*, p. 2). This conception is close to the notion of reasoning given by Heylighen (see the summary of his description of systems at the rational level in Table 4.1). Nevertheless, it seems that Edmonds recognises a certain additional independence in the agent's reasoning in terms of some 'creativity', e.g., when writing, 'The process of deliberation leading to a choice of action has to be free in the sense that it is not constrained to a particular 'script' - this means that there is also some choice in that deliberation, as well as choice in how to make that choice, and choice in how to make the choice in how to make that choice etc.' (*idem*). In this thesis, there is more interest in notions like Heylighen's idea of reasoning and Edmonds's suggestions for modelling a 'weak' conception of free will than in the strong philosophical and theological ideas of free will. This is so because these softer conceptions can be useful in simulation – indeed similar ideas have already been used for modelling computational agents (e.g., see Edmonds, 1999b).

Both Heylighen and Edmonds recognise that at this level of complexity the behaviour of an individual (a system) does not depend only on the circumstances, but also on internal reasoning and on a weak notion of self-awareness as the system might also have a simple model of itself. It is here where the notion of the subject-object dichotomy is useful, since the system becomes a subject able to reason and to make choices regarding aspects of his behaviour. Here that dichotomy can be noticed as given by, on the one hand, the reality when the subject interacts with his surroundings, and, on the other hand, his perceptions of that reality as they are captured by the agent's language.

As was stated above, the new variety at this level appears in a subject's cognitive model – the subject is able to play with mental constructs. The new and higher control in the system appears as a capacity for subject-driven rationality. At this level, its capacity to keep models of the environment appears along with its ability to anticipate possible consequences of its own behaviour, behaviour of the environment, and behaviour of other subjects. There is a certain mental flexibility granted by, to some degree, a context-

independent reasoning (as in imagination). However, such independence is not complete (as with the imagination) but rather controlled to a certain extent by the context and that subject's reasoning and values. For example, humans in modern society are in part controlled by social norms, work norms, common sense, religion, and media. These also bring into a subject's cognitive model new ideas, further associations, and reasoning.

All this gives a higher level of autonomy to a system than at level 2 and introduces more unpredictability - it is not easy for an observer (e.g., another subject or a modeller) to foresee either the alternative actions the system will consider or the final choice the system will make.

Individuals (systems) at levels 2 and 3 can group and interact into communities and societies. The dynamics of these interactions are even more difficult to understand than that of the components. Examples are communities of humans where organisations and institutions appear as a result of cooperation and competence among groups (Carley *et al.*, 1998). The complexity of social systems made up of individuals at this third level is tremendously high, as are the institutions and organisations these individuals create. The dynamics of these systems allow tendencies to appear which are difficult to understand by the interacting agents and by a modeller of these systems. Studying human systems is the concern of people working in social simulation. Existing methodologies for studying these systems are not totally satisfactory (see Chapter 2, and especially section 2.10). Among the aims of this thesis is to provide a methodology for studying the emergence of tendencies, especially those causing social complexity (see Chapters 5 and 6).

The sciences studying systems at this level include: economics, sociology (including social simulation), soft systems, and psychology. There is a great deal of argument in human research concerning systems at this level.

4.3.4 Level 4: The Level of Meta-beings

As in Heylighen's view (see Table 4.1), in this section a meta-human is seen as a social human who is better able to cope with the difficulties faced by a social human seeking to achieve 'good' behaviour in a modern society. These difficulties are fundamentally due to a subject's bounded rationality and to the large amount and variety of information this individual finds in a social system. In the next paragraphs, the origin of such difficulties will first be discussed. Usually, mechanisms in a 'meta level' are proposed for helping a human being to cope better with such complexity.

In general, difficulties in a human understanding of phenomena are explained as due to their bounded rationality (as pointed out by Simon, 1982). Heylighen sees the problem precisely in the subjectivity of humans' knowledge. He identifies as the main weakness in a human's knowledge the limitations of his conceptual codes: 'each conceptual code will be impaired by its (not admitted) subjectivity, by its limited number of concepts, and by its tendency to reduce holistic phenomena to combinations of discrete elements.' In his opinion, a move towards objectivism in a social agent's mental beliefs is convenient. He points out that this has already been a tendency in the historical accumulation of knowledge in philosophy, religion and science.

Heylighen associates increase in variety in society in the last century with cultural revolutions such as relativity and quantum theory, and with the massive use of communication and transportation media. In his view, these two factors, plus some changes in values, have brought a release of social and cultural constraints. Massive access to media by people has allowed a faster spread of, for example, knowledge, ideas, technology and fashions. The mental associations each individual makes increase the variety even more.

He proposes a technological framework for helping a human control this variety and for decreasing the level of subjectivity in humans' knowledge. Such a device would be especially useful for 'storing, transmitting and processing information' (see Table 4.1). The idea is to increase a human's rational capabilities indirectly by using computational devices able to capture and process information, and then abstract and filter it in conformity with an individual's goal. Mechanisms for helping in this direction at the present time include: MAS, database interfaces, and information systems (as conceived in computer science and management).

4.4 Subjective Notion of Complexity

Here a notion of complexity from the other side of the coin of the object-subject dichotomy, i.e., a subjective notion, will be presented. In this view, knowledge is basically relative to a subject's language. For example, what is obvious, simple, complex or emergent will depend on its judgement.

Edmonds (1999a) discusses a conception of complexity observing that different notions of complexity can be found in the literature. He points out many misinterpretations of the term and argues that complexity must be differentiated from other notions of difficulty

such as size, number, ignorance, variety, randomness, expressivity, irreducibility, dimension, and order. He gives the following definition of complexity:

'Complexity is that property of a model which makes it difficult to formulate its overall behaviour in a given language, even when given reasonably complete information about its atomic components and their inter-relations.'

This definition is closely linked to the notion of a subject's bounded rationality and identifies a subject's language (and hence its subjectivist character) as the final cause of complexity.

4.5 Emergent Tendencies

As in the case of the discussion about complexity, emergence can be seen either from an objective or subjective point of view. Referring to the discussion in the two previous sections, first an objectivistic notion of emergence, based on an objective notion of a systems' complexity, will be given. Afterwards, a subjectivist version of emergence of tendencies linked to a subject's bounded rationality will be presented. *Finally, a notion of emergence of tendencies, supported by a trade-off between the named subjective and objective factors on which each of the previous definitions rest, will be offered.*

4.5.1 Objectivistic Notion of Emergence of Tendencies

In the previous section, it was pointed out that the higher the level of complexity of a system, the higher its autonomy, the unpredictability of its behaviour, and the potential of the system to present regularities that are difficult to understand by an observer (subject). Hence, the higher the level of complexity of a system, the higher the likelihood that such a system presents emergent tendencies to that subject.

Less important factors for an objectivist view of emergence of tendencies in a system than its level of complexity can be identified. The most significant of these seems to be the nature of the changes the system is undergoing. A system experiencing only quantitative change is supposed to be easier to understand than a system suffering qualitative or structural change. An example of structural change occurs when a new controller appears in a system, bringing it from one level of complexity to a higher one (as was discussed in section 4.3 above). A case of this sort of change is the rise of the European Union (here a new geographical-political entity appears), along with which a controller, represented by the Council of Ministers, the European Commission, and the European Parliament, arises.

An additional factor of objective complexity and emergence seems to be the degree of coupling of a system's components, that is, the extent of the dependency of properties of

the whole on properties of the components. Consider, for example, how the life of an animal depends on different parts of its body. This property (life) is more dependent on certain of the animal's components and on properties of those components than on other components and their properties; e.g., it is more reliant on the animal's heart or blood system than on its hair or on its extremities.

This degree of dependency or coupling seems associated with the reversibility of properties of both the whole and the components with respect to disaggregation and aggregation of components. There are properties of the whole that are very intricately linked to the structure of the system. If, for example, components are disaggregated, the system loses that property and, in many cases, the property cannot be restored by aggregating the components. An example is life in an animal (the whole). An animal will loses its life if its heart is separated from its body. Furthermore, life cannot be returned by bringing the heart back to its original position. To keep the property 'life' of the whole is possible, in some cases, only in very special conditions, e.g., in a scientific laboratory, where each separated component is kept under similar conditions to those undergone by it before being separated from the whole. Likewise, a component might irreversibly lose properties when separated from the whole. For example, a heart separated from the body embarks on a decomposition process that changes its properties. This immediate change in the properties of a component, started as soon as the component is separated from the whole, seems also to be associated with the irreversibility in the properties of the whole. Similarly with the case of the whole, in some cases properties a component undergoes before being separated from the whole could be kept during a limited period of time if the component were held under special conditions. Notice that in the case of the named property of the whole, i.e., life, it can only be kept if the component (the rest of the body without the heart) is restrained to special conditions, but once the property is lost it cannot be restored again.

In this paragraph the level of complexity a system (object) is embedded in has been identified as the main factor for the emergence of tendencies in an objectivistic view. Other important factors in this view seem to be the nature of the changes that system is undergoing (qualitative or quantitative) and the degree of coupling of its components.

4.5.2 Subjectivist Notion of Emergence of Tendencies

In this section, emergence will be characterised as purely relative to a subject's cognitive model. The general idea is that a subject will consider a certain tendency perceived in a

system as emergent if, having information on a system's components, the subject finds it difficult to explain that tendency (following Edmonds's definition of complexity, 1999a). This notion of difficulty leaves open the possibility for a range of definitions of emergent tendency. The stronger ones will be defined in terms of a subject's language and will be closer to Edmonds's definition of complexity as well as to a conception of emergence presented in Edmund *et al.* (1999). These notions will be linked to a strong idea of explanation of a tendency: a tendency is explainable in the 'theory' content in a subject's language if it is deducible from that theory. On the other hand, the weaker notions of emergence of tendencies will be founded on a softer notion of explanation of a tendency, one based on a conception of satisfiability of the subject with a certain explanation of such tendency. This notion may be more appropriate for modelling some systems, e.g., a social system.

As an example of a strong notion, consider a subject, S, using languages L1 and L2 to reason about phenomena perceived in an object, O, defined as a system at a certain level of complexity. It will be said that a certain tendency observed by *S* in *O* is emergent when

> *descriptions given in the language (L2) the subject uses to express overall patterns in the object are not reducible to descriptions in the language (L1) the subject uses to describe the design of the system (see* Figure 4.2*).*

Using this stance, the notion of a potential for emergence of tendencies can also be described in the following manner:

> *the higher the level of complexity of the perceived system and the more bounded the subject's language, the higher the potential for emergence of tendencies in the subject-object interrelationship.*

For a *weaker notion* of emergence of tendencies, a notion of *satisfaction* will be applied as the criteria an agent uses for evaluating its own performance and satisfaction of its goals, following Simon's ideas (Simon, 1982). Replacing the notion of reduction among languages (used above) by a notion of satisfiability, a tendency observed by a particular subject in a particular system would be considered emergent if

> *descriptions given in the language (L2) a subject uses to express overall tendencies in an object (a system) are not satisfactorily explained by descriptions in the language (L1) the subject uses to describe the design of the system.*

An even weaker definition of emergent tendency is as follows:

> *a tendency observed by a subject in a system would be considered as emergent if the subject does not have a satisfactory explanation for that tendency.*

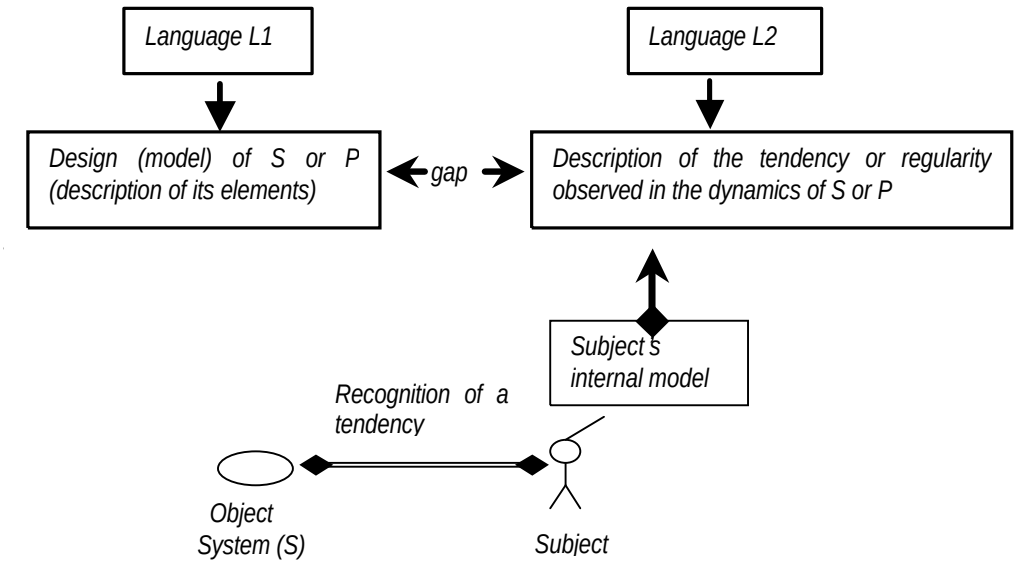

Figure 4.2. Emergence of tendencies for a subject observing a system

A tendency is no longer considered as emergent for a subject as soon as the subject finds a satisfactory explanation for it. A satisfactory explanation might appear either as a result of changes in the subject's explanation of the tendency, i.e., as a result of the subject's reasoning, through the gathering of more information related to that tendency and the updating of the subject's mental models, or, because the subject's notion of satisfiability is relaxed, e.g., owing to changes in the subject's goals. In none of these cases must the subject's language necessarily be altered.

In the case of the first two definitions (those using the notion of languages), the tendency loses its characterisation as emergent if, e.g., a new language *L3*, able to fill the gap, appears in the subject's mental model (see Figure 4.3). Additionally, the former languages, *L1* and *L2*, might be, in some sense, subsumed by the new language, *L3*; i.e., knowledge expressible in *L1 U L2* is also representable in *L3*.

Example 4.3

1. In this example the strongest notion of emergence of tendencies given above will be used. Consider the languages for describing movement of bodies in space, e.g., movement of planets in the solar system, allowed by Einstein's relativity law. This language had the role of the new language *L3* appearing for satisfactorily explaining regularities not understood using the languages available to the scientific paradigms existing in physics at the time it was conceived. One of these regularities was the

particular elliptical pattern a planet describes in its revolution about the sun. Language *L1* was that language given by the old paradigm, i.e., Newton's gravitational law, now replaced by *L3*. *L2* is the language used to describe mathematically that elliptical orbit a planet describes. This description might come first from observations of the planet in its movement about the sun; e.g., they might be a sequence of spatial positions the planet has occupied. A mathematical model of this pattern of movement (a tendency) can be built using, e.g., numerical methods for interpolating those points. How good this model is can be checked by comparing predictions of the model with new measurements. This model, described in a hypothetical language *L2*, is not well explained by using a language *L1* allowed by Newton's gravitational law. Nevertheless, it will be well explained by a model of the relation space-time in a language *L3* enabled by Einstein's general theory of relativity. *Tendencies* like this are *emergent* for a subject using language *L1* but not for a subject using language *L3*.

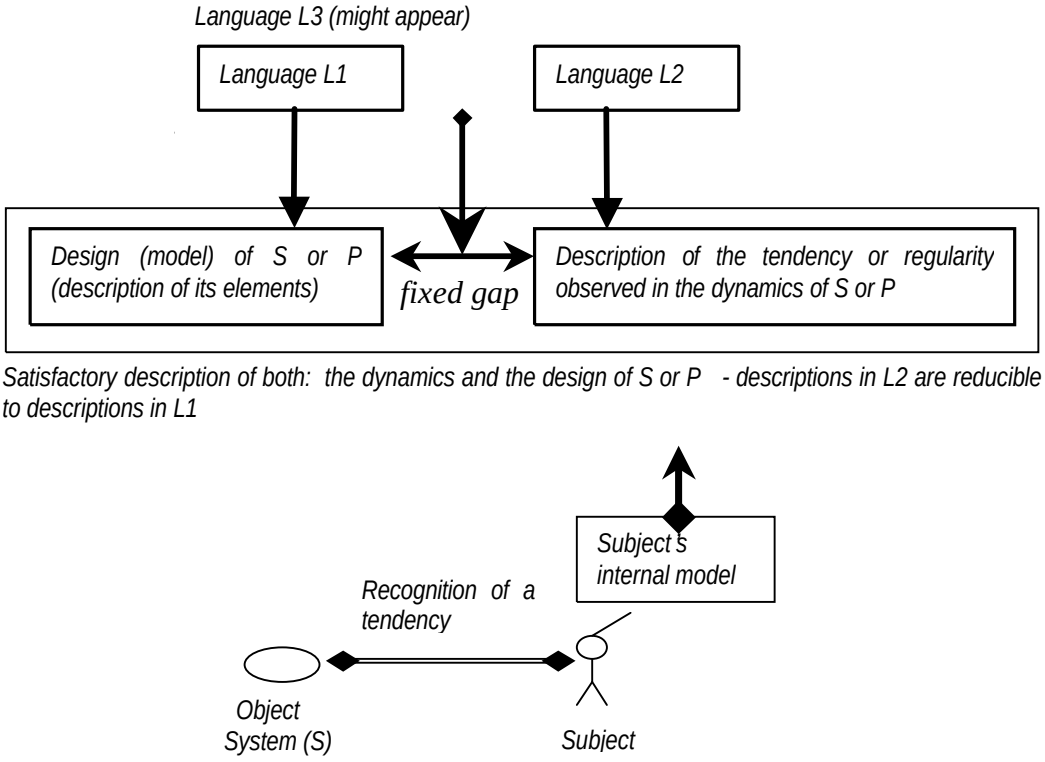

Figure 4.3. A new language *L3* is used by the subject and the emergent tendency is no longer considered emergent

2. Consider a certain geometric area, e.g., a circle, and the points it is made of. Suppose that a language *L1* is used to describe points and its properties and another language *L2* is used to describe the area and its properties (e.g., size of the area). Assume also that this language *L1* is 'poor' in the sense that it does not include mathematical notions such as differential and integral calculus. It might be impossible to explain the size of the area of the circle using language *L1, and then this property (size)* will be considered emergent. After 'discovering' the differential and integral calculus (language *L3*) properties of a geometric area, such as its size, can be explained satisfactorily in terms of properties of the points in this area and the language *L3*.

   In this example, a property of the whole (area) is explained by properties of the entities (points) in terms of which it is defined by using a mathematical model of the relation between the property of the whole and properties of the components. This model is given in the language *L3*. It seems common to find cases of emergent properties where the gap among descriptions has this sort of source: the whole and the components are of a different nature (mathematically). This different nature is sometimes expressible as a different dimension between the elements constituting the whole and the whole itself. Similar examples can be brought from applications of integrals, series, and other mathematical 'tools'.
3. An example where the source of difficulty for explaining a regularity has similarities to that shown in the previous example but where that regularity is not expressible mathematically will be examined. Consider an agent answering to the question: 'what makes a picture more than its separated lines?' (this example has been borrowed from Edmund *et al.* (1999)). For recognising a picture, a subject captures certain patterns of the outline of the picture and compares them with its 'mental' prototype (his model) of what is a picture. Assume the subject has a language *L1* used for describing lines (potential parts of a picture) and a language *L2* used to describe general regularities observed in the whole figure. It would be difficult for a subject to explain why that particular figure is recognised as a picture using *L2,* as this language only permits descriptions in terms of lines. Many regularities perceived as pictures for a subject using *L1* will be emergent. Assume now that the subject has a new language *L3,* allowing him to capture and explain more patterns in the picture, e.g., other patterns in addition to lines, than if using language *L1*. Patterns are sub-models the subject uses to define a figure. He may, for example, define the picture 'face' as that consisting of the following patterns: a whole closed area (having a certain shape

and representing the borders of the face), a short line perhaps slightly curved (representing a mouth), a point (representing the nose), and another two points (representing the eyes); and the constraints given the relative positions among these sub-models. A definition of a figure might be part of the definition of a wider figure, so that, for example, the definition of the figure 'face' might be used for defining the figure 'human'. Such patterns seem to be more helpful for explaining what a figure is than those offered by the language *L1*. Then, a subject's prototypes of figures are more flexible, as they include a wider variety of forms. In this case, the subject's new language, *L3*, can be used to express basic patterns useful to describe a figure as well as individual lines and the figure itself. It can be used to explain patterns, including lines, captured in the figure, and a picture in terms of those patterns and their interrelations. More likely, this new language rather than language *L1*, would allow an agent to find a satisfactory explanation for the question posed above.

4.5.3 Emergence of Tendencies: a Trade-off between Subjective and Objective Factors

The work developed in this section has produced a compromising notion of emergence, taking into account both the objectivist and the subjectivist views. The emergence of tendencies will be recognised as dependent on both objective and subjective causes. The conclusion is that, given a subject-object relationship, where the subject is a system at the third level of complexity or above and the object is a system at any level of complexity, *the higher the level of complexity of the system and the more bounded the cognitive model of the subject, the higher will be the likelihood that the subject finds emergent tendencies in the object.*

4.6 Tendencies in a Simulation

A weak (subjective) notion is chosen as the more appropriate to describe emergence for a modeller observing a simulation. A tendency observed by a modeller in a simulation would be emergent if:

descriptions given in the language (L2) the modeller uses to express overall tendencies in the simulation are not satisfactorily explained by descriptions in the language (L1) that the subject uses to express the design of the simulation.

5 Chapter 5 - Mapping the Envelope of Simulation Trajectories via a Constraint-based Exploration

5.1 Introduction

This chapter is aimed at offering an alternative methodology for exploring simulation trajectories, a methodology allowing one to implement stronger notions of morphism than those commonly found in the literature and associated with the notion of approximation discussed in 2.6.

This methodology is also intended to help a modeller (a subject) to understand aspects of interest in a simulation better (and, by extension, in a target system). A modeller (for instance, a computer scientist, a biologist, or a manager) usually seeks to behave 'well' either in an area of research related to the target system or in the target system itself. In the first case, his aim is to understand the target system better, while, in the second case, he is also interested in being well informed as an agent acting in such a system. For instance, a manager is usually interested in understanding his business environment better where he is an 'active' agent, while a biologist is commonly interested in studying biological systems where he is not usually an agent. Hence, the methodology to be proposed is intended to have both informative and 'instructive' orientations.

The need to better understand a certain kind of tendencies in the dynamics of a target system is what motivates a simulation in many areas of research. In areas such as social simulation, it is of particular interest to analyse processes and to better understand tendencies in social behaviour (Carley *et al.*, 1998). In management and policy analysis, simulation is valuable to guide and inform managers and policy analysts, assisting them in taking decisions (Wack, 1985a and 1985b; Domingo *et al.*, 1996). Information allowing more general conclusions will help in all these areas of research to test theories and hypothesise about the simulation and the target system, and will assist managers and policy analysts more convincingly.

It is of particular interest for such modellers to analyse the commonality of emergent tendencies in different simulation trajectories, as this allows them to draw conclusions about the theory implied in the simulation. However, there is usually a trade-off between the richness of the study in terms of the number of explored trajectories (sometimes related to how fine-grained the simulation model is) and the amount of required computational resources. The finer the simulation model, the more 'realistic' the simulation model will be, but also the more intricate the analysis of the simulation will be.

A typical case where this analysis is crucial is in the MAS-based simulation of social systems. There, modellers attempt to generate certain 'complex' tendencies in the dynamics of a whole population and in the behaviour of an agent as the result of the interaction of individuals (agents), where unforeseen behaviour of individuals and unpredictable tendencies in the behaviour of the whole population can arise (Edmonds, 1999).

A factor that limits the comprehension of emergent tendencies is the lack of alternative methodologies and tools for appropriate exploration and analysis of the dynamics of a simulation. As explained in section 2.6, present methods include examining individual trajectories, as in scenario analysis (Domingo *et al.*, 1996), and statistical sampling, as in Monte Carlo techniques (Zeigler, 1996). In both cases, the scope of conclusions is limited. Also, they have some drawbacks for modelling complex systems (see section 2.10).

This chapter is aimed at developing an alternative way of exploring and analysing a MAS-based simulation by systematically and automatically enveloping all possible trajectories in a substantial fragment of a simulation theory as a complement to existing methods. More specifically, this thesis proposes a complete search of trajectories for a range of parameterisations and agents' choices. This kind of search corresponds to a logical model exploration in theorem-proving (see section 3.5.2). Conclusions will be more general than when using the named alternative methods. In addition, the proposal will be made to analyse tendencies in a simulation by enveloping them for the subset of explored trajectories. An envelope places emphasis on the results from the worst investigated cases rather than on average measures.

The exposition in this chapter is as follows. First, in section 5.2, non-deterministic factors in, and driven by, the dynamics of a simulation will be pointed out. Basic aspects of the proposed methodology, namely a constraint-based exploration and envelope of a subspace of simulation trajectories, are discussed in section 5.3. In section 5.4 generalities about the sort of exploration of *post hoc* analysis to be proposed in this chapter are given. Afterwards, in section 5.5, more fundamental theoretical aspects with respect to the kind of exploration (namely, a logical model envelope of simulation trajectories) and the sort of proof intended (necessity) are elucidated. In section 5.6, the relationship between proving the necessity of a tendency in a simulation, on the one hand, and a modeller's satisfactory explanation and understanding of the tendency, on the other, are addressed. This discussion is useful for figuring out the usefulness of proving tendencies for understanding better emergent tendencies in a simulation. Section 5.7 depicts the sources of constraints on the

proposed constraint search. Both technical ones, related to computational modelling, and other, more pragmatic ones, associated with difficulties in modelling complex systems, are taken into account. Generalities about the proposed methodology for translating a MAS-based model into a constraint-based model for proving tendencies is covered in section 5.8. Section 5.9 examines in detail how such a translation process can be implemented in the simulation language SDML (there the relevant aspects of SDML for achieving this sort of proof of tendencies are listed). Finally, section 5.10 discusses the aspects of this translation that suggest it is isomorphic.

5.2 Factors Driving the Dynamics of a Simulation

Factors for experimentation in a simulation will be linked to causes of variety in a system's states. Factors defining variety at the structural level of a system will be called parameters. Each alternative parameterisation will define a different structure of the system. Variety at the dynamic structure means alternatives for the system's behaviour or branch points in the simulation trajectories. They stand for the options (non-deterministic behaviour) the processes the system undergoes can follow. These factors will be called choices.

Experimenting with a MAS-based simulation consists in generating different trajectories or alternatives for the behaviour of a system via changes of either parameters of the model or choices of the processes the system undergoes. Different parameters and choices will be linked to non-determinism. This non-determinism was seen in the description of the hierarchy of levels of complexity of systems (see section 4.3) as grounded in the autonomy and unpredictability of the system. There, factors conveying this unpredictability were also pointed out. Usually, in this research, choices will be represented by the alternative actions available to agents. Each choice becomes a branch point in the simulation, with each alternative in the choice representing a different simulation trajectory.

Another factor present in a simulation is the underlying logic of the simulation language.

5.3 Enveloping Outputs in Simulation Trajectories

This thesis proposes to analyse the dynamics of a simulation via enveloping outputs, the analysis of emergent tendencies being of particular interest for the work presented here. This represents an alternative to traditional methods such as those based on central measures (e.g., Monte Carlo studies).

Merriam-Webster's dictionary at http://www.m-w.com defines 'envelope' in the following terms:

Main Entry: envelope
Pronunciation: 'en-v&-lOp, ÷'än-
Function: noun
Date: circa 1714
1 : a flat usually paper container (as for a letter)
2 : something that envelops : WRAPPER <the envelope of air around the earth>
3 a : the outer covering of an aerostat b : the bag containing the gas in a balloon or airship
4 : a natural enclosing covering (as a membrane, shell, or integument)
5 a : a curve tangent to each of a family of curves b : a surface tangent to each of a family of surfaces
6 : a set of performance limits (as of an aircraft) that may not be safely exceeded; also : the set of operating parameters that exists within these limits

In 1, 2, 3, and 4, is presented the most common usage found in everyday life. There an envelope is conceived as a container or covering, consisting, e.g., of hard material (e.g., a cover of a letter) or of a softer one (e.g., the air around the earth); but no constraint is posited about its nature. There are three elements: a physical one clearly perceived, that enveloped; and a second physical one, the cover or container; and finally, a relation among these two: the enveloping relation.

In 5:, the idea is better formalised. There the mathematical notion is offered. In mathematics an envelope is conceived as a curve/surface tangent to a family of curves/surfaces. Now some constraints are added to the concept. First, both entities, the enveloped and the enveloping, are mathematically defined objects: a curve/surface, and a family of curves/surfaces, respectively; and second, the enveloping object must be tangent to the enveloped one.

Finally, 6 introduces a new notion about the concept; what is being enveloped is neither a physical object nor any mathematical function, but the behaviour or performance of something (let us assume that this something is a system). This performance can be characterised directly by observing the outputs of the system, or indirectly by placing constraints over some parameters that are supposed appropriately to anticipate such behaviour. This second method does not seem convenient for analysing the dynamics of a simulation, as the dynamics of a simulation is difficult to understand from the design (that is why a *post hoc* study of the simulation outputs is of interest in this thesis). However, it is of interest in this thesis to define in advance that space of the theory where the dynamics of the simulation will be explored, and this is achieved by placing constraints over the parameters of the model and over the choices of the processes. A typical study of the

relation between these simulation outputs and factors of the simulation, i.e., parameters of the model and choices of the processes, is data analysis, though its methods (e.g., principal components, factor analysis) are based on statistical results. It would be of interest to explore the use of envelope following methods like those of data analysis.

But, how could we envelope the simulation outputs?

The simulation outputs are functions over time represented by a set of values (one for each time instant considered). Using Zeigler's notation (see section 2.3) the collection of values for a variable is called *Y*, and its representation over time is refered to as *r*. Should several outputs be considered, say *n* of them, the collected set of outputs will be called Y_1, Y_2, $\ldots Y_n$ and their representations over time r_1, r_2, $\ldots r_n$.

It is important to remember that the intention is to make the output comprehensible for a modeller. Consequently, it does not seem convenient to use the strong concept managed in mathematics. Rather, an envelope will be chosen considering the trade-off between practical usefulness for a modeller and precision (by precision we mean how close it is to the ideal mathematical notion of a tangent curve/surface).

Enveloping one-dimensional outputs

First, consider the case of enveloping a single output *Y*. Each trajectory will generate a sequence of real values over time, *Y*. Calling y_{ij} the output value at time instant *i* for trajectory *j*, an envelope might consist of two sequences of values over time: E_{upper} and E_{lower} , which in some sense cover all trajectories. The value of E_{upper} at time instant *i* must be greater than or equal to y_{ij} for all *j*, and E_{lower} at time instant *i* must be lower than or equal to y_{ij} for all *j*. That is, the envelope would be given by two sequences of values over time, where for each time instant all values generated by the simulation trajectories are enclosed by the two values given by these two value sets. Putting this in other words, at each time instant, *t*, the smallests interval covering all points generated by the explored trajectories is included in the interval given by the two sequences E_{upper} and E_{lower} for instant *t*. This tactic is exemplified in the case presented in Chapter 7.

Alternatively, first an approximating function, *f*, for the value set *Y* that each trajectory generates of the output might be elaborated; then, the instances of these functions (one for each trajectory) might be enveloped in accordance with the mathematical definition or a related one (i.e., a tangent curve or other function enclosing all instances of *f* might be developed).

Enveloping multi-dimensional outputs

Until this point, the enveloping of each output independently of any other has been considered. However, this does not always have to be the case. We might also be interested in the interrelation among the outputs, in which case that we would have to consider all outputs as a multi-dimensional function over time. Assume that a value set Y_h is generated for each output. After generating all trajectories, there will be a collection of values $(Y_h)_{ij}$ representing each value for output *h* at time iteration *i* for trajectory *j*. This data could be arranged in different manners. For example, each trajectory can be seen as generating a matrix of values where each column would have the collection of values for a time instant, and each row the collection of values over time for each output variable (for trajectory *j*, the datum at the position *hi*, $Y_{hij,}$ would represent the value for the output *h* at iteration *i*).

Assume that two output variables are of interest. Considering them over time, there would be a three-dimensional collection of points. As for the one-dimensional case, the envelope could be discrete, when it is defined over the discrete outputs the simulation generates; or continuous, if an approximating surface for that three-dimensional collection of points is generated and then enveloped.

In general, either a discrete or a continuous enveloping surface for the multi-dimensional function hypothetically defined by the outputs of interest might be generated, taking into account the trade-off between precision and the modeller's goals.

Final remarks

Among the procedures of interest for enveloping tendencies in simulation studies might be the following:

- Enveloping certain *properties of the observed tendency rather than the tendency itself*. In this case, it is supposed that a certain mapping from the tendency gives the output of interest (the properties to be enveloped). It may be of interest for a modeller to prove that certain properties of a tendency are within certain bounds. This may be achieved by showing that their envelope satisfies certain conditions. It would allow a modeller to affirm that the tendency satisfies certain properties in accordance with the simulation model theory – defined by the envelope of the properties of the tendency achieved via a simulation. The results might permit one to relate the simulation results to theory developments and to elaborate conclusions with respect to the theory underlying the simulation model.

- Producing a *mathematical description* of some coarse borders of the space where the tendencies have been observed. This is useful if it is difficult to describe the subspace of the tendencies directly. Then, coarse borders are chosen as a first approximation to the envelope and, afterwards, these enclosing borders are expressed mathematically.
- *Using extreme cases of representative or typical instances* of a tendency. In this such cases it is assumed that the observed tendencies in the simulation can be grouped qualitatively as similar or close enough (in accordance with some criteria) to a finite number of typical tendencies.
- *Specifying a range of parameters and choices*. This is the original description of the subspace of explored trajectories used in the previous paragraphs (it is also similar to definition 6 in Merriam-Webster's dictionary presented at the beggining of this section).

5.4 (Logical) Model-Constrained Exploration of Simulation Trajectories

A simulation - either an event-driven, a finite differences, or a MAS-based - can be seen as a partial logical model generation (see section 3.5.2). Usually, in a trajectory only a partial set of all the facts of the logical model corresponding to the trajectory are produced. This partial set consists of those facts that are relevant, either because they are required for the modeller as outputs or because they are necessary to generate the simulation transition steps. The remaining ones are left as unknown (see Chapter 3).

There are different methods to specify a theory in a language. One commonly used in logic is via a set of formulas of the language that become the axioms of the theory (see section 3.2.1). In a declarative program a simulation model is specified via a set of rules and the underlying logic of the program. Potential trajectories are defined via non-deterministic factors of the simulation, e.g., parameters and choices.

In this chapter we propose analysing the emergence of tendencies in a simulation by exploring a subspace of the space of trajectories. This will be done via a (logical) model-based constraint search, where the constraints will stand for the selected parameters and choices. It will allow a modeller to explore that fragment of the simulation theory content over a range of parameters and choices (see Figure 5.1). Consequently, the resulting conclusions and proofs will be valid over that fragment of the theory and, under appropriate justifications, they can be extrapolated to the whole simulation theory.

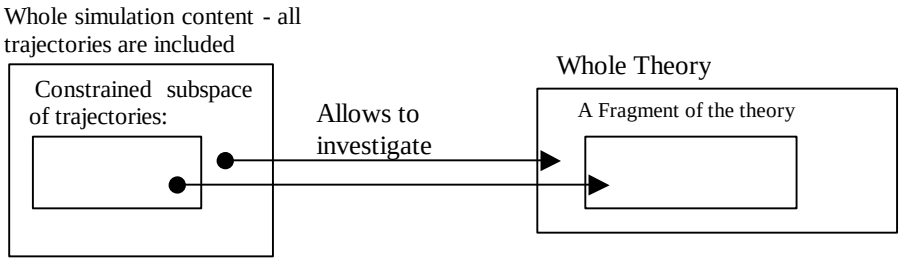

Figure 5.1. Theory given by simulation trajectories

Like scenario analysis, the idea is to generate individual trajectories for different parameterisations and agents' choices, but, unlike scenario analysis, the exploration is constrained to only a certain range of parameters and choices.

Akin to Monte Carlo techniques, it explores many possible trajectories, but, unlike Monte Carlo studies, it explores an entire subspace of trajectories (rather than some randomly generated sample of the whole) and is able to give definitive answers for inquiries related to the dynamics of the simulation in that subspace.

5.5 (Logical) Model Exploration for Proving the Necessity of a Tendency

The idea is to generalise about tendencies going from the observation of individual trajectories to observation of a group of trajectories generated for certain parameters and choices. In particular it is intended to know if a certain tendency is necessary or contingent in the explored trajectories. We understand a simulation trajectory (or, shortly, a trajectory) as a logical model embedded in a simulation program (a 'possible world' in semantic terms) and involving trajectories of entities (e.g., agents) inside the simulation and, hence, different from trajectories of these entities. It is a cross-product of all settings of the structure of the simulation model and all processes (e.g., agents' choices) into one path through a high-dimensional space (see Figure 5.2).

The character of the search will be predominantly (logical) model constraint, forward-chaining, and clausal ordered. A logical model will be generated for each combination of parameters and choices. Each combination of parameters provides a different structure of the simulation model (see Figure 5.3). 'Paths' representing trajectories are generated for each structure. Then, while the simulation is going on, choices produce branch points where alternative settings for each choice turn out into a different simulation trajectory. The order of the inference will be given by rule dependencies. Possible origins of rule dependencies will be explained below.

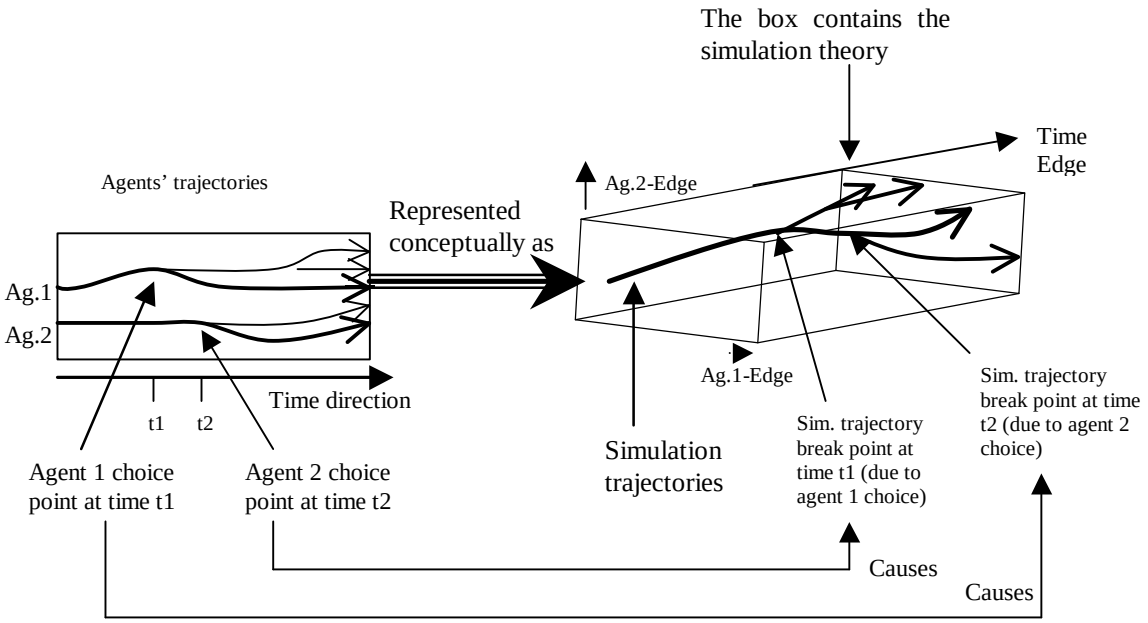

Figure 5.2. Representation of a simulation theory in terms of the simulation trajectories, and of these in terms of agents' choices (for a single parameter-setting and assuming there are two agents)

This exhaustive constraint-based search over a range of possible trajectories makes it possible to establish the necessity of postulated emergent tendencies. Following a procedure similar to that used in theorem-proving (see Chapter 3), a subset of the possible simulation parameterisations and agent choices is specified, the target emergent tendencies are specified in the form of negative constraints, and an automatic search over the possible trajectories is performed.

Thus, a subset of the possible simulation parameterisations and agent choices is specified, the target emergent tendencies are specified in the form of negative constraints, and an automatic search over the possible trajectories is performed.

Tendencies are shown to be necessary, with respect to the range of parameterisations and non-deterministic choices, by first finding a possible trajectory without the negative constraint to show the rules are consistent and then showing that all possible trajectories violate the negation of the hypothetical tendency when this is added as a further constraint. This is equivalent to showing that all possible tendencies obey the positive form of the constraint, i.e., that the positive form is true for all tendencies.

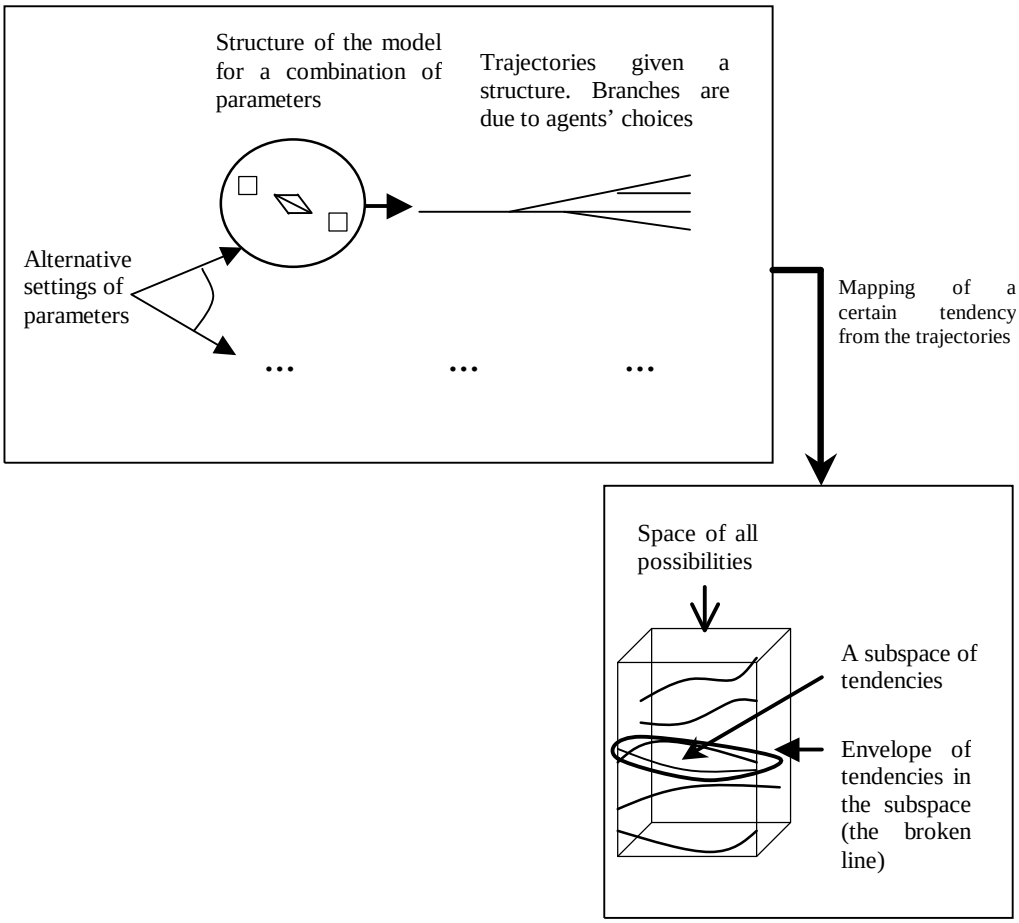

Figure 5.3. A model constraint-based exploration of the dynamics of a simulation

For instance suppose there are two cities - a first city, called the'origin city', *City-O*, and a second one, called the 'target city', *City-T* - and that, after some natural phenomenon (e.g., a heavy fall of rain), the routes communicating between these two cities have become badly affected (it is assumed there are several possible routes). As a result of the natural phenomenon, many, or possibly all, of the routes between the two cities have become blocked. Assume that there is a subject in C*ity-O* who believes all routes have been blocked and wishes to prove that this hypothesis is true. One way to implement the proof is to assume that the route is not blocked (e.g., to assume as true the negation of the hypothesis) and then send 'an explorer' to investigate each possible route (to follow a possible route will be equivalent to the exploration of a simulation trajectory). If the investigated route is found to be blocked, then there will be a contradiction. The contradiction results because, on the one hand, the negation of the hypothesis (i.e., ¬ ('all routes are blocked')) was assumed to be true (the negation of the hypothesis for a particular route would be ¬(the route is blocked')), and, on the other hand, the hypothesis

has been proved to be true for the investigated route (the hypothesis for a route is 'the route is blocked'). Once the contradiction occurs in a route, the 'explorer' goes back (this is equivalent to a backtrack in the simulation) to the most immediate branch point in the investigated route in order to attempt to reach *City-T* by following an alternative route, e.g., by chosing another branch at that branch point. If the 'explorer' finds a contradiction in all possible routes, e.g., all possible paths are found bo be blocked, then the hypothesis has been proved to be necessary.

Another example is the case study to be presented in Chapter 7. The exploration of a route in the previous example corresponds to a simulation trajectory there. However a simulation trajectory is more complex as it is a cross-product of all parameter- and choice-settings, as was noted in the previous section. The task of the explorer when following a route is simple: to check if that route is blocked. The task of the simulator generating a trajectory is more complex: to generate a logical model, that possible logical model (or possible world) associated with that parameter- and choice-settings. There would be a large amount of detail that might, or might not, be useful for a modeller. In the case to be given in Chapter 7, choices appear in a trader's price-imitating – for this purpose, a trader chooses another trader for comparing prices and sales. In a simulation trajectory, there will be three agents' trajectories with choices (there are three traders) and another three agents' trajectories without choices (there are three distributors). The number of possible simulation trajectories in this case is huge. An exploration of the trajectories would be useful, on the one hand, to prove a tendency, and, on the other hand, to inform about detail in a trajectory. However, there would usually be a trade-off between the richness of the study in terms of detail explored in single trajectories (sometimes related to how fine-grained the simulation model is) and the power and the number of trajectories explored – also associated with the limited computational resources. To explore a single trajectory in detail would help to explain why agents have made particular decisions and then, probably, to understand a tendency better, while an exploration involving more trajectories would help in proving it, i.e., in making conclusions more general.

The proof-based approach to be proposed in this chapter ihs similar to tableaux and different from OTTER in that it is logical model-oriented (see Capter 3). It is similar to OTTER and different from tableaux because it is clausal ordered, but it uses an order criterion different from OTTER's. It is similar to tableaux and a version of OTTER with splitting, as it searches each branch, explicitly backtracking once a contradiction happens (i.e., when the negative constraint is violated).

5.6 Modeller Beliefs and Proving a Tendency

5.6.1 A Review of the Concepts of Explaining, Understanding, and Proving

John Casti (Casti, 1992) considers the concept of *explanation* and compares it with the concept of *description*. For him, an explanation answers to the question *'why?'*, while a description answers to the question *'what is?'*. For instance, if a subject perceives a light, a description of the light might involve an account of its intensity, brightness, and colour; while an explanation might specify some of the reasons for the light's being perceived as it is, so that it might answer the question why it 'has' that specific colour, brightness, and intensity that the subject observes.

For Casti, 'an explanation involves giving an account of an already known fact on the bases of logical conclusions drawn from well-established general theories' (Casti, 1992, p. 387). He recognizes a hierarchical structure of perceived facts, where what is explained is at a higher level than the components of the explanation.

He affirms that giving a causal description for a process and providing a formal explanation of it are synonymous. From his position, the concept of description is wider than the concept of explanation.

The term 'to explain' is defined in Merriam-Webster's (http://www.m-w.com/home.htm) as:

> 1 a: to make known b: to make plain or understandable <footnotes that explain the terms>
> 2: to give the reason for or cause of
> 3: to show the logical development or relationships of

Thus, we see that an explanation is a sort of description, but a description where reasons supposed to be better known than what they try to explain, either more intuitively or based on other reasons, are given. Here, an important notion is stressed: that an explanation gives *meaning* to the explained phenomena. This meaning is supposed to be offered with respect to a cognitive model or to a logical theory.

Accordingly, an explanation usually comes as a justification in terms of known facts, which in turn are justified in terms of other facts, and so on. Ultimatelly, all facts are justified in terms of some 'primary' facts usually called 'principles' or *axioms*.

Consequently, an explanation of a tendency in a simulation should come in terms of the simulation principles or axioms, e.g., in terms of the simulation design. Ultimately, it should be given in terms of the design language. On the other hand, an explanation in a subject's cognitive model would be given in terms of elements of its language – facts at the pre-lowest level would be justified in terms of his more basic beliefs or intuitions.

Let us now turn to the concept of understanding. To understand is to know. There is implicit the idea of a subject as the know-er - as that agent who realises and finds the meaning of some phenomenon in terms of its internal language. There seems to exist a strong link between the notions of explanation and understanding. However, not all explanations of a phenomenon can be considered as part of a subject's understanding, but only those causal descriptions offering meaning to the observed phenomenon in terms of elements of a subject's cognitive model. On the other hand, not any understanding comes from an explanation. For instance, something could be understood using those principles in the lowest level of understanding - what in a subject's cognitive model would be the subject's intuitions and facts.

The *Oxford Advanced Learner's Dictionary* offers the following definition for the term 'to understand':

> 1(a): to know the meaning of words, a language, a person's character, etc.
> (b): to perceive the meaning or importance of something, to perceive the explanation for or cause of something.
> 2 to have a sympathetic idea of awareness of somebody/something
> 3 (a) to be aware from information received that
> (b) to assume that something is the case; to take something for granted
> 4 to supply or insert an omitted word or phrase mentally

In this definition, again, understanding is closely related to a subject's given meaning to something via its cognitive model. Once more, meaning could be based on either an explanation (e.g., definition 2(b)), or on intuitions and faith (e.g., definition 3(b)).

Summing up, in this thesis an explanation will be understood as a causal description offering the subject reasons and probably also givin sense and meaning to certain phenomena perceived in an object (in terms of using components from a theory or from a subject's beliefs). On the other hand, understanding will be conceived as the meaning the subject has for something in terms of its cognitive model. This meaning might come from an explanation (corresponding with a theorem when it is an explanation based on a logical model) or from an intuition (corresponding with an axiom in a logical model).

Finally, let us review the notion of proof and proving. The Merriam-Webster's dictionary defines 'to prove' as:

> 1 archaic: to learn or find out by experience
> 2 a: to test the truth, validity, or genuineness of <the exception proves the rule> <prove a will at probate> b: to test the worth or quality of; specifically: to compare against a standard -- sometimes used with up or out c: to check the correctness of (as an arithmetic result)
> 3 a: to establish the existence, truth, or validity of (as by evidence or logic) <prove a theorem> <the charges were never proved in court> b: to demonstrate as having a particular quality or

worth <the vaccine has been proven effective after years of tests> <proved herself a great actress>
4: to show (oneself) to be worthy or capable <eager to prove myself in the new job>

Alternatively, the *Oxfords Advanced Learner's Dictionary* says:

1 To show that something is true or certain by means of facts or evidence.

As is seen, the idea of proving is related to notions such as genuineness, trueness, and accurateness. Refereing again to our previus example of observing a light, it might be of interest to prove that the light has the quality an observer affirms he perceives, e.g., a certain degree of brightness, or specifications of colour. A proof has similarities with an explanation, as they both consists of some sorts of causal descriptions, but is different in that it is a causal description that pretends to show trueness and intends to be more objective as it is based on specific theories and methods (e.g., a logical theory) rather than on any theory or on a modeller's cognitive model.

Procedures can be used to implement a proof in accordance with what has to be proved (the nature of the phenomenon) and by the means we have for elaborating the proof (e.g., a computer, a mathematical model, a physical laboratory). In this thesis, it is of interest to prove a theorem defined as a tendency in the dynamics of a simulation in terms of the theory content in the simulation model. In this case, proof procedures are derived from logical theories.

The idea would be to show that, under the theoretical content in a simulation model a certain fact is true. That is, it would consist in showing that the theorem is a fact in the simulation model theory. This might be ahieved via an appropriately justified (as a proof) causal description.

A subject might take a proof procedure as an explanation if the subject can explain the tendency giving similar reasons to those involved in the proof. In this case, the subject is supposed to understand the proof procedure, in the sense that the subject is able to elaborate an explanation similar to the proof procedure and based on his cognitive model. It is important to remark that, in general, a proof procedure is only a means for achieving a proof. The important point is simply to show that the theorem is true (the proof) rather than the proof procedure itself.

A more precise definition of the notion of proof to be managed in this thesis is given in Chapter 3 (see particularly section 3.2). There a proof is defined as a computational procedure for verifying the validity or inconsistency of a formula or theorem with respect to a theory.

5.6.2 A Proof of a Tendency in a Simulation Theory and a Subject's Knowledge

In the light of the previous discussion and also of Chapter 3, proving the necessity of a tendency in a simulation informs a modeller that the tendency is a fact in the simulation theory by showing that it appears in all possible simulation trajectories or logical models of simulation model theory; i.e., the tendency is a fact in the theory content in the simulation model (see section 5.5).

On the other hand, a subject understands a tendency if it can give meaning to the tendency, that is, if it finds an explanation or an intuition in his cognitive model giving sense to the occurrence in the simulation.

A computational proof of a tendency might, or might not, help the modeller to understand that tendency. A proof procedure works upon facts on a computational model, while giving meaning to a tendency observed in the simulation on which a modeller's understanding is based works upon facts in the subject's cognitive model.

Proving and understanding can correspond to different modeller's goals and, in general is achieved via different strategies. For example, for a modeller aiming better to understand a tendency, it might be enough to explore a single simulation trajectory. In this case proving the tendency would waste computational resources by needlessly exploring the whole range of trajectories and might provide too much information to help improve the modeller's understanding. Also, the level of detail and the how the information is offered to a modeller plays a role here, as it will be easier for a modeller to find an explanation for a tendency if the simulation offers outputs which the subject can relate to knowledge he already has in his cognitive model (e.g., those coming from his observations from and experience with the empirical system).

5.6.3 Interaction between a Subject's Cognitive Model and a Simulation Model

Part of the structure and the interaction of these two systems are presented in Figure 5.4. The modeller's given cognitive model is in accordance with the discussion of section 8.6, where ideas from Salomon's (1996) interpretation of aspects of Kant's philosophy are presented. Other features of this figure come from the previous discussion about explanation, proofs, and understanding and from the discussion presented in Chapter 4 about emergence of tendencies.

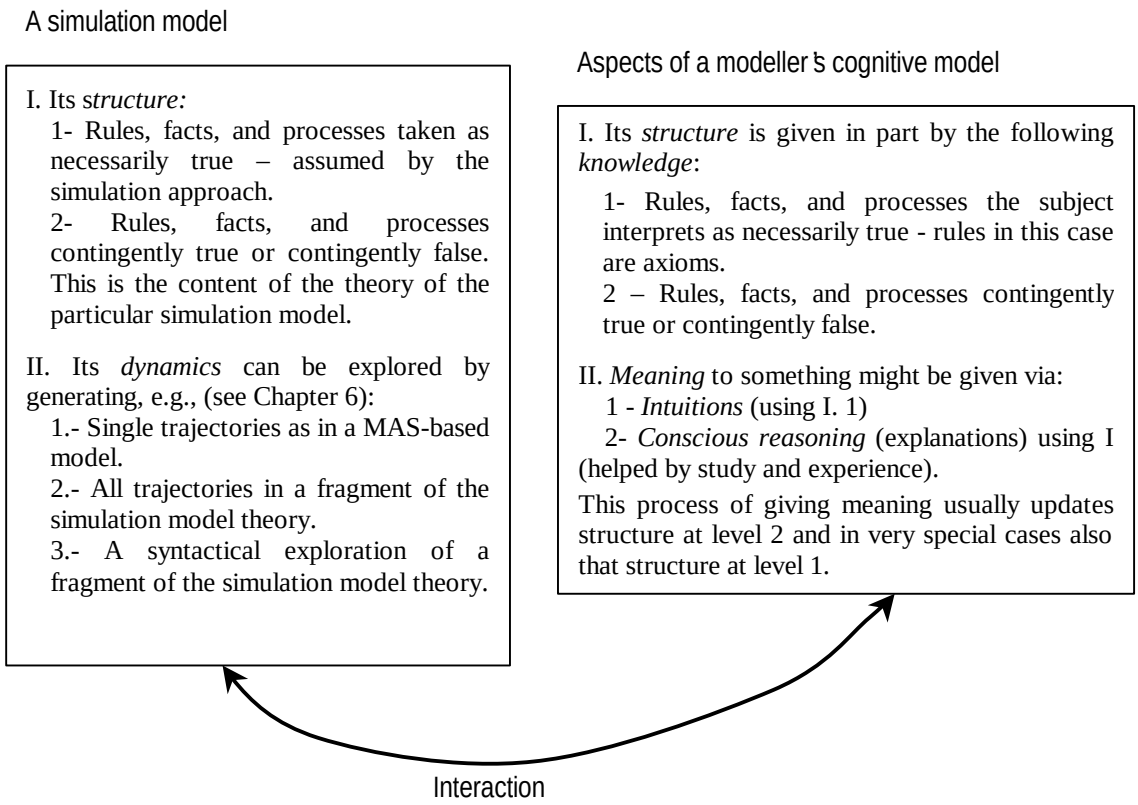

Figure 5.4. Interaction between a subject's cognitive model and a simulation model

5.7 Sources of Constraints: Bounds of the Searched Space of Trajectories

Since emergence of tendencies and the complexity of a system are based on both the objective complexity of the object and the bounded rationality of the subject, the difficulties for computational modelling of a complex system can be understood as having a subjective source (the subject in this case would be the computer, and its mental model would be given by the simulation program): the limited computational resources; and an objective basis: the objective complexity of the simulation model and the nature of the modelled changes (changes in the simulation model might, for example, be qualitative or quantitative; state transitions could be defined in terms of numerical or symbolic manipulation). These difficulties could also be understood as constraints for simulating a complex system. More specifically, we characterise the possible origins of constraints in simulations of complex systems as beloning to one of two groups:

- Those sourced in the *limitations of a modelling device*. In this thesis the modelling device is a computer. Here are the 'subjective' constraints, which, in this case, are *technical constraints.*
    - Limited *computational resources*
    - Manipulation of *real numbers*: making it difficult to evaluate the similarity between paths (i.e., between different simulation trajectories) and to prune the search.
- Those sourced in the *complexity of a simulation model*, and probably ultimately rooted in the nature of the changes a modeller perceives the target system is undergoing.
    - Nature of the modelled change. It is especially difficult to model a system undergoing structural change (see sections 2.9 and 4.3).

5.8 Towards an Efficient Constraint-based Search in MAS: Transformation of MAS into (Logical) Model Constraint-based Models

In order to make possible a theorem-proving procedure about the theorems in the theory of a MAS-based model (see section 5.9), we will attempt to carry out a homomorphic translation (see Chapter 2) from the original MAS into a platform where the alternative trajectories can be unfolded more efficiently. The MAS will be translated along with the range of parameterisations and agents' choices. The platform it is translated into is described in the next section. Each trajectory in the new platform will correspond to a possible trajectory in the original MAS. This transformation seems to be a particular case

of isomorphism, the two systems being equivalent in many respects (see section 5.10.1). The equivalence between the two systems is based on a stronger notion of morphism than the weak specification commonly used in simulation, which is based on the idea of approximation (see Chapter 2). Specifically we translate the original MAS-based model into a simulation model with a single database-rulebase pair (see Figure 5.5).

The translation is useful because it allows the computationally efficient exploration of trajectories, and the achieving of restricted proofs of theorems in a MAS-based model. This translation is practical for dealing with several difficulties for modelling in MAS. Among these difficulties we have, first, the limitation in computational resources and the complexity of the task (usually too many trajectories have to be investigated), and, second, the technical hitches in experimenting using a particular MAS model directly, since data and rules are encapsulated in different abstractions and hierarchies like those of agents and time levels. This encapsulation makes it difficult and sometimes even impractical to experiment directly with simulation models of complex systems in a MAS.

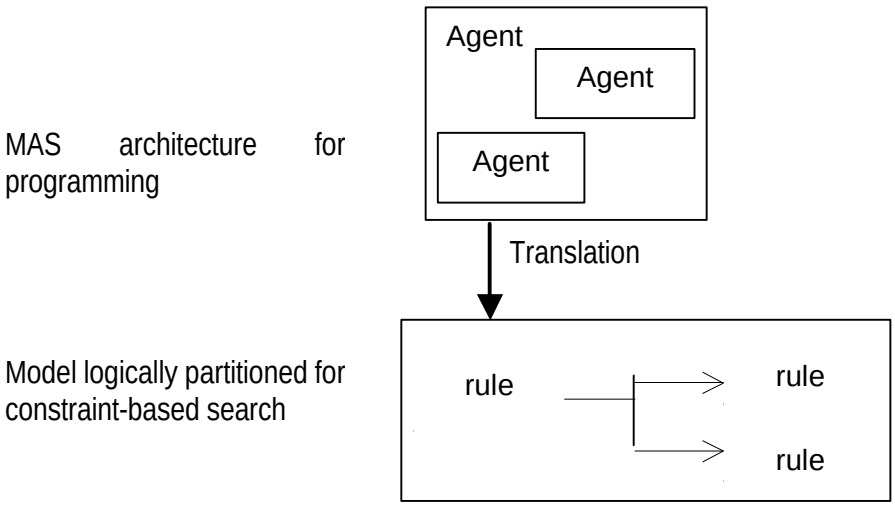

Figure 5.5. Transformation of a MAS into a single database-rulebase pair

This methodology is useful for enabling proofs to be made. Nevertheless, the proof procedure is usually inefficient because aspects of the structure of the rulebases and databases inherited from the MAS inhibit efficient proof, and so further efforts to achieve efficiency might be needed. For such a purpose it might be convenient to take advantage of dependencies between rules. The translation process might already have been useful for

revealing rule dependencies. Afterwards, unhidden dependencies can be unwrapped and then exploited to make the modelling exploration process more efficient.

5.9 Implementation of a Method for a Constraint-based Search of Tendencies in MAS

In this section it will be shown how to achieve the translation specified in the previous section, using the simulation language SDML.

5.9.1 Using SDML and Declarative Programming Paradigm

SDML is not only suitable for the purposes of this thesis, as it is a declarative language for programming MAS-based simulation models, but also because it offers facilities for (logical) model constraint-based search (Moss *et al.*, 1997). Among the advantages it shares with declarative programs, there are the following:

- *Modularity*. Any part of the simulation model is constructed as a group of standardised units (i.e., rules) allowing flexibility and variety in use. The declarative paradigm facilitates a greater level of modularity than the imperative paradigm because the control of the program is separated from the content. This flexibility is useful both when representing the static structure of the system and when generating the dynamics of the simulation. It facilitates the introduction of alternatives for agents' choices and parameters of the simulation model.
- *Expressiveness*. Effective conveyance of meaning is a consequence of the representation of the system as linguistic clauses on a set of databases. It facilitates the interpretation of a set of social phenomena into a simulation by allowing the dual interpretation of clauses as pseudo-linguistic tokens and as entities to be computationally manipulated.
- *Easier analysis*. Context-situated analysis of detailed data, and tracks of trajectories, as well as analysis of groups of trajectories, is much more straightforward than in imperative programs because the resulting databases can be flexibly browsed and queried.
- *The Possibility of Formal Proof*. The data generated by the dynamics of the simulation can be analysed as a logical extension under the particular logic of the simulation language. It opens the possibility of achieving proofs related to the logic of the language and the constraints imposed by the allowed choices of the agents and parameters of the simulation model.

Among the particular advantages SDML offers for implementing a forward-chaining, (logical) model, semantic-driven search that is clausal ordered, and explicitly split, are the following:

- *Good underlying logical properties* of the system. The core of SDML's underlying logic is the Strongly Grounded Autoepistemic Logic (SGAL) described by Kurt Konolige (1995).
- Its *backtracking mechanism* facilitates the exploration of alternative trajectories via the splitting of simulation paths according to the agent's choices and the simulation model's parameters.
- *The efficient forward-chaining assumptions manager* in SDML tracks the use of assumptions. Assumptions result from non-deterministic rules.
- A collection of useful *primitives relevant to social simulation* is available.
- *A meta-agent for automatic translation of rules.* A meta-agent (meta, for the purposes of this presentation) is an agent 'attached' to another agent as a controller; it is able to write rules for that agent. This is used here *not* as an agent *per se* but as a module used to 'compile' rules into an efficient form as well as to monitor and control the overall search process and goals.
- *A mechanism* for an *automatic and static analysis of rule dependencies.*
- *Simple negative contradiction generation* via a false predicate: P => ?.
- *User defined backward-chaining clauses* useful as demodulators.

5.9.2 SDML's Inference Mechanism

This sub-section is intended to clarify how SDML's inference mechanism works, in order to help in better understanding the translation procedure of the MAS-based model into a (logical) model constraint-based architecture.

The SDML inference mechanism is forward-chaining (though it allows backward-chaining rules used to implement auxiliary manipulations needed by other rules - these rules can be conceived as the demodulators that are found in OTTER; see Chapter 3). A simulation is usually implemented in SDML in a way such that the antecedent retrieves instances of data valid for the past simulation time in order to generate data for the present simulation time (and perhaps the future):

antecedents of the rules *instantiate* existing facts in the database =>

new facts are added into the database in accordance with the *consequents* of the rules whose antecedents have successfully matched facts in the database.

The exposition will be guided by examples. They will be used to describe key points such as: what happens when a rule fires, why a rule dependency is created, how SDML knows when a dependency should be (or might be) placed between two rules. Also, the usefulness of the assumption mechanism and of using a meta-agent will be addressed.

The idea is progressively to introduce features of SDML's inference mechanism. In this sense, a latter example will display aspects of this mechanism not exposed in a former example. For facilitating the discussion, we will first depict informally an algorithm very similar to that used in SDML. This allows one to show the main characteristics of SDML's algorithm. Additional features will be shown progressively by the examples. The exposition to be followed will allow us, on the one hand, to show drawbacks of this first algorithm, and, on the other hand, to introduce additional features of SDML's algorithm (as changes made in this first algorithm in order to add efficiency). Among the aspects to be considered later are 'backtracking' and 'rule dependencies'.

First informal approximation to SDML's algorithm

Introducing the forward-chaining mechanism and the idea of tagging assumptions to facts in the database,

(a) For each rule, it is checked whether the antecedent holds;

(b) For a rule whose antecedent holds, all bindings (e.g., all sets of values for its variables that match the database) are found;

(c) For a rule whose antecedent holds, the consequent with respect to each of these bindings is asserted to the database;

(d) If it is possible that the antecedent will be later contradicted by the action of another rule, tags 'are added to the relevant facts in the database in the form of an assumption; and

(e) Once all rules have fired as often as they can, the assumptions are checked for consistency. If no contradiction exists, the simulation of that rulebase successfully finishes.

1st Example.

Assume the following two rules have been written in SDML:

R1: True => P(a); P(b);

R2: P(?x) ^ P(?y) => Q(?x, ?y);

where:

- *P, Q* and *True* are predicate names (*True* permits a rule to fire unconditionally, i.e., it is used to place facts in the database);
- *a* and *b* are constants;
- and *?x* and *?y* are variables

Initially the database is empty, so by step (a) only the first rule fires - its antecedent always holds. According to (b) and (c), the database will contain:

Database: P(a); P(b)

As there is no rule with a consequent that can contradict the antecedent of the rule that has fired, e.g., the predicate *True*, no contradiction is possible. Step (d) does not find it necessary to 'tag' the generated data with an assumption.

Step (e) finds that the database has been updated and there is new data that can make *R2* fire, so that the simulation cannot be stopped. The steps of the algorithm are followed again from the beginning (there is no need for this rule to check assumptions as there are none).

According to (a), SDML checks again the antecedent of the rules using information from the updated database. Rule *R1* does not give new information. Now the antecedent of rule *R2* holds. Then, by steps (b) and (c), and after the corresponding unifications, the database becomes:

Database: P(a); P(b); Q(a,b); Q(a,a); Q(b,b)

For a second time, there does not exist any rule whose consequent (action) could contradict the antecedent of the rule that has fired (*R2),* so no assumption is necessary.

Step (e) finds that there it is no possibility of generating new information - all rules have fired as often as they can. In addition, there is no need for checking consistency of assumptions, as there are none. In consequence, the simulation is finished.

2nd Example. Introducing the dependencies mechanism

Assume the rulebase consists of the rules:

R1: True => P(a) ^ P(b);

R2: True => R(c) ^ R(d);

R3: P(?x) ^ R(?y) => Q(?x, ?y);

Following the algorithm given above, we check the rulebase. The two first fire and the database becomes:

P(a); P(b); R(c); R(d);

In a second application of the algorithm, the system finds that only *R3* gives new information and the database grows to:

P(a); P(b); R(c); R(d);

Q(a, c); Q(a, d); Q(b, c); Q(b, d);

Then the simulation stops as 'all rules have fired as often as they can' and there is no data with a linked assumption tag.

Notice that for each trial of the algorithm, all rules have been checked out and only some of them have fired. There were two applications of the algorithm. In both of them the three rules were checked, but not all of them fired: an the first attempt only *R1* and *R2* fired and at the second attempt only *R3* fired. Because of these failed attempts, the procedure is inefficient.

A more efficient algorithm will be that where rules are checked in some order according to some dependencies between the antecedent of one rule and the consequent of another rule. In this example, data generated in the consequents of *R1* and *R2* are used in the antecedent of *R3*. It is convenient to consider firing rule *R3* only after rules *R1* and *R2*. Dependencies exist between these rules and can be explicitly flagged (see Figure 5.6). In general, dependencies are detected between a pair of rules if the consequent of one of them is instantiated by the antecedent of the other one.

After dependencies are added, the algorithm given above can be slightly modified and made more efficient. Now, steps (a)-(d) are applied to each rule in the database following the order of the dependencies, and then, after all rules have been checked in accordance with that order, step (e) inspects the consistency of the assumptions and the end of the simulation.

This new procedure is more efficient than the previous one. It can be better explained below when applying it to an example. First, any of the rules *R1* and *R2* can be checked, as their antecedents do not depend on other rules' consequents. After steps (a)-(d) have been applied to *R1* and to *R2*, the database becomes:

P(a) ^ P(b); P(c) ^ P(d)

Only then is *R3* inspected, e.g., steps ((a)-(d)) are applied to *R3* (this is in accordance with the order given by the dependencies shown in Figure 5.6). This rule fires and the database becomes:

P(a); P(b); P(c); P(d);

Q(a, c); Q(a, d); Q(b, c); Q(b, d);

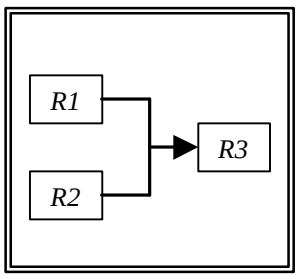

Figure 5.6. Rulebase dependencies for the 2nd example

As we reach the end of the dependencies graph (and all rules have been checked), then step (e) is carried out, and, in conformity with this step, the simulation is finished. Following the new algorithm, each rule is inspected for only once (and the simulation finishes after following the algorithm only one application of the algorithm), while in the previous algorithm each rule needed to be inspected several times – and in this sense rule dependencies add efficiency into the inference engine.

3rd Example. Showing how the assumptions mechanism works and its usefulness

In this case, in order to make the explanation clearer, we will consider three cases.

Case 1: Showing how the predicate 'not inferred' (¬) can be used

Assume the following two rules were written in SDML:

R1: True => P(a) ^ P(b);

R2: P(?x) ^ P(?y) ^ ¬(?x = ?y) => Q(?x, ?y);

where the symbol ¬ means *not inferred*. The proposition *not inferred (?x = ?y)* can be taken as conditionally true if *(?x = ?y)* is not inferred from the database. Should the rule fire under that supposition, an assumption 'tag' will be added to the data placed into the database. SDML tags a datum with an assumption when it can change in the future, as is the case when the predicate not inferred (¬) is used. Another way for generating assumptions (in addition to the use of the predicate ¬) is when the antecedent of a rule can be contradicted by the consequent of another rule.

The main stages in the simulation of this example would be (in this case the algorithm will not be explicitly referenced (as above), but its basic steps are followed)*:*

1.- SDML's inference engine partitions the space of rules and builds the rulebase (see Figure 5.7). In the example, the consequent of rule *R1* is referenced in the antecedent of rule *R2* (by P(a) and by P(b)). Then a dependency is placed between these rules.

2.- After creating the rulebase with the (static) rule dependencies, the simulation

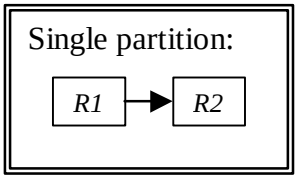

Figure 5.7. Rulebase dependencies for the 3r example (Case 1)

begins. Rules are checked for firing in accordance to the generated dependencies and rulebase partitioning. First, the rule *R1: True => P(a) ^ P(b)* is considered. As it fires, the rulebase will consist of:

P(a); P(b)

3.- After firing *R1* and updating the database, the inference engine passes on to check the rule *R2*: *P(?x)* ^ *P(?y)* ^ ¬*(?x = ?y)* => *Q(?x, ?y)*. The first part of the rule fires as the predicates *P(a)* and *P(b)* are already in the database. In addition, as the constants *a* and *b* can be assumed different (there is no information in the database contradicting this), the remaining part of the antecedent of *R2*, e.g., ¬*(?x = ?y)*, can be taken as true under this assumption. Then the rulebase and *assumption set* (which until this point was empty) are updated (see Figure 5.8).

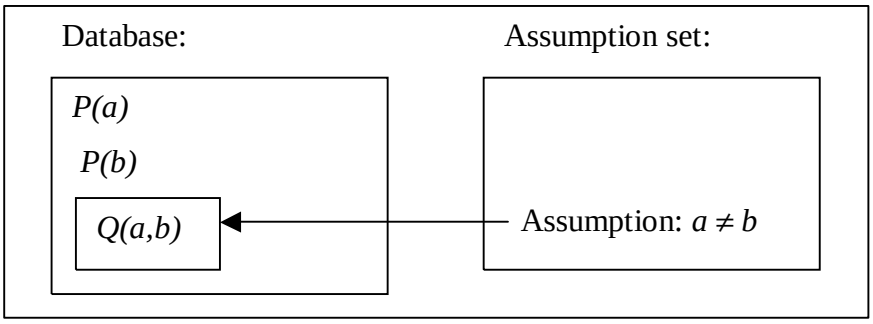

Figure 5.8. Database for the 3[r] example (*Case 1*)

4.- As 'all rules have fired as often as they can' and the assumptions are consistent, the simulation stops. If there were a rule generating data contradicting the assumption made in the previous step, then the associated data (e.g., *Q(a,b))* would be discarded. In such a case, the simulation would backtrack to the point where the rule generating the assumption (*R2)* was fired, and the simulation would be restarted from that point on.

Case 2: showing how the 'no inference' mechanism can be used to generate different logical models for the rulebase

Consider the rules:

R1: ¬*Rainy => Sunny*

R2: ¬*Sunny => Rainy*

The graph of dependencies is given in Figure 5.9. This set of rules (*R1* and *R2*) forms a (logical) '*partition'* as there is a cycle in the dependencies. The antecedent of a rule placed late in the dependencies graph (*R2* in this case) depends on the consequent of a rule preceding it in that graph. This cycle defines a sort of iteration when firing the rules. It is not only rule *R1* that can generates data yielding new firings of rule *R2*, but the reverse can also happens.

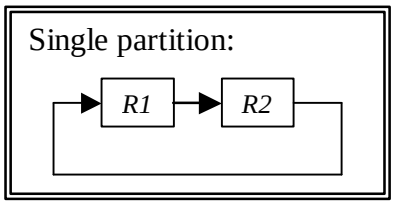

Figure 5.9. Rulebase dependencies for the 3^r example (*Case 2*)

Assume *R1* is considered first by the inference mechanism; then the data:
Sunny under the assumption: ¬*Rainy*
would be generated. However, if *R2* is considered first, the database would consist of:
Rainy under the assumption ¬*Sunny*

We would say that there are two possible (or allowed) worlds (*logical models*) for this rulebase, as there is no additional information for conceding one of them and discarding the other. Some inference mechanisms would accept one of these logical models (using some criterion) while others would accept both of them. SDML's inference mechanism would arbitrarily choose between the two rules and then would accept that logical model generated by the chosen rule. However, should a contradiction arise in the explored logical model, SDML would attempt to find and alternative logical model by backtracking to the point where the assumption was generated and making another choice (in this case there would be only one left). This is the sort of mechanism that we will use in the example to be given in Chapter 7 for investigating all traders' choices for price-imitating. These aspects of SDML's inference mechanism are better explained below in Case 3.

Case 3: Showing how SDML's backtracking mechanism works
Assume, the rulebase consists of:
R1: ¬*Rainy* => *Sunny*
R2: ¬*Sunny* => *Rainy*
R3: Sunny => ^
where the symbol ^ stands for *contradiction* (or, for *false).*

The dependencies graph is shown in Figure 5.10. According to this graph, rules *R1* and *R2* fire before rule *R3.* Assume *R1* is chosen first, then the database will consists of:
Sunny under the assumption: ¬*Rainy*

This allows *R3* to fire and generate the contradiction: ⊥. This contradiction will be found by step (e) of the algorithm given above. There (in that algorithm), it was not specified what the program has to do if a contradiction is found. To include this possibility,

the step can be rewritten (it will allow the algorithm to consider backtracking when a contradiction is found, as is the case in SDML):

(e) Once all rules have fired as often as they can, the assumptions are checked for consistency. *Should* no contradiction exist, the simulation successfully finishes;

otherwise, the system backtracks to the point where the assumption generating the contradiction was generated and attempts another alternative at that simulation branch point (then another simulation trajectory is set off).

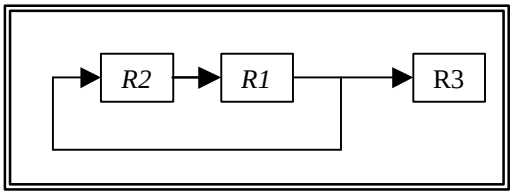

Figure 5.10. Rulebase dependencies for the 3^r example (*Case 3*)

In consequence, as a contradiction appears, the system backtracks. The alternative the system finds is to fire *R2*. Then the database will become:

Rainy under the assumption ¬*Sunny*

In this case the assumption prevents *R1* firing. As a consequence, the predicate *Sunny* will not be in the database and this prevents *R3* firing. At this point where rules have fired as often as they can, then step (e) of the algorithm (rewritten above) checks consistency. As, in addition, there is neither inconsistency among the assumptions nor has the *false (^)* predicate been generated, the simulation is stopped. Finally, the logical model generated has the single predicate: *Rainy*

4^{th} Example. Introducing the meta modules (an agent in SDML) as a module that can use the rulebase of other modules (agents) as its database

Assume that the following rule is in the rulebase of the module *meta*, which is part of the module *prover*, and so can write rules into the rulebase of this module (i.e., into the module *prover*'s rulebase):

```
R1:
{
{  (antecedent of the rule)
clauseList ?antecedent [true];
clauseList ?consequent1 [P(a)];
clauseList ?consequent2 [P(b)];
clauseList ?consequent and [?consequent1 ?consequent2];
namedInstance ?rule RuleName 'R1-1st example'
}
=>
{ (consequent of the rule)
rule ?rule ?antecedent ?consequent
}
```

This rule *R1* in *meta* will write in *prover* the rule (named '*R1-1st example'):*

R1-1st example: True => P(a) ^ P(b);

In the following, it will be explained how this rule works.

There (in *R1)* the predicate *clauseList* is used to build clauses in SDML (the clause created might consist of only one predicate). The first element is the name of the variable where the created clause is going to be kept. The second element is the name of the predicate. The third one, if given, indicates the operator to be used for concatenating the clauses to be used to create the new clause. These clauses to be used to create the new clause are listed as the fourth component of this predicate. The example given by its application in rule *R1* will make its use clearer:

In *R1,* first the predicate *clauseList* applied in the clause:

clauseList ?antecedent [true]

assigns to the variable *?antecedent* the predicate *true.* This will be used as the antecedent of a rule to be created.

Then the same predicate is used to add to the variables ?*consequent1* and *?consequent2*, the respective predicates *P(a)* and *P(b).*

Afterwards the same predicate is used to conjunct the content of these two variables into a new clause; the variable is called *?consequent* and the result would be *P(a) ^ P(b)*. This result will be used as the consequent of the rule to be created (*R1-1st example)* by *R1*.

Then, an instance of rule is created using the SDML's predicate *namedInstance*. There the following elements appear:

- *?rule*: the name of the variable the module uses to identify the created rule.

- *RuleName*: the name of the standard object used in SDML to identify the type of data 'rule'.

- *'R1-1st example'*: the name given to the rule to be created. It will be the name the rule will have when written in the rulebase of prover.

Bringing the strands together

All these features of the inference mechanism that SDML presents are shown in Figure 5.11, where can be seen on the left side the controllers of the whole system: the inference engine, which gives the core control of the system and is a built-in mechanism the usercan not modify; and a meta module (programmable as a meta-agent), which the user can employ to write rules after the simulation has started. These two parts are responsible for building (in part) and partitioning the rulebase, carrying on the simulation (by using the rulebase), backtracking and assumption manipulation, and managing the database. In the right part of Figure 5.11, we can see the rulebase written by the user and by the meta-agent (programmed by the user), as well as the database and the assumption set. The database contains the facts that are true when the simulation is going on. The rulebase is partitioned in accordance with rule dependencies.

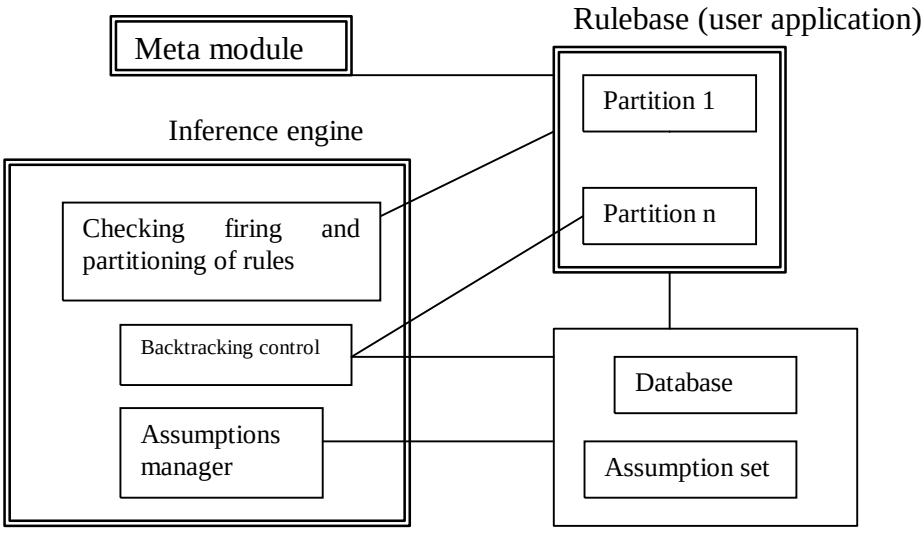

Figure 5.11. SDML's inference mechanism

Assumptions consist of indexes placed to data conditionally valid, e.g., links specifying the conditions under which data is contingently valid. Thus if an assumption becomes false while the simulation is going on, data supported by the assumption is discarded and the simulation backtracks to the point where the assumption was generated.

5.9.3 An Efficient Translation of a MAS-based Model into a Single Database-Rulebase (DB-RB) Pair

We intend to transform a MAS-based model into a model with a single DB-RB pair. This can be seen as a sort of 'compilation' process, which undoes the agent encapsulation allowing a more efficient exploration of the total system behaviour. The technique was applied to a simulation model originally built in SDML and is intended to be an example of proving tendencies in a MAS-based model, and particularly an illustration of the translation procedure. This translation will be accomplished in two steps. First, hidden rule dependencies will be revealed, after which, in order to deal with a certain problem to be explained below, a second step is carried out, namely an unwrapping of dependencies.

5.9.3.1 First Step. Revealing Dependencies

The dependencies in a MAS built in SDML are hidden in the hierarchies of agents (each agent has its own DB-RB), modules, and time levels (see upper left side of Figure 5.12). First, the hierarchy of agents consists in sub-agents contained in other agents. Sub-agents' rules for a certain time level fire after the rules of its container-agent, updating data for the same time level. This gives an order to how rules are fired and, hence, some dependencies between container's rules on the one hand and sub-agents' rules on the other. Second, time levels give an additional order among rules. For instance, if there were the time levels *month* and *week*, rules of an agent in time level *month* would fire before rules of the same agent in time level *week*. In addition, rules in time level *week* would fire several times (e.g., for each simulated *week* in a *month*) to update data valid for a *week* before rules updating data for a *month* fire again (this can be seen in the predicate and rule shown in the left side of Figure 5.12).

For the first task, i.e., to reveal hidden dependencies, we will translate the MAS into a single DB-RB pair. For a signature (namely each rule, each type of predicate, or function) and each instance of data generated during the simulation in the MAS model, there will be an equivalent signature and data instance in the new model (see the illustration in the right side of Figure 5.12). Nevertheless, the structure and dimension of predicate and function constants in the new (simulation) model are expanded as fields are added in order to explicitly refer to agents, objects, and time levels that were previously implicit in the MAS structure. This is shown in Figure 5.12. When eliminating the hierarchies of agents and time levels in SDML, the agent where the datum was placed in the MAS model and the time level the datum is valid for, which are implicit data in the MAS model, have to be referenced explicitly. So, for instance, predicate price in the MAS model *Price(?amount*)

becomes, in the new simulation model, *Price(?amount,?Trader,?iteration)*. The last two fields are added to reference explicitly to the agent and the time level.

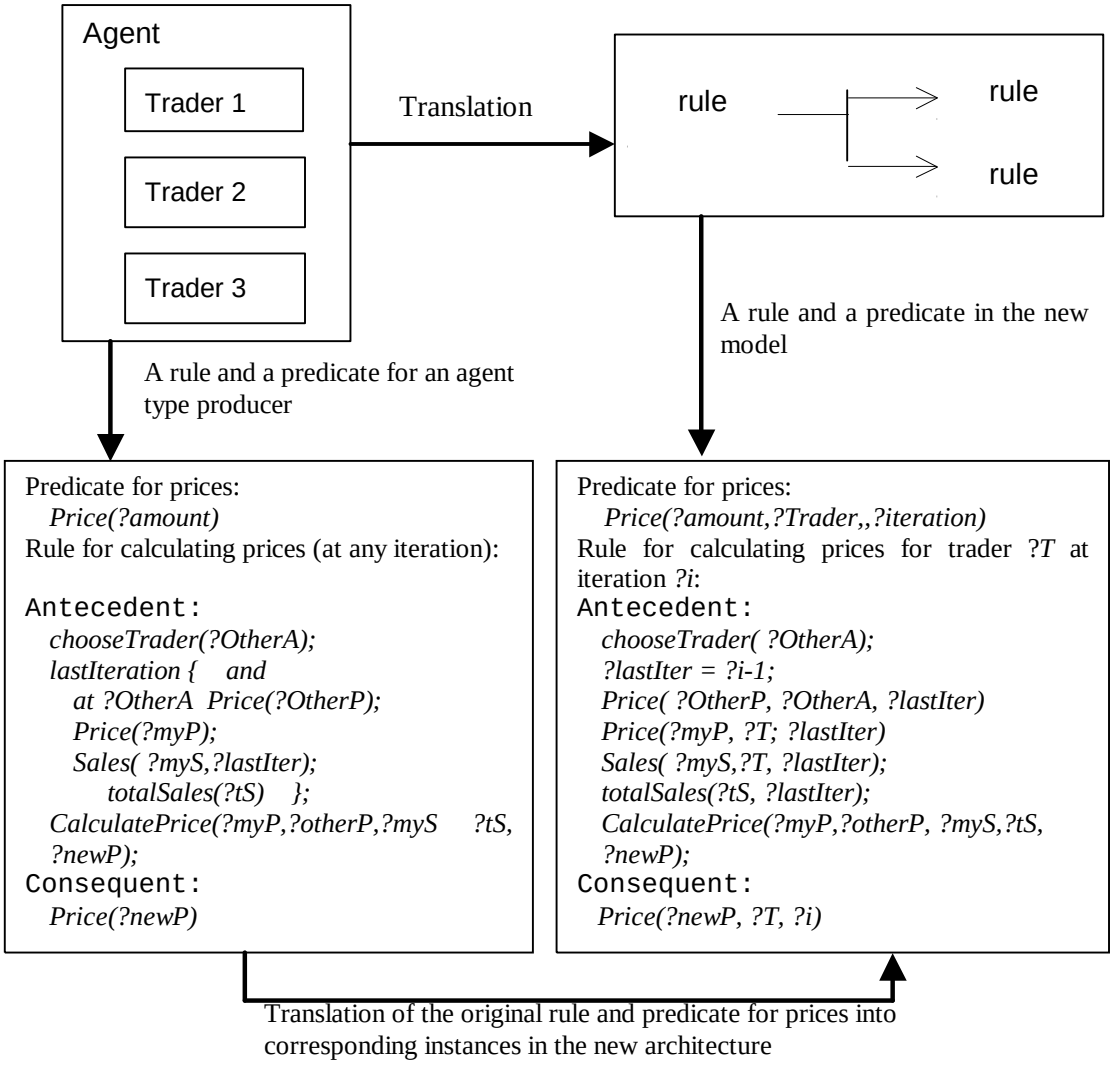

Figure 5.12. Illustrating how agents and time levels become explicit in the new architecture

For the first task, i.e., to reveal hidden dependencies, we will translate the MAS into a single DB-RB pair. For a signature (namely each rule, each type of predicate, or function) and each instance of data generated during the simulation in the MAS model, there will be an equivalent signature and data instance in the new model (see the illustration in the right side of Figure 5.12). Nevertheless, the structure and dimension of predicate and function constants in the new (simulation) model are expanded as fields are added in order to explicitly refer to agents, objects, and time levels that were previously implicit in the MAS structure. This is shown in Figure 5.12. When eliminating the hierarchies of agents and time levels in SDML, the agent where the datum was placed in the MAS model and the

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After the rule in the two implementations is illustrated in Figure 5.12, in Figure 5.13 the aspects that make iterative the rule in the MAS system (shown in the left side of Figure 5.12), as well as the origin of rule dependencies, are exhibited. Notice that a single rule is used; i.e., it is applied at all transitions. Also, observe that references to instances of agents and iterations are implicitly alluded to, and, in some sense, 'hidden'.

The sources of dependencies (the named data now given explicitly) are shown in **Figure 5.14**. There, it is assumed a rule for calculating prices for *agent ?T* at *iteration-i* in the MAS model has been transformed into a rule where the involved agents *?T (the rule's owner),* ?*otherA* (an agent chosen by *?T*), and the iteration at which the rule is applied (data from *iteration-(i-1)* is taken to generate data at *iteration-i)* are referenced explicitly.

As a result of making data in the predicates explicit, some new dependencies between rules previously 'screened off' from each other can be statically recognised.

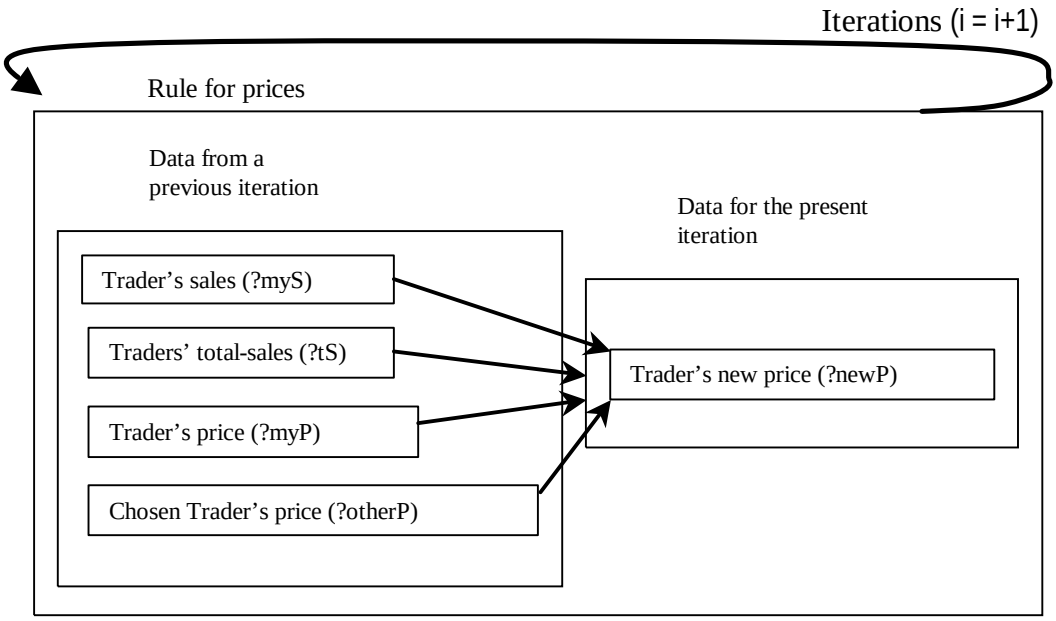


Figure 5.13. Showing origin of rule dependencies for the rule for prices. Dependencies are due *only* to the iterative character of the rule

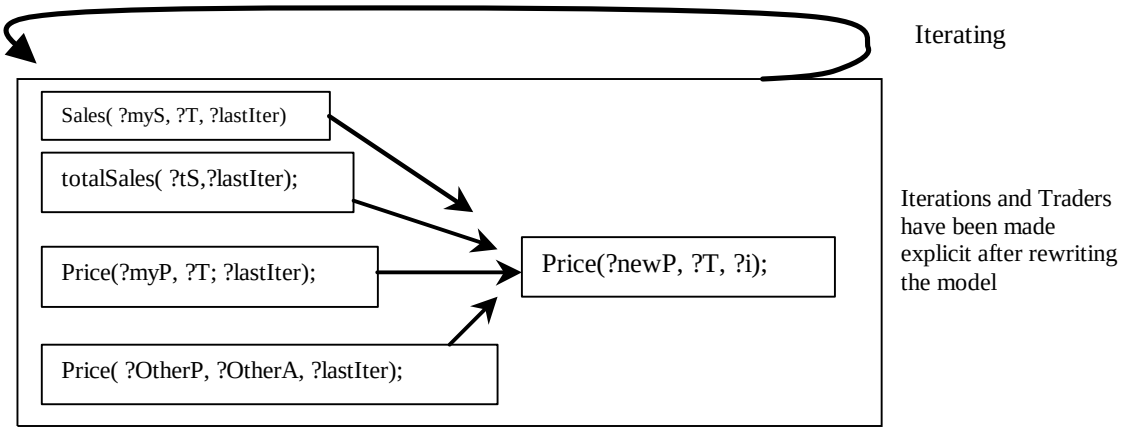


Figure 5.14. Revealing dependencies, e.g., agent *?T* price-setting at time *?i* in accordance with the rule in the right side of Figure 5.12 (*?T* and *?lastIter* create new dependencies)

5.9.3.2 A Problem Appears: The Growth of the Space of Searched Data

Nonetheless, a difficulty appears after revealing dependencies: the space of data a rule searches to check rule-firing grows linearly with the number of iterations, even though most of the attempts are failed. This drawback seriously slows down the simulation over time.

The origin of this drawback can be explained using the example for revealing dependencies given above. Note that it is intended to use the same rule at any iteration, as was the case in the original MAS. At *iteration-1 (e.g., i = 1),* the initial data is given, then at *iteration-2* rules use data written for the single *iteration-1* and generate data for *iteration-2* (see Figure 5.15). There only the references of the predicate *price* to *agents* and to *time iterations* are shown, indicated only by numbers (e.g., *price(2,1)* references price of *agent-2 (e.g., Trader-2)* at *iteration 1*). After this, the antecedent of the rule matches data for *iteration-i, i=1,2* but it can only generate new data for *i = 2.* Similarly, once data for *iteration-i, i = 1, 2, …k,* has been generated, the antecedent of the rule matches instances of data for all these values of *i,* but it can produce new data only when *i = k.* As the simulation time goes on, the simulation slows down because of the discrimination the program has to carry out among the (linearly) growing amount of data matching the antecedent. In addition, note that as the same rule is valid for any agent of type *A1,* the program has to discriminate among agents. This is another factor that can be improved where agents are explicitly referenced (in the example it is assumed there are three agents of type *A1,* and consequently there will be three instances of price). In order to deal with these drawbacks, rules are 'unwrapped'.

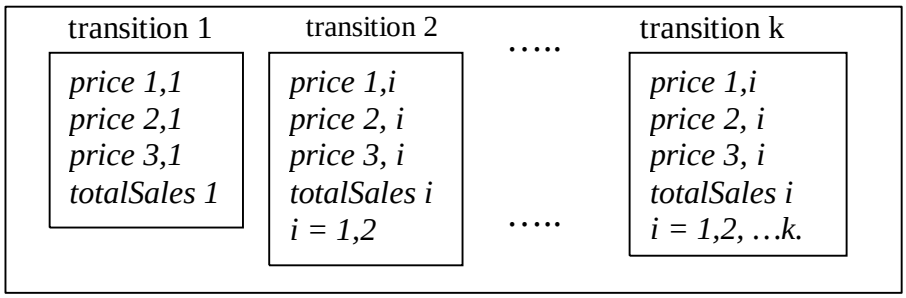


Figure 5.15. The *growth* of the space of searched data. Notation: *price i, j* denotes the price of Trader *i* at iteration *j*

5.9.3.3 Dealing with this Difficulty: Unwrapping the Rules

Rules will be split for time iteration and agent. Figure 5.16 shows the splitting of the rule for prices for the case presented in Chapter 7. When possible, there will be a rule for time iteration and agent, in order to make the data to be instantiated by the antecedent of the rule as explicit as possible (see Figure 5.17). Each rule will have an explicit reference to data given at a previous, or in the present, iteration. For example, in Figure 5.17, where one of the references to price is *Price(?myP, T-2; (i-1),* there we can see the explicit reference to a specific agent, *T-2,* and to a specific time iteration, *(i-1).* As can be seen, the number of rules can by huge as now they are specified by iteration and by trader (and probably by trader's choice). An automatic way of doing this splitting would be helpful.

This process of splitting needs a semantic manipulation of rules allowing the reference of data to occur more explicitly. The idea is to instantiate data already set up in the database or whose introduction during the simulation can be foreseen. Parameters of the simulation model are an example of data given at the beginning of the simulation. Choices, like an agent's selection of another agent for price imitation, or decisions can also be predicted. This was the case in the applications.

Compilation of rules can be implemented in SDML using a *meta agent*, or a *meta module*. SDML's meta module was used in the case given in Chapter 7 only at the beginning of the simulation. In other applications, its usefulness would be furthered if it were employed to write rules while the simulation is going on. It might be used, for instance in an iteration, for writing transition rules for a next iteration. It might be helpful for driving the search conveniently, for example, to choose rules to guide the search according to certain criteria (e.g., as in OTTER when using the weithing criterion for selecting clasues from the 'set of usable'). It would also help if it of interest to elaborate

the theorem to be proved during the simulation (e.g., a theorem defined in terms of an envelope to a certain simulation output).

Original dependencies graph for the sales and price rules

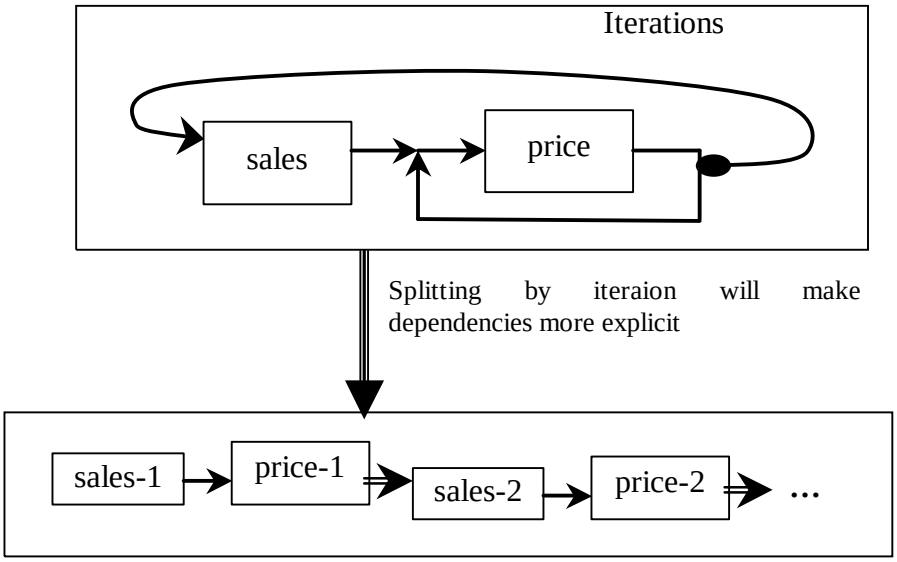
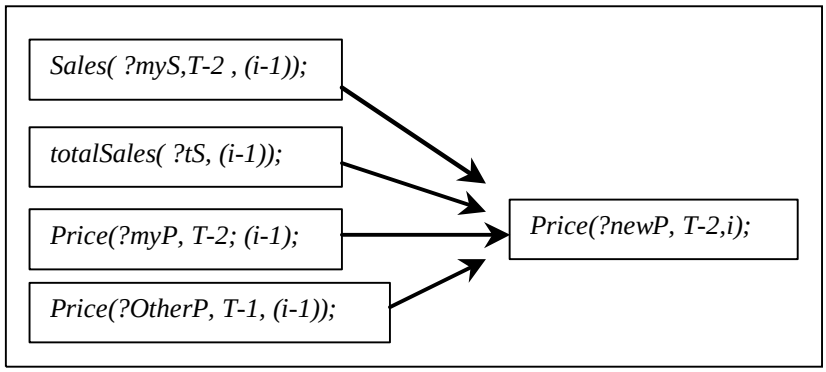


Figure 5.16. Splitting of rule for prices

One rule is written for each instance of prices. In the illustration, at time *i* (in the current trajectory) Trader *T-2* (the trader the rule is written for) chooses Trader *T-1* for price-imitating. Changing the trader the rule is written for (e.g., *T-2*), or the chosen trader (*T-1*), will correspond to another rule



Instantiated data:

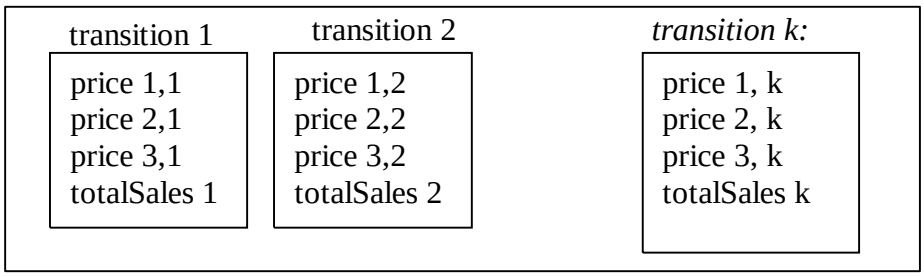


Figure 5.17. Above, 'unwrapping' of dependencies is shown. Below, the data-space searched by the rule-setting for a trader is illustrated

5.9.4 Overview of the System

In the light of the discussion so far, it seems convenient to distinguish and to model as distinct entities three basic elements of a simulation: the *static structure* of the simulation model, that part where parameters and initial data are given; the *dynamics* of the simulation where transition rules are fired; and a *meta module* responsible for writing transition rules as explained above. Thus, each one is programmed in a different module, as follows:

model, sets up the structure of the simulation model, that is, it gives the environment of the simulation: range of parameters, initialisations, alternative choices, and basic (backward-chaining) rules for calculations.

prover, generates the dynamics of the simulation. This is a sub-module of *model* (i.e., it is contained in *model*). This will basically contain the transition rules, auxiliary rules for generating pre-processing required data, and the conditions to test the necessity of the theorem. All of them are rules to be executed while the simulation is going on.

meta, is responsible for controlling the dynamics of the simulation. Its meta-rules write the transition rules and the theorem in (as well as others required by) the module *prover.* A picture of the system is given in Figure 5.18.

Program dynamics:

Modules are executed in the following sequence (see Figure 5.18):

1. *model: initialising* the environment for the proof (setting parameters, etc.).
2. *meta:* creating and *placing the transition rules* in *prover*, using data given by *model* to explicitly reference instances.
3. *prover*: carrying on the simulation by *executing the transition rules* and backtracking while a contradiction is not found.

The program *backtracks* from a path once the conditions for the theorem are verified, after which a new path with different choices and/or parameters is picked up.

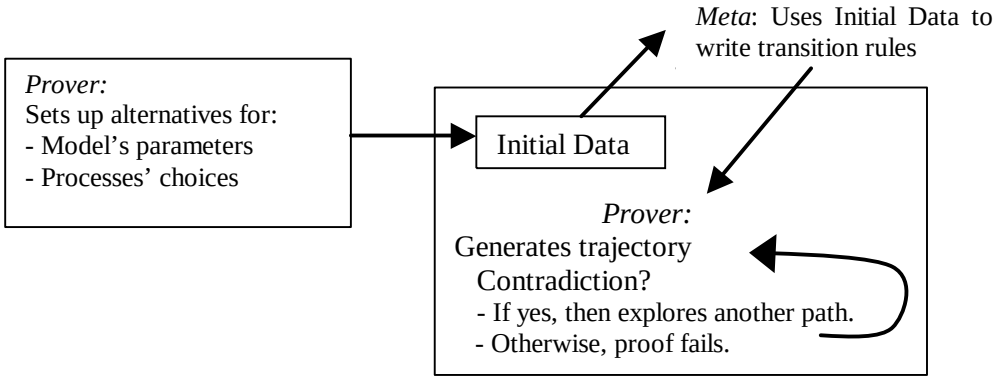


Figure 5.18. Overview of the efficient implementation

5.9.5 Speeding up of the Simulation

Comparing the original MAS-based simulation and the constraint-based translation, a *speeding up* of a factor *O(N)*, where *N* is the average number of agents instantiated by a rule, is gained in the later implemententation,. Notice that all these values are only estimations because it is assumed rules give up trying to fire after checking all their clauses, since in fact a program stops trying to fire a rule as soon as one of its clauses becomes false. For more details about these estimations, see Appendix 5. Advantages of this technique for exploring the simulation space of possibilities, such as making the backtracking more specific and the space of updated data between backtrackings as small as possible, are also explained in Appendix 5.

There are additional benefits associated with the more explicit exposition of the data, as given in a database and accessible from a meta-module. These may allow the theorem to be established over a range of theorems as a result of the adaptation of the original specification to the results of the simulation.

5.10 Morphism and Valid Translation of Simulation Models

5.10.1 Translating a MAS-based Model into a (Logical) Model Constraint-based Model

The translation from a MAS-based model $S = <T, X, W, Q, Y, d, l>$ to a (logical) model constraint-based model $S' = <T', X', W', Q', Y', d, l'>$ seems to be *isomorphic*, though the modelling platforms might be different. For example, in the case study presented in Chapter 7, the MAS-based model *S* is implemented in SDML and the (logical) constraint-based model *S'* is written in two platforms: in OTTER and in SDML. The comparison is more direct if the two models are written in the same platform. If *S'* is written in OTTER and *S* in SDML, the underlying logic of the languages is different. On the one hand, SDML follows a sort of non-monotonic logic, as, for instance, it accepts not only two-valued predicates, e.g., predicates that cannot only be two-valued as *true* or *false* but which can also be assumed as *unknown* (for instance, those preceded by the predicate *not-inferred* (¬)). In addition, in SDML the inference mechanism is driven by a set of rules following and order given by the rule dependencies. On the other hand, OTTER's underlying logic is two-valued; its predicates can only be either *true* or *false*. Besides, in OTTER the search is driven by the set of usable, which, in the case given in Chapter 7, contains the translation of what in the original model *S* (written in SDML) were the rules and the theorem to be

proved. In that case given in Chapter 7, hyperresolution is used to drive the search. Despite all this, however, the generated simulation is very similar because of the following reasons:

- Each rule and each data assertion in *S* has a corresponding rule or data assertion in *S'* (tough predicates in the (logical) model constraint-based platform have additional arguments to make explicit the *agents*, *objects*, and *time levels*).

- Assumptions can be introduced in OTTER by adding an argument to predicates to keep trace of the assumptions (as was the case in the application). The split procedure permits one to create branch points and to choose different alternative values for some variables (e.g., it allows an agent to choose different agents for behaviour imitating) via an explicit search.

- The isomorphism function *h*, as well as the functions *g* and *k* defined in section 2.3.3, would be appropriately defined to filter the differences between the simulation models given by the addition of arguments into predicates of *S'* to make explicit agents and time levels.

- In the simulation a trajectory in *S* has an equivalent trajectory in *S'*. The state transitions are equivalent. For example, a state transition in one of the models has an equivalent state transition in the other model; there should exist a one-to-one translation between the states of the two systems.

- Because of this, any *proof* in *S'* (such as that achieved in the case presented in Chapter 7) is also valid in *S*. In more general terms *S'*, seems to be a valid translation of *S* under any experimental frame $< W, Y, l, V >_E$. The isomorphism would be *valid* under any experimental frame.

Notice that Zeigler was interested in a valid simplification (see Chapter 2) while in this thesis the interest is more in valid translations. In both cases, there should exist a morphism between the two simulation models valid under an experimental frame. However, Zeigler was aiming at finding a simulation model S' simpler than the original simulation model S (it is assumed that a simulation model is a system) in accordance with a certain measure of complexity, while the interest in this thesis is in finding a simulation model S' that is more efficient for proving tendencies than S. This efficiency is measured in terms of the computational resources the simulation model requires for exploring a fragment of the simulation theory.

In the (logical) proposed model constraint-based exploration, the investigated fragment of the theory is defined using a range of parameters and choices. This new simulation

model is more efficient in terms of its potential for proving than the original MAS-based model. In particular, *conclusions* from a (logical) model constraint-based exploration defined by a range of parameters and choices are valid for both simulation models in the experimental frame $< W, Y, I, V>_E$ (for an explanation of this terminology, see section 2.3) where the set V contains the range of constraints (parameters and choices) defining the subspace of explored simulation trajectories. V is the key variable for defining an experimental frame. Notice that V also defines the explored fragment of the simulation theory.

Nevertheless, the new architecture might be less appropriate for purposes other than theorem-proving. For example, an efficient simulation model for proving might give less detail about the simulation. This is not the case for this (logical) model constraint-based architecture as it explores each simulation trajectory. Nevertheless, the information it generates might not be in an easily understood format. For example, the links between data and agents might be subtle and more difficult to understand, as more of the references in the rules have to be managed by the user himself rather than being implicit in the structure of the MAS-based model. Similarly, it might be more difficult to program the output a modeller requires.

Summing up, the simulation model in the (logical) model constraint-based architecture seems to be equivalent (isomorphic) to the MAS-based model. The proofs it offers seem to be valid in both simulation models under the experimental frame and for the fragment of the theory defined by the range of parameters and choices determining the subspace of trajectories to be explored.

5.10.2 Translating Models from an Architecture Offering a (Logical) Model-based Exploration into other Architecture Characterised by a Syntactic-based Exploration

The two architectures of programming described above, namely a MAS-based model and a (logical) model constraint-based architecture, are both logical model-oriented; the dynamics of a trajectory are generated explicitly. Because of this, the needed amount of computational resources is huge. The second architecture adds efficiency to the search and allows restricted proofs of tendencies. Nevertheless, it would be convenient to explore other alternatives, in a search for even more efficient procedures for proving tendencies in a simulation. Syntactic manipulation of rules and constraint logic-programming are areas of knowledge showing potential for assisting in this task. This section suggests the

investigation of methods related to these two approaches and speculates about morphism between a MAS-based model and a hypothetical syntactic constraint-based model (for more about a syntactic constraint-based architecture, see Chapter 6).

One of the factors making the comparison between the above (simulation) models *S* (the MAS-based model) and *S'* (the logical model constraint-based model) less difficult than the present one is that the two architectures offer a logical model-based exploration of the dynamics of a simulation. If we were translating the simulation model into a more syntactic-based exploration (given a simulation model $S'' = \langle T'', X'', W'', Q'', Y'', d', l'' \rangle$), then not all the semantics of the simulation would be explicitly generated and the comparison would not be so direct.

Similar to the translation to a constraint-based logical model search, if the aim is facilitating theorem-proving, the translation should be *homomorphic* under the experimental frame defined by $\langle W, Y, l, V \rangle_E$, where *V* is a set containing the range of parameters and choices for which the dynamics of the simulation are going to be explored. However, the functions *h, g,* and *k* would be more difficult to define. Here, neither do all the semantics of the simulation have to be generated, nor does a one-to-one correspondence between the original simulation model *S* and the new simulation model *S''* necessarily exist. For example, the set of transition states in the simulation models, *W* in the MAS-based model and *W''* in the constraint-based syntactic model, might be different. In constraint logic-programming (see section 3.7), notions different to unification are applied for firing rules. Advance in the search, i.e., a state transition in *W''*, might be due, e.g., to a syntactic manipulation implicitly involving a range of semantic values (perhaps a range of choices or a range of parameters) associated to the state of the system at different time instants (this is different in a semantic search where each possible world or logical model is explicitly generated). This means that a syntactic state transition in *S''* might resume changes corresponding to several state transitions when using only unification and exploring single simulation trajectories (this is the case in a logical model constraint-based architecture and in a MAS-based simulation). A *search in S'' will hopefully be cheaper in computational terms and more efficient for proving than S' and S (for more details about this proposed architecture, see Chapter 6).*

6 Chapter 6 - Transforming MAS to Improve Efficiency of Constraint Logic-programming

6.1 Introduction: A Hierarchy of MAS Architectures

In this chapter a hierarchy of architectures for improving the understanding of MAS-based simulations of complex systems is proposed (this hierarchy is illustrated in Figure 6.1; a discussion about what we mean by a subject's understanding and how it changes as the modeller's experiences with a simulation model was given in section 5.6). This will go further than the constraint-based architecture proposed in Chapter 5. The idea is to provide modellers with several architectures informing them about complementary aspects of the MAS-based simulation. They will each have different modelling purposes. The level of programming, the searched space of trajectories, and the power of the conclusions drawn at each level will be different. In addition, in the following discussion, the advantages of investigating MAS dynamics through such transformations, using a mechanism allowing such a translation automatically, or semi-automatically, and interactively will be highlighted.

The main intention of this hierarchy of architectures is to help a modeller to search, identify, and prove tendencies in a MAS-based simulation (this is represented as the level of single exploration in Figure 6.1). For searching tendencies, a high architectural level is proposed. It is intended to assist the modeller in capturing relevant macro aspects (tendencies) of the simulation phenomena. It allows an active participation of the modellers in order to identify tendencies they are interested in. The idea is to exploit those capabilities that make MAS a special paradigm for generating macro aspects of phenomena at a high level in a way that is easy for the modeller to understand and easy to associate with what he observes in the modelled system. Additional details about the advantages of this architecture are given in section 6.2.

After some tendencies have been identified in a MAS, a more careful (systematic) exploration of trajectories and investigation of tendencies might be implemented using a (logical) model constraint-based search under a relevant range of parameters and choices, such as that proposed in Chapter 5 (this is represented by the intermediate level in Figure 6.1). This search will be complete for certain ranges of parameters and choices. It will help a modeller to do scenario analysis, as the exploration is logical model-oriented (the semantics are shown), as well as to execute proofs over restricted domains. The advantages of this architecture are summarised in section 6.3.

Ideally, we would be able to provide a modeller with an even lower level of programming (see the lowest level in Figure 6.1). That would be a third architecture, proposed in section 6.4, aimed at increasing the potential for proving. This would be a syntactic procedure. At this level proofs for a wider range of parameters and choices and/or for a longer time period (a larger number of time steps) than in the previous level would be possible. The proof procedure will be, on the one hand, subtler from the modeller's point of view but, on the other hand, the results will be more powerful, permitting the modeller to elaborate more general conclusions.

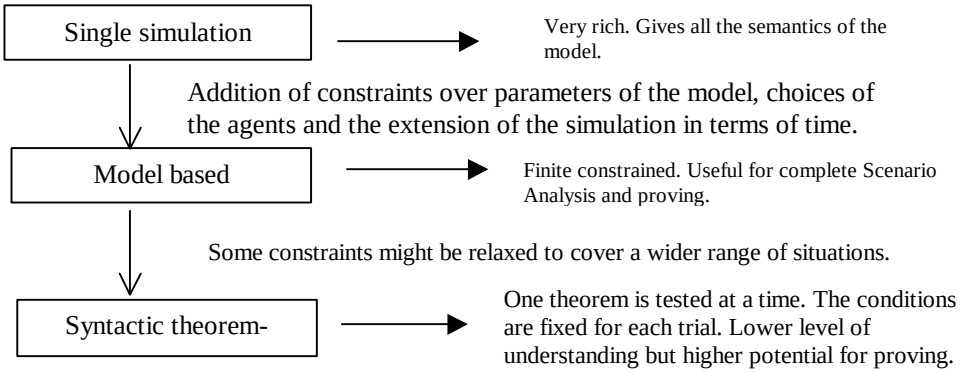


Figure 6.1. Sequence of modelling architectures

The idea is not to move only in one direction in this hierarchy. Going down from a higher level of programming towards a lower one might be useful for proving tendencies already observed in a higher level. Likewise, going up to a higher level of programming will help the modeller understand aspects related to the proofs he has achieved in a lower level.

To close the chapter, in section 6.5, different aspects of a simulation that each architectural level informs about and the complementariness of this information are highlighted.

Examples of applications where architectural transformations would be useful are found in the social simulation community. In Axtell *et al.* (1996), alignment of models is proposed as useful for comparing models and the theories behind the models. Those compared models might represent different architectures. Models in the different architectures might represent aligned models. Moss (1998) compares several MAS modelling approaches in social simulation. It would be useful to have an automatic 'translator' of models in different architectures for assisting a modeller when doing this sort of comparison.

6.2 Programming and Experimenting in MAS to Gain Advantage of High-Level Simulation Paradigm – Higher Architectural Level

The idea here is to execute a MAS-based exploration of simulations in a modeller's search for tendencies by inspecting single MAS trajectories. A MAS offers a high level programming paradigm giving and appropriate environment to observe agents' interactions. A MAS provides facilities to program interacting agents as observed in the empirical system. In advanced MAS, like those that can be written in languages such as SDML, both data and behaviour are conveniently distributed in the hierarchy of agents.

A MAS allows the modeller to trace both the quantitative and qualitative evolution of the agents' interaction. Hopefully, the level of phenomena observable in MAS allows a modeller to find emergent tendencies, some of them resembling emergent behaviour that is also recognizable in the empirical system. A further investigation of an emergent tendency can be done by translating the simulation model into a lower level of programming than a MAS. The idea at this point is to allow a wider exploration of simulation trajectories as well as a broader investigation of fragments of the simulation theory. An example of architecture at a lower level of programming than MAS is that given in Chapter 5 (e.g., the (logical) model constraint-based architecture). The next sections will outline a hierarchy of architectures that includes these two levels and a possible even lower architectural level.

6.3 Translation to Constraint-based Paradigm for Systematic Exploration of Possible Logical Models – Intermediate Architectural Level

For this level an efficient (logical) model constraint-based exploration is proposed. An example of this sort of architecture was given in Chapter 5 using a logical model-based, clausal ordered exploration with a forward-chaining inference procedure, and a case is presented in Chapter 7. The idea is to explore each logical model of a fragment of the simulation theory and prove facts when possible.

In those chapters (5 and 7), the feasibility of this idea and a technique for its implementation are illustrated. Such architecture is a lower level of programming than a MAS. The hierarchy of agents disappeared as the simulation model consisted in a single DB-RD pair. At the MAS programming level, a modeller is better informed about the explored trajectories, as he can more easily follow over time the evolution of the agents' interaction for each trajectory. This new architectural level was intended to allow constrained proofs of tendencies. Hopefully, it will help a modeller to understand better the dynamics of a simulation model (and hence of a target system). At this intermediate second

level of programming, the modeller is less informed about individual trajectories, but he obtains information from a wider space of trajectories than a MAS can explore about a specific theorem. However, information about a fragment of the whole theory embedded in the simulation will be collected. The proposed technique can be seen as falling in between inspecting single runs in a logical model-based exploration of trajectories and syntactic theorem-proving.

The proposed architecture enables a modeller to implement a complete logical model exploration of the dynamics of a simulation for a range of parameters and choices though encoding it in a lower information level than that which a MAS offers. It allows a modeller a different trade-off between two characteristics of the information the architecture provides a modeller: the size of the explored simulation theory and the detail of data in a trajectory. In this sense this architecture is complementary to a MAS.

After experimenting in a constraint-based architecture, a modeller might find it worthywhile to go back again to the higher level of programming again to increase the understanding of proved tendencies, as this gives more detail about the agents' interaction. That is because simulations at the higher MAS level are less complex from the modeller's point of view and are generally more helpful for increasing understanding of macro aspects of a simulation.

6.4 Possible further Translation for Attempting Syntactic Proofs – Lower Architectural Level

Given the difficulties for implementing a logical model-based constraint search (principally that the amount of data generated is huge), it would be convenient to implement an even lower level of programming: one more efficient for implementing proofs (though probably particular details of the search process become less meaningful for a modeller). Specifically, we propose a syntactic constraint-based exploration of the simulation theory.

This level would be similar to the logical model constraint-based architecture in the sense that the exploration would be constrained by the range of parameters and choices. It would be different to this architecture (and to the MAS-based model) because the search would be syntactic-based rather than logical model-oriented. A syntactic method would be based more on the syntactic manipulation of elements of the simulation rather than in the semantics of the simulation. The two methods given above generate explicitly all the semantics of a simulation; each trajectory is explored at a time. The idea at this level would

be to explore the relevant aspects of the simulation theory in a more efficient way. It is believed that constraint logic-programming is an area where helpful ideas could be found (see Chapter 3).

To clarify the idea of efficiency behind the proposal when compared to a logical model-based exploration, consider an example of numerical mathematics. In example 6.1 we will compare a numerical manipulation (a kind of logical model-oriented strategy) with an algebraic manipulation (a sort of syntactic approach).

Example 6.1.

Assume it is intended to prove the proposition *P* under the constraints *c1* and *c2:*

P: X + 100 < Y
c1: X = 1, 2, …, 80; c2: Y = 200, 201, …, 1 000

First, numerical manipulations (following the logical model approach) are used. For proving the proposition, each of all possible values for *(X, Y)* is chosen. In total, there are: 80 * 800 = 64 000 possible combinations (these possibilities are: *(1, 200), (1, 201), …(2,200), (2, 201), …, (80, 200), (80, 201), … (80, 1000)).* After all possible substitutions of the variables have been examined, the proof is done. In this case a lot of manipulations are needed and all possible semantic values of the variables have to be explicitly tested.

Now, assume that algebraic manipulations are allowed in a syntactic attempt to prove *P*. In this case, *algebraic axioms* are used for reasoning. First, the axiom *a1 given below is used*.

a1: for all integers X, Y, Z: X + Y < Z if and only if X < Y – Z

By *a1*, the proposition *P* becomes: P: X < Y – 100

Now suppose we use another axiom, *a2*:

a2:{ if
{(X_{max} = Maximum of the values X can take) and
(Y_{min} = Minimum of the values Y can take)}
Then X < Y – Z if X_{max} < Y_{min} - Z}

Using this axiom the proposition *P* is true if the following proposition *P'* is true:

P': 80 < 200 – 100 = 100

Clearly P' is true (80 < 100), hence also P is true.

This second proof is more efficient than the logical model-oriented one, as the syntactic approach permits one to achieve the verification without the need for generating explicitly all the semantics of the variables. This is the sort of advantage that would be won if using a syntactic oriented architectural level for proving tendencies in a simulation.

In a syntactic exploration of a simulation theory, on the one hand, it might be more difficult for a modeller to find a satisfactory explanation of proved tendencies and other macro aspects of the simulation. On the other hand, however, it might be more powerful by proving in the sense that it would allow the relaxation of bounds imposed over the search, namely the limited range of parameters, choices, and/or time iterations.

It is believed this can be implemented via syntactic backward-chaining inference manipulations. The idea is to prevent exploration of each single logical model and, consequently, the need to lay down explicitly all the semantics of the explored trajectories (which is computationally expensive), and instead to explore a subset of trajectories in a more concise manner (as in example 6.1). The plan is to implement the syntactic manipulation of rules with an implicit introduction of constraints. More concretely, the conditions established by experimenting with logical model-based exploration would need to be added to the MAS specification and all this translated into axioms for a syntactic theorem-prover to work upon. In this sense, less data would be generated and less manipulation necessary than in the two higher levels of programming.

Compared with the two previous methods, this approach would offer a user a different trade-off between the level of detail of information and the scope of the exploration. The user would find it more difficult to understand particular results the method gives, but the conclusions it allows would be valid for a larger fragment of the simulation theory. This architecture might be used interactively with the others; after some proofs a user might go back again to a higher level in order to explore aspects related to which the proved tendencies he wishes to understand better.

6.4.1 Comparing the Architectures

ASPECT	*SCENARIO ANALYSIS*	*LOGICAL MODEL-EXPLORATION*	*SYNTACTIC PROOF*
Typical paradigm	Imperative	Constraint	Declarative
Typical deduction system	Forward-chaining, logical model-oriented	Forward-chaining using efficient backtracking	Backward-chaining or resolution-based
Nature of the manipulations	Possibly semantic	Range of semantics (logical model)	Syntactic (clausal-ordered)
Limitations	Not constrained. Very rich. Too much information could mislead.	Finite constrained. Still quite rich. Suitable for scenario analysis.	Constrained. Valuable for proving specific tendencies.
Search style	Attempts to explore all simulation paths.	Limits the search by constraining the range of parameters and agent choices.	Can be efficient in suitably constrained cases, typically impractical.

Table 6.1. A Comparison of the Architectures

6.5 Modelling Process using Architectural Transformations

The main aspects distinguishing the architectural levels are the generality of the allowed conclusions and the level of programming. The higher the architectural level, the more a subtle design and dynamics can be practically supported; but the easier it is to understand the generated dynamics, the more general are the conclusions that can be drawn. This process has been illustrated in Figure 6.1 above.

A modelling process where a modeller uses all three architectures interactively is exemplified in Figure 6.2. Some of the benefits of a mechanism allowing an automatic translation among these architectures are improving understanding of and better informing a modeller about the dynamics of a simulation. A modeller moving among architectural levels according to interest, learning, and experience would benefit greatly.

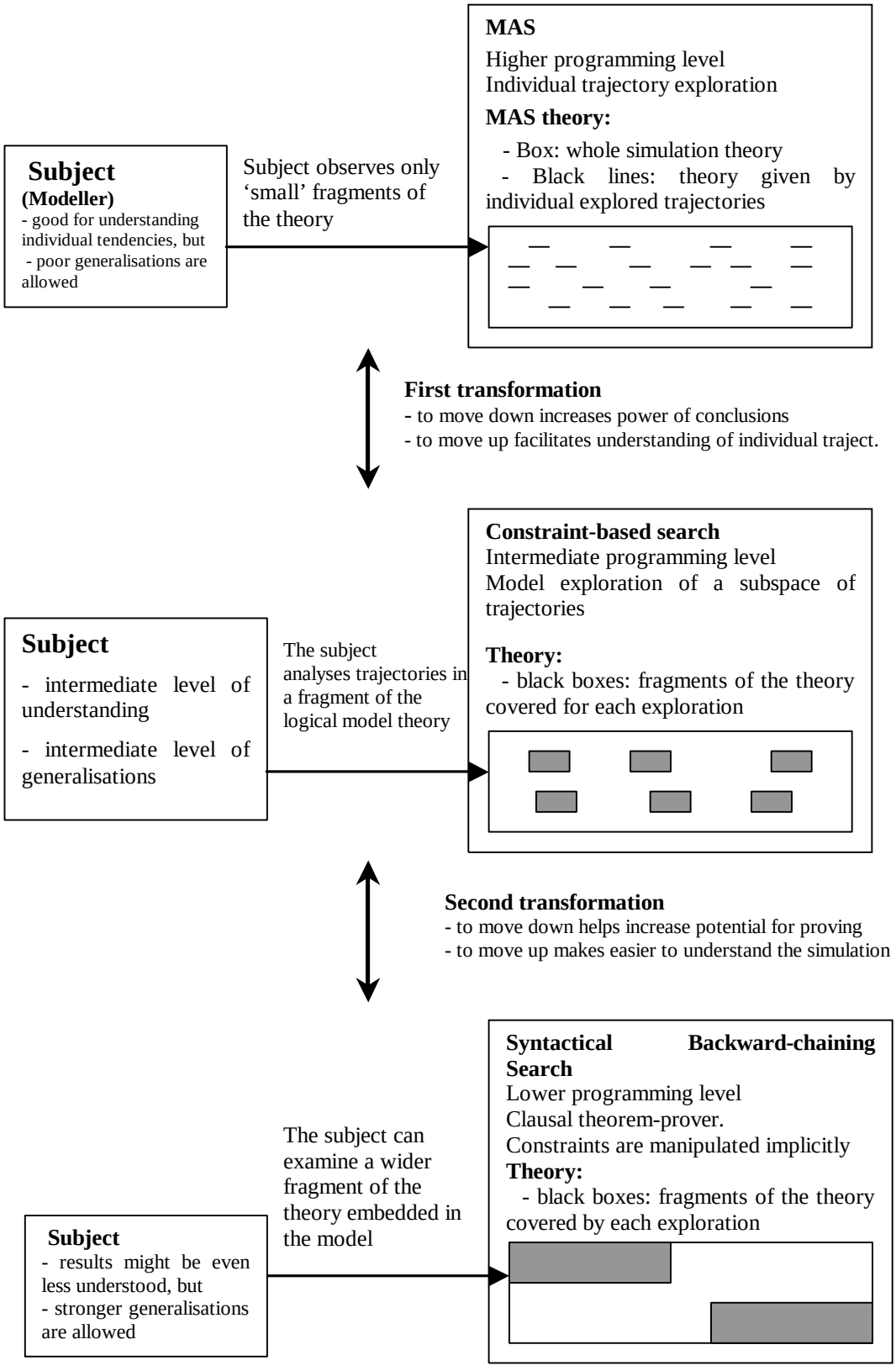


Figure 6.2. Interactive use of the architectures by a modeller

7 Chapter 7 - A Case Study: A Simple Trader-Distributor Model

7.1 Introduction

The *aim* of this chapter is to communicate the experience of developing the methodology presented in Chapters 5 and 6. Details of the simulation case and its different implementations in SDML and OTTER will be presented.

The example consists of a trader-distributor (simulation) model with six agents: three traders and three distributors. The agents' main tasks are: for traders, price-setting, price-imitating, and sale-setting; and, for distributors, order-setting. The example resembles basic characteristics that can be observed in many empirical systems, but it is ideal in the sense that it is not a representation of any particular empirical system. The idea has been to elaborate a typical simulation model to develop the methodology for exploring the dynamics of MAS-based simulations.

The first model (we use the world model for simulation model) is a MAS-based model built using the language SDML. Basic aspects of this model are presented in section 7.2. This model is used to explore the dynamics of the simulation. At this level the idea is to explore the dynamics by generating trajectories at a high level of programming, since a MAS is an architecture which gives detailed information about individual trajectories. This experience is discussed in section 7.3. A tendency concerning the behaviour of prices is observed in the MAS-based simulation. This tendency was expressed as an envelope rather than as a central measure, as would occur if Monte Carlo techniques were used. How the tendency was enveloped is reported in section 7.4.

A MAS level of programming can provide rich information about the evolution of state transitions in each trajectory. However, it is difficult to prove the commonality of a particular tendency over a range of trajectories. Because of this, a different architecture of programming, namely a theorem-prover, is chosen for attempting to prove a tendency in a simulation. Among several theorem-provers available, OTTER seems to be one of the more suitable for this task because of its success in other areas of research and because it presents more facilities for numerical manipulations (arithmetical manipulations are common in simulation of social systems) than other available theorem-provers (e.g., among the facilities are demodulators; see Chapter 3).

The experience from a first attempt to prove the tendency is offered in section 7.5. A translation that seems to be isomorphic to the MAS model is built in the theorem-prover OTTER (why it is believed that the translation is isomorphic is explained in section 5.10). A more exhaustive investigation of the dynamics of the simulation is carried trough in an

attempt to prove the necessity of a tendency for a range of trajectories. Though OTTER supports these sorts of proof, it is not totally appropriate. On the one hand, OTTER (like most of the existing theorem-provers) is oriented at proving in first-order logic via symbolic manipulation. However, on the other hand, it has poor user interface, limited facilities for numerical manipulation, and does not permit change over the structure of the model or meta-reasoning while the simulation is going on. As was noted in Chapter 3, OTTER is not flexible for user-driven manipulations of the database and rulebase.

It seems convenient, then, to attempt the proof in a more suitable language allowing not only a more flexible manipulation of data and rules, but also the implementation of a proof procedure close to that followed in OTTER. For instance, the idea is to take advantage of a proof procedure like that implemented in the OTTER model and of facilities that a higher architecture of programming offers. This is also convenient because a common platform is necessary if we wish to implement automatic translation of a model among different architectures. We choose SDML as that language. SDML offers a meta-agent to write rules on other agents. Also, SDML tenders facilities for a wide range of data manipulation and for implementing proofs. Some facilities for proving that were initially not present in SDML were added (e.g., false predicate in the consequent). Section 7.6 reports these SDML features.

Section 7.7 presents the first attempt to prove in SDML. In this first effort drawbacks in data manipulation, inherited from the MAS implementation, were found to slow down the simulation. These drawbacks are referred to in section 7.8 and have already been explained in section 0. Then a more efficient structure was implemented to deal with these drawbacks. Generalities of this architecture are given in section 7.9, as a more detailed review was already presented in section 5.9.3.3.

These proofs in SDML and OTTER are examples of the constraint-based logical model exploration of a simulation proposed in Chapters 5 and 6. This represents a lower architecture of programming than a MAS-based model.

A comparison between the MAS-based and the (logical) model constraint-based architectures is presented in section 7.10. Characteristics of the information each architecture provides with respect to a simulation trajectory and to the whole simulation theory, as well as the allowed scope of the conclusions drawn from that information, are emphasised.

In section 7.11 the importance of both studying the dynamics of a MAS and proving tendencies in a constraint-based architecture, for a modeller's understanding of tendencies

in a simulation, will be addressed, and special consideration will be given to how this analysis relates to the notions of a subject's explanation and understanding discussed in section 5.6.

Finally, in section 7.12, the (logical) model constraint-based approach will be contrasted with other procedures used for exploring the dynamics of simulations and for theorem-proving.

7.2 MAS Model using the Strictly Declarative Simulation Language (SDML)

The first model was a MAS-based one, built using the language SDML. The more relevant aspects of this model are shown in Figure 7.1. There are six agents: three distributors and three traders. The agents' main tasks are: for traders, price-setting, price-imitating, and sale-setting; and for distributors, order-setting. In addition to the agents some objects are used representing the traders' warehouses. The dynamic structure of the model is represented using rules and the static structure, in part, via predicates, objects, and agents.

The facilities SDML offers for modelling MAS include those for manipulating agents, time levels, and modules *hierarchically*. The hierarchy of agents is useful for representing agents as sub-agents contained in other agents. A sub-agent inherits facilities and properties such as predicates, clauses, and types of objects from its containers. The hierarchy of time levels is helpful for writing data valid for periods of time of different length. For example, if there were the time levels 'week' and 'day', there would be some transition rules for updating facts valid for a whole 'week', and other rules for updating data valid for only a 'day'. A hierarchy of modules is valuable for defining modules at different levels, where those at a lower level are more basic than those at a higer one. Those at the lower level will be used for defining more basic types of agents and objects to be used by the modules at the higher level for defining more complex entities.

Using these hierarchies, certain information is given implicitly. For example, there might exist (internally in SDML's design) an indexation of data per agent and time level, so that it is not necessary to indicate these explicitly at each predicate during the time period it is going to be valid for and at the agent holding it, but it will be enough to place it in the appropriate area of a database. If the instance of price, Price(10), is placed in the database of Trader-1 in the sub-area of data for time levels *week = 3, day = 2*, it would mean that agent Trader-1 has set his product's price at *value 10* on *day 2* of *week 3*. This is an example of how the hierarchy of time levels and agents hides data (making explicit this

data would create additional rule dependencies; see Chapter 5 and, more specifically, section 5.9.2).

The main predicates used in the model are listed below along with their domain. Predicates represent relationships among agents and/or objects. For example, the predicate

sale(Trader, Distributor, Amount)

is used to indicate the Amount of good a trader, Trader, sells to distributor, Distributor. Below, a list of the most significative predicates used in the model is given. There it can be seen that the *sale* predicate is placed at agent *Model*. The arguments of the predicate are: the two agents involved in the transation, e.g., *Trader* and *Distributor*, and the Amount of good sold. If this predicate had been asserted in agent *Trader*, the explicit reference to this agent would not have been needed.

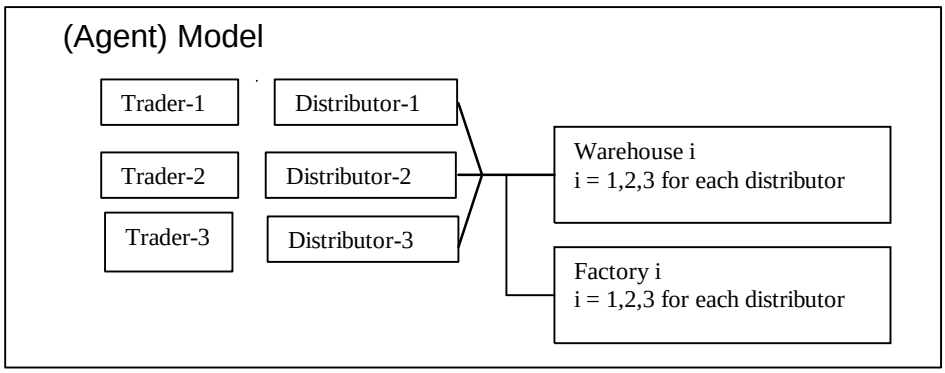


Figure 7.1. An overview of the model

7.2.1 List of the Most Relevant Predicates used in the Model

Model (general or environmental variables):

listChoiceTrader(Trader, Trader,Trader, i);

Description: gives the choices of traders to be used in price-setting. Element-j in the list is the choice for trader-j at day i. There will be eight of these predicates per day (the eight different choices).

listSelTrader(Trader, Trader,Trader);

Description: lists the traders ordered by their prices. It is used for calculating the amplitude of the interval of the prices and for a distributor placing orders (they place orders to the trader with the lower price).

order(Distributor, Trader, Amount);

Description: Amount is the quantity orderer by distributor Distributor at trader Trader. The time (day) is implicit, that is, there will be one of this predicate per iteration (day). This will be a fact in the following cases.

orderDistributor(Distributor, Amount); (total order the distributor places among all traders);

Description: Amount is the total quantity of good an agent *Distributor* places at current time (day).

orderTrader(Trader, Amount) (total order a trader has from all distributors);

Description: Amount is the *total* quantity of good ordered by all distributors from trader Trader.

sale(Trader, Distributor, Amount);

Description: Amount is the quantity of good trader Trader sells to distributor Distributor, at current time.

listSelTrader(Trader, Trader,Trader);

Description: places the traders in order accordancing to their prices (one listed earlier has price lower than or equal to that of one listed later). This is used for calculating the amplitude of the interval of the prices and for distributors placing orders (they place orders to the trader with the lower price).

Trader:

price(Value);

Description: Value is the price trader *Trader* gives to his product at current time.

saleTrader(Amount); (total sales the trader makes in a day);

Description: Amount is the *total* sales trader *Trader* makes at current iteration.

Distributor:

Demand(Amount);

Description: Amount gives the total demand distributor *Distributor* has at current time.

saleDistributor(Amount); (total sales made to this distributor by all traders).

Description: 'Amount' gives the total quantity of sales made to distributor Distributor at current day.

supply(Amount); (it resembles a factory production).

Description: Amount gives the quantity of good the *Trader* owner of the warehouse *Warehouse* gets at the current iteration.

Warehouse:

level(Amount);

Description: Amount indicates *Warehouse*'s level of good.

7.2.2 Outline of the Most Relevant Rules used in the Model

All rules are supposed to generate data for the present iteration at which the simulation is going on, unless the contrary is explicitly specified (the present iteration will be called `?i` in some rules, but it is usually referenced implicitly). The previous iteration is referenced as `lastIteration`. In the model, a rule is always part of an agent rulebase and it will write, by default, data in the dababase of that agent.

1) Rules in the environment given by the agent Model

1.1) Rule used by a trader for choosing another trader for price-setting

This iterative rule makes a choice for `iteration ?(i+1)` after the choice for `iteration ?i` has been made. It divides the task of making the choices for the whole simulation to one per day, which helps in decreasing the computational complexity of this task.

```
Antecedents:
and
    listChoiceTrader(?previousChoice, ?i); {instantiates choice for iteration ?i}
    randomChoice ?randchoice ?allchoices 2
       listOfPossibleChoices ?allchoices; {number 2 is a randomisation factor }
        {traders' choices are listed below in section 7.2; this clause gives alternatives when the
         system backtracks after the theorem has been proved in a trajectory }
Consequents:
  listChoiceTrader(?randchoice,?i+1);
```

1.2) Rule for creating a list of traders ordered according to the prices they offer

```
Antecedents:
  backSelProd(?i = ?currentIteration, listSel);
          {this backward-chaining rule orders the traders according to their prices at
           iteration ?i and returns them in the list ?listSel }
Consequents:
  listSelProd(?listSel);
```

1.3) Rule for calculating distributors' and traders' total orders

```
Antecedents:
and
  order(Distributor-1,Trader-1,?s1);
  order(Distributor-2,Trader-1,?s2);
  order(Distributor-3,Trader-1,?s3);
  order(Distributor-1,Trader-2,?s4);
  order(Distributor-2,Trader-2,?s5);
  order(Distributor-3,Trader-2,?s6);
  order(Distributor-1,Trader-3,?s7);
  order(Distributor-2,Trader-3,?s8);
  order(Distributor-3,Trader-3,?s9);
 ?order1  = ?s1 + ?s4 + ?s7;
 ?order2  = ?s2 + ?s5 + ?s8;
 ?order3  = ?s3 + ?s6 + ?s9;
 ?ordert1 = ?s1 + ?s2 + ?s3;
 ?ordert2 = ?s4 + ?s5 + ?s6;
 ?ordert3 = ?s7 + ?s8 + ?s9;

Consequents:
and
 orderDistributor(Distributor-1,?order1);
 orderDistributor(Distributor-2,?order2);
 orderDistributor(Distributor-3,?order3);
 orderTrader(Trader-1,?ordert1);
 orderTrader(Trader-2,?ordert2);
 orderTrader(Trader-3,?ordert3);
```

1.4) Rule for calculating traders 'total sales

This rule calculates 'totals' in a similar way to the rule listed above, and generates the data in the predicates: `saleDistributor(Amount)`and `saleTrader(Amount)` (the former predicate is placed at the distributor's rulebase and the second at the trader's rulebase, which is why such agents are not explicitly given in these predicates).

1.5) Rule for theorem-checking

The hypothesis we have attempted to prove is: *the amplitude of the interval of prices does not increase over time for the iterations (1,…,k) as the simulation is generated*. If the theorem is true in a simulation trajectory, the predicate false is generated and the system backtracks to explore another simulation trajectory.

```
Antecedents:
and
  calculateAmplitudeIntervalPrice(1,?amIntPrice1); {1: stands for the iteration}
  calculateAmplitudeIntervalPrice(2,?amIntPrice2);

  ...
  calculateAmplitudeIntervalPrice(k,?amIntPricek);
  notInferred greaterOrEqueal ?amIntPrice2  ?amIntPrice1;
  notInferred greaterOrEqueal ?amIntPrice3  ?amIntPrice2;
       ...
  notInferred greaterOrEqueal ?amIntPricek ?amIntPricek-1;
Consequents:
  false
```

2) Rules in the agent type trader (traders are sub-agents of the agent Model)

2.1) Rule for traders' price-setting

```
Antecedents:
and
  lastIteration {and
    saleTrader(?mysales);
    totalSales(?totalSales);
    price(?oldprice);
  }
  listChoiceTrader(?choices, at ?i = currentIteration);
  ?choices = [?sel1 ?sel2 ?sel3]; {Trader in position k in this  list is the choice for
                                    trader with name 'Trader-k'}
  otherPrice(?otherprice,?choices,?i,self {sending its name});
       {Knowing the trader, the rule is calculating the price for (e.g., Trader-j), the iteration (?i), and
       the list  of choice of traders (the choice for distributor-j would be ?selj), this backward-
       chaining rule returns the price of the chosen trader
                  (i.e., at (?selj,?i) it gets price(?otherprice);
  calcprice(?newprice,[?oldprice ?otherprice ?mysales, ?totalSales]);
Consequents:
  price(?newprice);
```

{if the agents and the iteration were explicitly referenced, the rule would be (after the appropriate changes in the definitions of the predicates):

```
Antecedents:
and
  saleTrader(?Trader,?mysales,?i);
  totalSales(?totalSales,?i);
  price(?Trader,?oldprice,?i);
  listChoiceTrader(?choices,?i);
    ?choices = [?sel1 ?sel2 ?sel3];{Trader in position k in this  list is the choice for
                                          trader with name 'Trader-k'}
  otherPrice(?otherprice,?choices,?i,?Trader);
       {Knowing the trader the rule is calculating the price for (e.g., Trader-k), the iteration (?i), and
        the list  of choices of traders, this backward-chaining rule returns the price of the chosen
        trader at iteration ?i (e.g., price(?selk,?otherprice,?i));}
  calcprice(?newprice,[?oldprice ?otherprice ?mysales ?totalSales]);
Consequents:
    price(?Trader,?newprice,?i+1);
```

{this is the sort of transformation that will be necessary if the hierarchy of agents and time levels is eliminated, as will be seen below} }

2.2) Rule for traders' sale-setting

```
Antecedents:
and
      level(?level);     {in fact, level is consulted at the trader's store's database}
      order(?Distributor,?Trader,?orderDistTrader);
      orderTrader(self, ?orderTrader);
      calcSale(?level,?orderTrader,?orderDistTrader,?sale)

Consequents:
  at Model write: sale(?Trader,?Distributor,?sale);
```

2.3) Rule for trader's production (or supply) and level updating

```
Antecedents:
and
 lastIteration {and
   level(?level);
   saleTrader(?sales);
   orderTrader(?order);
 }
 calcSuppyAndNewLevel(?order,?level,?Sales,?newLevel,?newSupply);
Consequents:
 and
   level(?newLevel);
   supply(?newSupply);
```

3) Rules in the agent type Distributor (distributors are sub-agents of the agent Model)

3.1) Rule for distributors' demand and order-setting

This rule is written at each Distributor. It is valid for any iteration (e.g., it is applied) recursively. Notice that the agent (`Distributor`) and the iteration are implicitly referenced.

```
Antecedents:
and
  lastIteration {and
     demand(?oldDemand);
      sale(?trader,self,?oldSale);
      saleDistributor(?oldSaleDistributor);
      listSelTrader([?traderSelected ?other1 ?other2]);
     order(self,?trader,?oldOrder)
   }
   calcNewOrder(?trader,?traderSelected,?oldDemand,?oldSale,?oldOrder,
                ?oldSaleDistributor,?newOrder, ?newDemand);
Consequents:
and
  demand(?newDemand);
  at Model write: order(self,?trader,?newOrder);
}
```

{if the agent and the iteration were explicitly referenced, the rule would be (after the appropriate changes in the definitions of the predicates):

```
Antecedents:
and
  demand(?distributor,?oldDemand,?i);
  sale(?trader,?distributor,?oldSale);
  saleDistributor(?distributor,?oldSaleDistributor);
  listSelTrader([?traderSelected,?other1,?other2]);
  order(?distributor,?trader,?oldOrder)
  calcNewOrder(?trader,?traderSelected,?oldDemand,?oldSale,?oldOrder,
               ?oldSaleDistributor,?newOrder, ?newDemand);
Consequents:
and
  demand(?distributor,?newDemand,?i+1);
  order(?distributor,?trader,?newOrder,?i+1);
}
```

7.3 Exploration of Simulation Trajectories using the MAS-based (Simulation) Model

The first exploration of the simulation trajectories was performed at the MAS level using a model built in SDML. This stage of observation corresponds to what in Chapter 6 is proposed as the first level in a hierarchy of architectural transformations. A sort of scenario analysis was made generating several simulation trajectories.

These trajectories corresponded to different values of parameters of the model and agents' choices. Parameters of the model were, for example, the capacity of the warehouse and the rate of supply from a trader to a distributor. The non-deterministic choice modelled was the trader's price-setting. Traders imitate other traders' prices. For this purpose, each trader chooses another trader each day. This was implemented using the predicate listChoiceTrader(Trader, Trader,Trader, i). At this stage of the experimentation, only one alternative for trader's choices is allowed per iteration and a single trajectory is explored per run. If all alternatives for choices were allowed in the model, as the total number of traders is three, each of them would have two choices, and there would be a total of 2 x 2 x 2 = 8 alternative price-settings at each iteration. The alternative choices for price-setting (traders' choices of another trader) at day i were:

Choice 1: listChoiceTrader(Trader-2, Trader-1, Trader-1, i);
Choice 2: listChoiceTrader(Trader-2, Trader-1, Trader-2, i);
Choice 3: listChoiceTrader(Trader-2, Trader-3, Trader-1, i);
Choice 4: listChoiceTrader(Trader-2, Trader-3, Trader-2, i);
Choice 5: listChoiceTrader(Trader-3, Trader-1, Trader-1, i);
Choice 6: listChoiceTrader(Trader-3, Trader-1, Trader-2, i);

Choice 7: listChoiceTrader(Trader-3, Trader-3, Trader-1, i);

Choice 8: listChoiceTrader(Trader-3, Trader-3, Trader-2, i);

where the trader in position i is the choice for Trader-i. For instance, in case of the fourth choice (Choice 4: listChoiceTrader(Trader-2, Trader-3, Trader-2, i)), Trader-1 chooses Trader-2, Trader-2 chooses Trader-3 ,and Trader-3 chooses Trader 2.

A tendency was 'discovered' when examining the results of the simulation: the range of traders' prices stabilise over time and its size decreases monotonically over time. This tendency was common to all generated trajectories. Trajectories were explored for 20 or 30 iterations. Now that a tendency has been found, the idea is to investigate it better. The following step is to examine the commonality of the tendency in different trajectories and, if possible, to prove it.

7.4 Envelopes of a Tendency in a Subspace of the Simulation Trajectories

In a trajectory, there are three agents of interest for the analysed tendency (the traders). Each agent has its individual trajectory in each simulation trajectory, as was graphically represented in the left side of Figure 5.2 for the case of two agents. Along each trader's trajectory there is a trajectory of the variable prices: that trader's product's price (a typical example of the three price trajectories for a single simulation trajectory is shown in Figure 7.3). These three outputs are obviously interrelated. The idea is to study them either by enveloping them as shown in the lower part of Figure 5.3, e.g., by generating a single envelope for the whole subspace of the relevant simulation outputs (the tendency about prices) obtained from the exploration of all simulation trajectories; or, by enveloping them for each simulation trajectory and then examining all envelopes generated for all explored simulation trajectories, as will be the case in this particular application.

In the case relevant in the present study, there are three price trajectories resuming the aspects of interest for each simulation trajectory. Each sample of these price trajectories a simulation trajectory gives is the result of the interplay of agents' choices for a single parameter-setting during a finite number of time iterations. The idea is to envelope these three price trajectories and to study them better, not only for a single simulation trajectory but for a subspace of them - that defined by range of parameters, agents' choices, and a finite number of time interations.

The specified output of interest, traders' prices, was then enveloped in accordance with the discussion in section 5.3. This is the case of a one-dimensional output and it is of interest to envelope the observed tendency, *Y,* using two value sets E_{upper} and E_{lower} for each simulation trajectory, as was explained in section 5.3 for the discrete case.

In accordance with this, per each iteration in a simulation trajectory traders' prices might be delimited, e.g, by the highest and the lowest traders' price for that iteration. (i.e., the envelope value set at *t* is defined by the interval: [highest traders' price at time *t*, lowest traders' price at time *t*] = [$E_{upper}|_t$, $E_{lower}|_t$]). As was stated above, in this case, it is of interest to generate an envelope of the tendency for each simulation trajectory. Should the tendency be enveloped for all explored simulation trajectories, again the tendency would be given by the worst cases; i.e., its bounds for time iteration *t* would be given by the union of all intervals obtained for *t* (one for each simulation trajectory).

E.g., if we knew traders' prices for two days in a trajectory:

- for day 1: 0.40, 0.45, 0.43, and,
- for day 2: 0.42, 0.43, 0.41;

then the output of interest, i.e., the enveloping set of values for this trajectory, would be:

- for day 1: 0.40 and 0.45;
- for day 2: 0.41 and 0.43.

The relevant property of the tendency is a measure of this envelope, so that, for day *t*, what is of concern is the difference between the maximun and minimun prices on that day. Remember that the tendency identified in the previous section was that the amplitude of the interval for prices is decreasing over time. For the example, this output would be:

- for day 1: 0.05;
- for day 2: 0.02.

The values of this measure for a typical simulation trajectory are presented graphically along the trader's prices as the lowest curve in Figure 7.3.

Therefore, the interest in this case is not in studying a common envelope of the tendency for all simulation trajectories, but in analysing envelopes for each simulation trajectory. The concern is to prove that a property of the envelope of the tendency in a simulation trajectory holds for all instances of the envelope in all explored simulation trajectories, rather than (what is an alternative) to prove a property of a single envelope covering all instances of an output for all explored simulation trajectories.

7.5 A First Attempt to Prove in OTTER

In order better to explore the simulation dynamics and the possibility of proving tendencies in a subset of the trajectories, the model is translated into the theorem-prover OTTER. This seems to be isomorphic (discussed in section 5.10.1), since each instance of data and rule in the original SDML model had an equivalent instance in OTTER. Obviously, the

implementation, though equivalent, has differences, as OTTER gives different facilities and other levels of programming. In OTTER, it is necessary to express all data and all rules in a single DB-RB pair, as OTTER does not offer either the hierarchy of agents or the hierarchy of time levels.

Hence, SDML's hierarchy of DB-RBs becomes a single database–rulebase in OTTER. Because of this, information implicit in SDML has to be made explicit in OTTER. For example, predicates in OTTER have to be modified in order to add data about agents and time levels given implicitly in SDML. For instance, predicate Price(Value) at agent 'Trader-1' has to be written in OTTER as Price(Trader-1,10, iteration). That is, agents, objects, and time iterations have to be explicitly specified in predicates.

The main characteristics of the procedures used in OTTER for proving have been described in Chapter 3. OTTER is clausal-ordered (syntactic) and forward-chaining. It uses the set of support strategies. The old version of OTTER allows a user to implement only implicit split, but the newer UNIX versions also permit a modeller to implement explicit split (see Chapter 3). Most of the experiments were made in traditional OTTER using implicit split but as soon as the option for explicit split became available, it was also used.

In fact, implicit split has to be somewhat user-driven; the user himself has to program the search and exploration of the different simulation branches generated by the agent's choices. This can be implemented by adding a field into the predicate for identifying simulation branches. Different branches generated at a choice point are identified with a different assumption label – also user-handled. The idea is that different instances of data in a branch share the same assumption. In the model, each trader's choice was identified with a different number (from 1 to 8). For example, the predicate:

sale('Trader-1', 'Distributor-2', 10, 6, [1 5 3 8 2]);

indicates that 'Trader-1' has sold 10 units of good to 'Distributor-2' at time interation 6 in the simulation trajectory correspondng to assumptions [1 5 3 8 2], i.e., agents have made choices 1, 5, 3, 8, and 2 on days 1 to 5 respectively.

The model in OTTER resulted from translation of the original simulation model built in SDML that seems to be isomorphic (see Chapter 5). Inference is applied using a single set of clauses, placed in the support set (this is part of the programming strategy in OTTER; see Chapter 4). While OTTER's inference mechanism is clausal, the implementation in SDML is logical model-oriented. Rules in SDML were rewritten in OTTER as clauses in the support set and then hyperresolution was used as the inference mechanism. Forward-chaining was applied over data in a path that advanced over the simulation time. This

mechanism also propagates assumptions. New branches are generated as alternative results of a resolution (e.g., a set of resolvents, rather than a unique resolvent, results from applying hyperresolution to a set of clauses). The inference mechanism is then forward-chaining, driven by the support set and hyperresolution.

The proof procedure implemented in this model in OTTER has similarities to that followed in tableaux in the sense that the negation of the theorem (tendency) to be proved is added. A simulation path is closed as soon as a contradiction appears. This happens when the tendency turns up in the path. If all possible paths are closed, then the tendency is proved. On the other hand, if at least one path remains open and no more data can be generated, the proof fails – the open path providing the counter-example to the tendency. How the simulation paths are generated (for the parameters of the model and choices of the agents) is illustrated in Figure 5.2 of Chapter 5.

As soon as the proof is attempted, its complexity becomes evident. The number of simulation paths increases tremendously with the number of iterations allowed in a trajectory. A huge amount of computational resources is required. This become clear considering the number of simulation paths to be investigated as a function of the number of days, remembering that there are eight alternative choices (the traders' choices) at each branch each day. For instance, if the simulation is carried out for 2, … , 10 simulation days, there will be: 8, 64, 512, 4096, 32768, 262144, 2097152, 16 777 216, 134 217 728, 1073741824 paths. The experiments were made in a PC with 256 MB RAM, which was able to simulate for up to 6 iterations and 32768 simulation paths (8^5 simulation step transitions).

At this stage it becomes clear that this sort of proof has to be restricted. Clearly, the principal constraints are the limited computational resources and the number of iterations. More generally, constraints have the following sources (at least in the simulation of social systems):

- Technical
  - *Limited Computational Resources* given by a PC and its 512MB of RAM
  - Manipulation of *real numbers*: the presence of real numbers makes it difficult to evaluate the similarity of paths and prune the search. Conditions assuring a similar behaviour of prices can be found, but it is very unlikely that these similar conditions repeat during a simulation, as they will be given in terms of floating point numbers (basically in terms of prices and sales, because the instances are driven by the traders' price-setting). If the intention had been to

prune branches, information about all particular conditions found while the simulation was going on would have had to be kept in order to compare them with new conditions found, which is expensive in terms of memory and time. Moreover, it is not easy to manipulate a database in theorem-provers like OTTER (see Chapter 5). In fact, it would have been necessary to add special facilities into OTTER's code and to recompile it. Thus, this alternative would result in relatively little advantage even when compared to high-level languages such as those for programming MAS.

- *Complexity* of modelled systems
  - Unforeseen (especially qualitative) changes in 'empirical systems'. Qualitative changes are generally present in MAS applications as, for example, agents can be introduced and eliminated. Nonetheless, they were not present in the experimental model.

The first models in OTTER using implicit split were built in 1998. Additional versions of the model in OTTER using explicit split were built at the end of 1999. When using explicit split, at each choice point OTTER saves the state of the system in order to backtrack and explore alternative choices. Once a path is followed successfully, OTTER reports the relevant details about the result in that branch and backtracks until the latest choice point in order to follow another path.

7.6 Facilities for Proving Tendencies into SDML

SDML's characteristics and facilities, and SDML's inference mechanism for proving theorems in the theory of a simulation model, were discussed in sections 5.9.1 and 5.9.2. Most of those facilities were present at the moment of implementing the model discussed in this chapter. The only new one added was:

- *A simple negative contradiction generation* via false predicate: P => ?

This new facility was conceived by Bruce Edmonds and aggregated into SDML by Steve Wallis.

7.7 A first Attempt to Prove in SDML: Resembling the Experiments in OTTER

The idea at this point was to build a new model in SDML. The new model consisted of a single DB-RB pair for facilitating reasoning about the whole simulation model. The primary reason for having a single DB-RB pair is because SDML does allow backtracking over time levels or between agents (a side effect of this was the huge number of

assumptions generated). The plan is to experiment in SDML with a similar model to that built in OTTER and, in a following step, to improve the efficiency of this new model. The main aspects of this first translation are shown in section 5.9.3.1, which, as will be seen in section 7.8, turns out to have some serious drawbacks; then, in section 7.9, a more efficient implementation will be discussed.

7.8 Drawbacks of this Implementation: A More Efficient Implementation is Needed

A difficulty appears after revealing dependencies: the space of data matching the antecedent of the transition rules grows linearly with the number of iterations. However, most of these attempts are redundant, in which case no additional data is generated by the rule. These redundant rule-firings slowed down the simulation. These drawbacks are discussed in section 0. In order to deal with these drawbacks, the rules will be 'unwrapped'.

7.9 An Efficient Implementation in SDML: a (Logical) Model Constraint-based Architecture

The next step is to unwrap the rules. To write rules for iteration i+1 referencing explicitly data at iteration i, it would be necessary to know such data previously. That is possible, for example, if, at the beginning of the simulation, the names of the predicates alluding to the data to be instantiated are known and if the language permits one to write the rules using those predicates. This is possible in SDML using a meta agent. Another way of splitting rules would be by writing them even more dynamically while the simulation is going on, e.g., writing the rules for iteration i+1 once the data for iteration i is known. This option is not available in SDML when using backtracking. When using backtracking, a *meta agent* acts in advance of the agent it is writing to. So, the information the *meta* agent uses has to be given in advance of the target agent's activation (e.g., in this case during the initialisation of the simulation).

The idea, then, is to 'declare' (give explicitly in the model) as much as possible of the semantics of the model. The intention is to declare all instances of agents and objects in order to make possible the identification (in the program) of the range of agents' choices and model's parameters, and to declare the names of predicates referencing data so that this information can be used dynamically by transition rules in advance of the activation of the meta agent (a meta agent operates in this application as a module and will be called

meta). The module *model* is responsible for setting these *initial conditions* in the simulation. An overview of the efficient model built in SDML is given in Figure 5.18.

Afterwards, *meta* uses this semantic information in order to write a set of transition rules for each iteration, making the information to be instantiated by each rule as explicit as possible and eliminating the problem described above. Figure 7.2 pictures the whole translation process from the original MAS-based model in SDML to the efficient *(logical) model constraint-based implementation* (it is constraint-based because the trajectories are generated for a (complete) range of parameters and choices).

Trajectories in the four models, the two models implemented in OTTER, the first MAS-based model, and this new (efficient) implementation seem to be equivalent, since they generate the same instances of data for a single iteration. That is, *the models appear to be isomorphic* to each other under the experimental frames defined by the data a modeller wants to observe (namely price-setting, order-setting, and price-imitating) and the range of parameters and choices taken, in the strict sense defined by Zeigler (this is discussed in section 5.10).

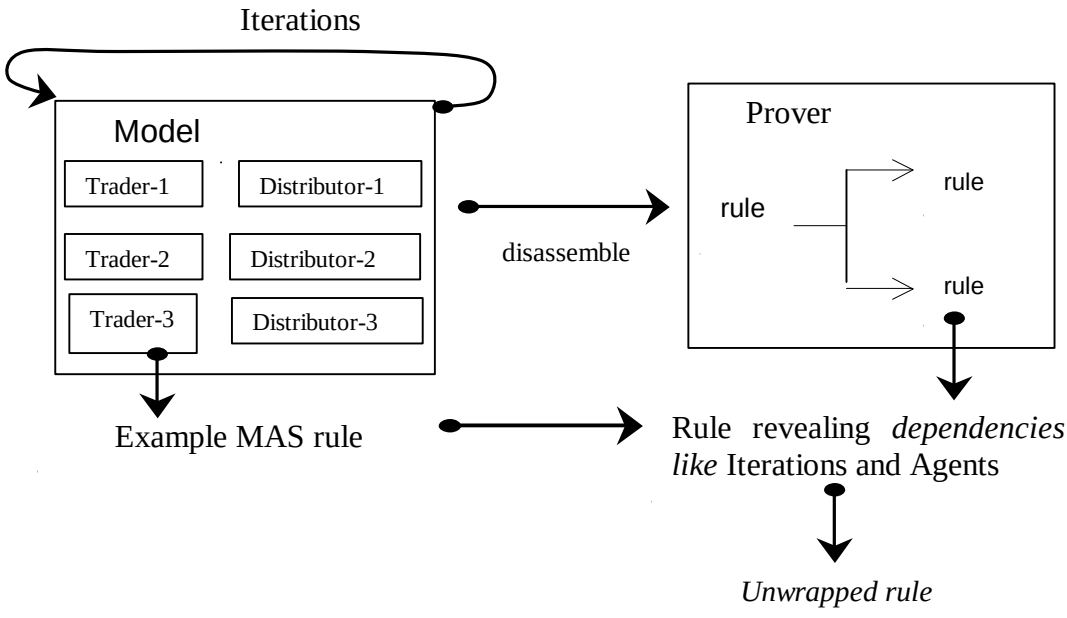


Figure 7.2. The whole procedure of revealing dependencies and unwrapping rules

7.9.1 Towards an Automatic Translation of a MAS-based Model into a Constraint-based Model

The translation process of a MAS-based model into a constraint-based architecture that might be implemented automatically would be:

1. Identify MAS constructs hiding rule dependencies (e.g., time levels, agents, or objects). Look for 'general' descriptions of the dependencies. E.g., a trader's price at iteration i+1 depends on traders' sales and prices at iteration i.
2. Redefine predicates by adding fields (e.g., for time levels and for agents) in order to make the hidden dependencies explicit. In order to do this, it is necessary to define predicate-types more specifically; e.g., rather than declaring only the predicate *price*, the idea is to declare, for example, $price_1$, $price_2$, .., representing price at iteration 1, 2, …. - the hidden rule dependency *iteration* is revealed. This is useful for referencing instances of price more specifically.
3. To facilitate reference to data when writing meta-rules in 4, it is useful to create a list of references or links to instances of predicates involved in these new dependencies. This is given by the predicate's names. Taking the example of the rule for prices, lists containing the names of the predicates for prices and sales are created (e.g, lists like [$Price_1$, $Price_2$ …, $Price_n$] and [$Sale_1$, $Sale_2$, …, $Sale_n$]). Lists containing objects' and agents' names might also be specified.
4. Initialise the simulation. This includes the range of parameters and choices, and the state of the simulation at iteration 1. It is a task of module *model* (see above). It creates data to be used by *meta* in order to write the transition rules for *iteration 2*.
5. Using these settings and knowledge about rule dependencies, meta-rules are written at *meta* which are responsible for writing (at the module *prover)* the transition rules (referencing as explicitly as possible with respect to the data), which are, in turn, responsible for executing the proof.

7.10 Comparing the Traditional and the Efficient MAS-based Implementations

The original model was useful for generating single trajectories. There a scenario analysis can be accomplished but the user has to execute and observe each trajectory individually. As was said above, the advantage of the MAS architecture is the high level of programming. It allows the modeller a good understanding of individual trajectories, as a lot of information about the quantitative and qualitative evolution of the simulation, both written and graphical, can be generated.

On the other hand, using the constraint-based architecture, a restricted proof of the tendency was achieved. All trajectories were generated for a combination of parameters and all traders' non-deterministic choices over six simulation time iterations.

In principle, there were eight 'types' of rules (see Table 12.1) in the original model involved in the application of the technique: those associated with generating the dynamics of the simulation (namely the transition rules, of which there are seven), the rule for checking the theorem, and the rules for setting up the traders' choices. The last two did not suffer additional split. The seven transition rules were split into 85 rules in the new model. Each of them was split by iteration, and two suffered additional split by trader. Distributor also split one of these rules. This gives: *(5 + (1 + (1 * 3)) * 3) * 5 = 85* rules in the logical model constraint-based SDML model, replacing those seven named rules in the MAS model. For more details about the sets of rules in the two models, see Appendix 5.

Other facts about the case study are:

- With the computational resources available at the moment, the efficient constraint-based program executed the proof in twelve hours, while the original one needed seven days.
- There were 8^5 = 32768 explored paths in the (logical) model constraint-based model.

7.11 Proving Necessity and Understanding of an Emergent Tendency

This section intends to make clearer the interrelationship among the concepts of explanation, understanding, proof, and changes in a modeller's internal model, discussed in section 5.6 (more precisely, see Figure 5.4), by using the experience of discovering and proving an emergent tendency in a simulation presented in the previous sections.

As was said in section 5.6, to understand a tendency a modeller would have to find meaning for it by himself (i.e., in his internal model); he has to bring about his own explanations or insights about how the tendency arises.

As mentioned in section 5.6, changes in the modeller's cognitive model might be either light, like those additional details a particular simulation trajectory can produce, or stronger, such as those created by new insights that a new simulation approach, a new paradigm in science, or the process of giving meaning to an emergent tendency can create. For example, as was discussed in Chapter 4, if a subject finds an explanation for an emergent tendency, then a new language might appear in the subject's cognitive model. This might bring to the subject not only new explanations for related phenomena but also new ways of approaching them, new insights, and probably a new world view in different matters. A new language in a subject's cognitive model might work like a new paradigm in science.

In the MAS-based model single trajectories can be observed. At this level, it was noticed that traders' prices became closer over time and practically similar after a certain number of iterations. This finding was helped by appropriately presenting the information graphically (see Figure 7.3). This fact, observed in single isolated trajectories, has two important components: those outputs about which hypothesis is arising, i.e., traders' prices; and a property the hypothesis is referring to, i.e., that one given by the differences among prices, or, equivalently by the size of the interval defined by the smaller and the higher prices per day. The hypothesis is that such an interval decreases over time. Why this tendency arises is not clear. A causal 'explanation' in terms of the computational model would be given by the sequence of transition steps the MAS carries out for generating this tendency. However, this is not an explanation for a modeller and it is a fact concerning only a very tiny part of the simulation model theory (that a single trajectory offers).

At this point the tendency could be considered emergent since it has not been understood by the modeller; the modeller has neither and explanation nor an insight to give it sense. On the other hand, as the tendency has been observed only in isolated simulation trajectories, no conclusion with reference to the simulation theory can be drawn.

Further investigation of this tendency is made by implementing a constraint-based exploration of all simulation trajectories in a subspace of the simulation theory. This space is supposed to be relevant in accordance witht the modeller's goal, e.g., including those parameter-settings that seem more likely to happen in the empirical system or those involving what the modeller believes to be the most significant ones because of a certain critical value of some output.

Using the logical model-based constraint architecture, a restricted proof of the tendency was achieved for a single parameter-setting, all agents' choices, and six time iterations. This involved the exploration of 32 678 simulation trajectories. As the tendency was observed in all explored simulation trajectories, it informed the modeller that the tendency was a fact in that fragment of the theory given by the simulation model constraint under that single parameters' setting, all agents' choices and six time iterations. This result gives a more general conclusion than that offered by the MAS-based model.

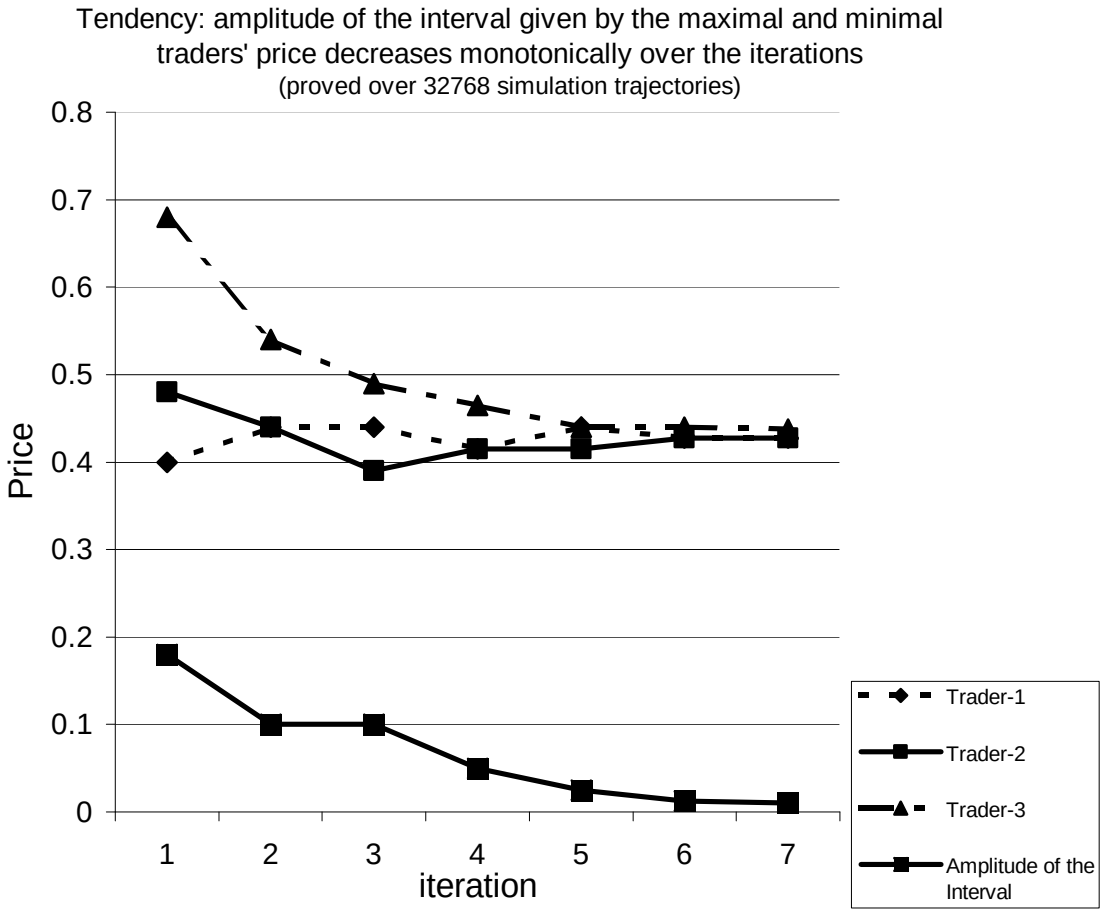


Figure 7.3. Tendency observed in a trajectory
The envelope of the tendency is given by the maximal and minimal price in the three upper curves, and a measure of it is offered below as the amplitude of the interval

A drawback of this procedure might be that, as more data is involved (that of the 32 678 explored trajectories), it is likely to make it more difficult for a modeller to find his own explanation and to understand the tendency. On the other hand, however, such an exploration of a whole fragment of the simulation theory might allow the modeller to elaborate theoretically valid conclusions, to bring in new insights for some analytical theory, and afterwards probably to update some of his own basic beliefs and intuitions. In this case, it would bring more essential knowledge than the MAS-based model, as more basic aspects of the modeller's cognitive model would be changed.

Likewise, proofs at an even lower level of programming would give even less detail about single trajectories, but they might offer a modeller stronger conclusions about the theory in the simulation model and bring more fundamental changes in his knowledge (more likely, they would generate structural change in his cognitive model) than those the model-based constraint architecture offers. Nevertheless, it is likely that the former

architecture would be less useful than the later one in helping a modeller to understand in a trajectory particular details associated to an emergent tendency.

The more involved the modeller is with a model and the better validated he feels the model is, the more likely it will be that simulation results from these architectures of programming modify the modeller's understanding about the empirical system. In addition, several characterisations of the simulation results via constraint-based proofs of tendencies will allow a modeller to characterise a range of behaviour in the model via a set of relations involving the range of parameters and choices on the one hand and the envelopes of the simulation outputs on the other; i.e., he will collect sets of envelopes of the simulation outputs for different fragments of the theory. All this will assist in training the modeller about what to expect from the empirical system. Such training will inform the modeller and change his expectations about details related to how events unfold (by observing single trajectories) as well as about more general and deeper issues (that the constraint-based architecture offers).

7.12 Comparison with other Approaches

The idea of this section is to briefly contrast the constraint-based approach exhibited above with other procedures used for similar purposes, namely to explore the dynamics of simulations, to prove theorems, or, in general, to investigate tendencies in the theory given by a computational program. That is, the aim is to explore the consequences (the dynamics) of computational programs so that a modeller might improve his understanding of aspects of the program itself or/and of an empirical system modelled. Some of these computational programs were reviewed in Chapters 2 and 3. In particular, the interest is in those computational tools related to this thesis: either using a declarative approach for agent simulation or with a potential for theorem-proving. This will include: the theorem-prover OTTER, the experience of people working with DESIRE (Engelfriet, 1998), and the declarative system METATEM (Fisher, 1996).

7.12.1 Theorem-provers: OTTER

OTTER is a theorem-prover (see Chapter 4) using clause-ordering (weighting) and a support set for efficiency. In SDML, the criterion for firing rules is well understood, and procedures like weighting and subsumption used in theorem-provers such as OTTER are not usually needed. In SDML the main source of efficiency is the exploitation of rule dependences. Information about predicates and their temporal order is exploited in SDML

using an automatic mechanism for the static analysis of dependences (see Chapter 5 and Moss *et al.*,1998a).

7.12.2 Proving in MAS: DESIRE

Among other approaches for the practical proof of MAS properties, the more pertinent appears to be the case conducted by people working in the DESIRE project. They proposed a hierarchical verification of MAS properties, and succeeded in doing this for a system (Engelfriet, 1998). The first part of the proof consists in proving that agents at the lower level of the hierarchy (those not containing sub-agents) satisfy the goals they are supposed to achieve. That is, for each type of agent at the lower level, it is proved that its goal is a theorem of the theory given by the rulebase of the agent. Then properties about agents at the next, higher level up (i.e., those whose sub-agents are all in the lowest level) are proved. Their properties depend on their own rulebase, and on their sub-agents' rulebases and properties. This procedure is continued until proving that the whole system (the agent at the highest level) has the correct behaviour.

The DESIRE group's aim is the *verification* of properties of a computational program, by proving that the program behaves in the intended way. Proofs are about aspects that are supposed to be introduced in the structure of the simulation model rather than about 'new' features observed in the dynamics of the simulation.

7.12.3 Constraint Logic-programming

Satchmo's and other constraint programming systems offer facilities similar to those of SDML, for example, *backtracking* and the *false* predicate. However, they also have built-in facilities for the manipulation of constraints, something SDML lacks. Instead, SDML presents facilities to introduce alternative values for the manipulated entities (e.g., predicates, clauses, integer variables), which can be used as *constraints* (clauses for choosing, e.g., randomChoice) as well as a *meta module* able to reason about terms or rules. As was seen above, a *meta* module can build rules to take advantage of the simulation semantics. A *meta* module able to act while the simulation is going on could adapt the search to the simulation results. Though SDML's approach is different from constraint logic-programming languages, it is able to control the manipulation of constraints flexibly and transparently for the user via the mechanisms described.

It should be possible to re-implement the example model in some of these systems, for instance in CHR[v]. This is an area of further research that will help in developing an even more appropriate methodological approach for proving in MAS.

7.12.4 MetateM

MetateM and concurrent MetateM are languages implementing executable temporal logics. They are in the family of languages following the imperative future paradigm (past and present ? (*implies)* future) (Barringer *et al.*, 1991; Fisher *et al.*, 1995; Fisher, 1996). Consequently, their execution mechanisms seem to have many similarities with SDML's and, in general, with forward-chaining oriented languages. Nevertheless, there are several differences between MetateM and SDML. These are related to, on the one hand, the way agents and their interactions are conceived, and, on the other hand, their capabilities for exploring and proving tendencies about the dynamics of a simulation.

In MetateM the order of the rules depends on a temporal order of eventualities; the rule with the oldest outstanding eventualities is chosen as the rule to be executed (this is explained in Fisher (1996), p. 2, section 2.2, in the Postscript version of the paper). It depends dynamically on both the rulebase and the database. Instead, SDML's rule dependencies management is static - rule dependencies do not change over time. This implies both advantages and disadvantages.

A disadvantage for MetateM seems to be the difficulties for tracking those eventualities, as it implies a dynamic modification of certain links to the database. This must be both time- and memory-consuming. In some sense this presents similarities to event-driven simulation, where a future event list determines at each time the next transition states to be generated. Nevertheless, in event-driven simulation each type of event as well as its consequences over the state of the system are assumed to be well known in advance. Eventuality-driven rule-firing might be difficult if the dynamics of the 'eventualities' are not well known. This sort of exploration suggests a sort of time-driven simulation, still not flexible enough for implementing efficient theorem-proving mechanisms (a single and fixed criterion is driven the search at a meta-level).

MetateM allows backtracking and branch exploration in agent database in case no data have been sent to other agents or to the environment. This sounds inflexible and inconvenient for theorem-proving. SDML has a similar inflexibility when using backtracking where several agents (and thus several databases) are used. This is one of the reasons a single DB-RB pair has been proposed for a constraint-based exploration of simulation trajectories. In fact, if backtracking of the simulation in a population of agents were allowed even after messages have been transferred among agents, then ideas would have to be brought from parallel simulation and distributed programming to maintain the feasibility of the computation. The difficulties for backtracking when using concurrent

processors or parallel databases (the usual case in a MAS) obviously vanish once the model is translated into a single DB-RB pair.

Transference and access to messages is controlled in SDML by using the hierarchy of agents, data attributes, and by writing data to other agents; e.g., one attribute of an agent's predicates can be either public (accessible to any other agent) or private (accessible only by the agent itself and by its container). This is less flexible in MetateM as there it is tied to other aspects of the model. In MetateM, it is necessary explicitly to specify in the definition of an agent which types of data this agent can accept as input, and which types of data this agent can send to other agents.

Though METATEM is appropriate to implement agent simulation and theorem-proving, facilities might be added or existing ones improved (e.g., for allowing backtracking even after agents have interchanged messages) to make it even more suitable (this is also the case with SDML).

8 Chapter 8 - Some Implications of this Research

8.1 Introduction

The *aim* of this chapter is to discuss some of the *implications* of this thesis in related areas of research, such as: complex systems, social simulation, MAS, policy analysis, and logic-programming.

First and as a preview, in section 8.2, conditions of how this thesis works in relation to other methods will be discussed. The idea is to situate better the results obtained in this thesis with respect to other research areas and their methods as well as to discuss the feasibility of the proposed methodology and techniques in other areas of research and with respect to the computation techniques available at the moment. This will also involve discussions about the trade-off between the complexity of the proposed methodology and techniques and its usefulness for studying complex systems.

Second, in section 8.3, advantages of a hierarchy of architectural transformations (such as that proposed in Chapter 7 for dealing with the difficulties behind modelling *complex systems*) will be discussed. In addition, the benefits of using a language permitting changes in the structure of the (simulation) model at the level of the rulebase and at the level of the components (e.g., allowing introduction and destruction of agents) will be discussed.

Then, in section 8.4, benefits of a hierarchy of architectural transformations (e.g., that given in Chapter 6) and of a more specific notion of emergence of tendencies (like that presented in Chapter 4) for the *social simulation community* will be considered.

Afterwards, in section 8.5, advantages of using hierarchies of architectures rather than a single one (the MAS itself) for different *MAS communities* in accordance with their needs and goals are discussed. Work in this area can already be found in the MAS literature, though generally those hierarchies are not intended for exploring tendencies in a simulation. Usually they are aimed at facilitating a MAS implementation; different architectures allow a programmer to specify a MAS at different levels of programming.

The next section (8.6) is aimed at recapitulating how the *conception of emergence* has been elaborated in this thesis, underlining its usefulness for decreasing controversy in areas of research such as social simulation. The relevant aspects of the research stances from which this conception originated are brought in.

In section 8.7, the importance of a hierarchy of architectures as an informative and 'educative' tool for studying *soft systems*, and in particular for *policy analysis and management* (where the subject studying the simulation is usually situated in the target system), is considered.

Finally, in section 8.8, the convergence of, on the one hand, communities of modellers working on MAS-based applications (e.g., MAS-based social simulation) progressing towards architectures allowing proofs and, on the other hand, communities working on logic-programming starting to use the more flexible programming tools of constraint logic-programming, will be discussed.

8.2 Discussion about the Conditions of how this Thesis works in relation to other Methods

8.2.1 How this Methodology would work in Simulation Platforms Different from SDML

The particular technique was developed for SDML, but the methodology could be adapted to other simulation platforms. The idea is to allow, in the corresponding platform, the implementation of a proving procedure by adding facilities (e.g., for assumptions management, splitting, backtracking, context-reasoning) and of how to rewrite the (possibly MAS-based or event-driven) simulation procedure in a single DB-RB pair in order to facilitate reasoning and to overcome particular limitations due to abstractions (like agents and time level hierarchies in SDML) that the particular platform presents. The general idea then would be, first, to reveal and, second, to exploit some of the language abstractions and facilities in order to build a proving procedure. The ideal step would consist in allowing automatic translation of simulation models from the original architecture into the proving architecture.

8.2.2 How Realistic is it to Implement an Automatic Platform for Translating and Proving Tendencies in a Simulation Model at Present?

Implementation of a platform for automatically translating MAS-based models into constraint-based models seems difficult at present, at least for ‘sufficiently’ rich models (e.g., those with many agents which are continuously eliminated and introduced in the simulation). A wider application of the methodology proposed in Chapter 5 would require computational procedures allowing simulation of the weak structural change that MAS-based platforms permit, e.g., introduction and elimination of agents.

That such difficulties exist is guessed because of the struggles in other areas of research where similar computational techniques are needed (e.g., in parallel simulation). In addition, theorem-provers are not flexible about modifying the structure of the model, so that it would not be possible to drive hints from this area of research.

To implement a constraint exploration of a simulation in rich models will involve dynamical manipulation of the rulebase and database from a meta-level. To do so, the database has to be appropriately designed in order to facilitate distinction among data associated with those entities whose structure can change or with entities that can be introduced in and eliminated from the simulation. The complexity of this computational work, plus that of the exploration the proof procedure requires (discussed in Appendix 6), would benefit from introducing parallel computational techniques, as these facilitates complex tasks by distributing computational sub-tasks among several processors. Nevertheless, parallel simulation is an area of research still struggling to develope appropriate computational techniques (see, for example, Nicol *et al.*, 1994). In conclusion, there are sufficient techniques difficulties in the way of creating a platform making possible constraint-based proofs of theorems, that we may guess it will take some time before an implementation accessible to modellers will be put into practice.

8.2.3 Trade-off between Complexity and Usefulness of the Techniques Proposed in this Thesis

With reference to the complexity of the search shown in Appendix 6, the most critical factor for the high complexity of the search is the number and frequency of the processes' choices. If this frequency is low (e.g., null for some time iterations), then the complexity of the exploration this thesis proposes would not be too high, as happened in the case presented in Chapter 7. This is the case in many simulation applications where the complexity is rooted in the simulation process rather than in the number of choices. For example, in the case study presented in Chapter 7, traders' price-setting might be based on a more complex cognitive model by taking into account and modelling additional factors of their environment, as is the case in existing models commonly used in organisational and social simulation (see for example, Moss, 1998; Carley *et al.*, 1998; Edmonds, 1999c). It will make an agent's decision more subtle but also more deterministic. In the other extreme case, if processes' choices are frequent, as in the case presented in Chapter 7, then the complexity of the search will be high and it will be costly to implement the proposed model constraint-based exploration. In such a case, a more syntactic approach might reduce the computational complexity of the search in terms of time and memory needed. In addition, to allow alternative settings for the environment (environment's choices) will increase the complexity of the exploration, though the analysis of such complexity can be

made using the study presented in Appendix 6 (e.g., environment's choices could be placed together with agents' choices in the '*or*' nodes of Figure 15.1).

8.2.4 Complexity of a Constraint Exploration of Simulation Trajectories in Some Applications

8.2.4.1 Robot Agents (as different from computational agents)

Usually, robots are situated in environments where they are supposed to interact physically among themselves and with their environment (examples of such applications can be seen at: http://www-poleia.lip6.fr/~drogoul/). They not only have capabilities for communicating via message-passing where physical interaction is abstracted and performed 'symbolically' (as trader-distributor interaction has been abstracted in the example of Chapter 7), but they also have capabilities for physical interaction; e.g., they might be able to move, to perceive noise and sound, to visualise, and to send sounds. In addition to other agents' behaviour, the events occurring in and the structure of its physical surroundings are relevant for an agent's cognition.

Possibly the complexity of the simulation model would be subtle, but this subtleness would be more due to the structure of the model than to the agent's choices. Consequently, there will hopefully not exist a high frequency of agent's choices and the complexity of a constraint-based exploration will not be too critical (see Appendix 6).

8.2.4.2 ALife and Microsimulation

In the ALife project and in Microsimulation applications (see, e.g., Langton, 1989; Nagel *et al.*, 2000), simpler agents are modelled than those considered in social simulation. They are simpler agents in the sense that their cognitive model does not generally correspond with elaborated theories of cognition. While in social simulation, evolution of both the agent's cognition and the model's structure are important phenomena, usually in models used in ALife and in Microsimulation the main interest is in the evolution of the agents and their interaction. The complexity in the experimentation is deeply rooted in the number of agents and their interactions rather than in the evolution of agents' cognitive models. Generation of the dynamics of the model commonly involves the introduction and elimination of agents in very dynamic populations. Consequently, even a few choices for each agent would make a constraint-based proving procedure difficult because of the large amount of branch points to appear (there are many agents), making the usefulness of the technique limited.

8.2.4.3 Event-driven Simulation

At present, some event-driven simulation languages allow some sort of structural change, e.g., limitedly in SLAM (see Pritsker, 1995) and, as has been shown in Terán (1995), more broadly in GLIDER. Allowed changes include elimination and introduction of elements of the structure of the model. For example, in Terán (1995), the exploration of several structural paths was executed in a single simulation. There, the alternative structures and conditions for the structural change to happen are predefined at the beginning of the simulation. In this sense the change is somewhat fixed in advance. This restricts 'creativeness' and other context-reasoning capabilities. In a rich environment this might be a very strong and even obstructive constraint. In addition, as these languages are procedural, it is not easy 'to evolve' agents' function (this is different in a declarative language like SDML). These are limitations for implementing context-sensitive changes in the agents' and model's structures and for constraint-based exploration of trajectories (either logical model or syntactic-based).

8.2.5 A More Practical Notion of Emergence: Considering Subjective Aspects in Addition to the Objective ones

This thesis attempts to find a more practical notion of emergence (considering subjective aspects) than those objectivist ideas traditionally used in science, e.g., in physics and in computational modelling. The idea is not to dispute the usefulness of the objectivistic conceptions, but to suggest alternative approaches more likely to be supportive in some modelling applications.

But, why is a subjectivist notion likely to be more practical in some cases? Consider a typical example where an objective notion of emergence is typically used: emergence of life on earth. Emergence of life is usually 'objectively' described as dependent on the object itself as, e.g., the result of chemical reactions. This seems a good approach as far as it works satisfactorily for the modelling goals. However, there are research areas such as those involved in studying social systems where good 'objective' approximations to key notions are missing – probably because of the high level of complexity of these systems - and more practical definitions would be valuable. An example is the notion of emergence of tendencies in social simulation.

In these cases, it seems that sometimes after a modeller adopts a simulation and modelling methodology, his subjective judgment of the simulation results and of the dynamics of the empirical system plays an important role in making theory. Because of

this, in the view of the present writer, it is important to uncover the subjective factors involved in studying simulation models, e.g., for studying emergent tendencies in social and organisational simulation. In addition, a formalisation of these ideas would help in explaining what happens in an agent cognitive model (see, e.g., section 4.5.2) when 'understanding' its environment.

This idea is not new and the distinction of subjective factors in the modelling process can be found in the simulation and modelling literature, for instance, in Zeigler's definition of experimental frame and his differentiation between real system, base model, and lumped model (Zeigler, 1976), and in Crutchfield's examining of subjective and objective factors present when modelling a complex system (Crutchfield, 1994b). However, usually little attention is accorded to the subjective aspects when defining key concepts for studying such systems. For example, it is not common to find reference to the involved subjective factors in definitions such as 'complexity of complex systems' or 'emergence in complex systems'. One of the few examples in this line is Edmonds's definition of complexity (Edmonds, 1995; 1999a).

8.2.6 Enveloping Tendencies: A New Approach for Characterising Simulation Outputs

Enveloping allows a modeller to prove properties of the theory in the model and, having the appropriate validation and verification of the model, also about the theory against which the model has been verified, and to bring in new insights to the subject's cognitive model about the empirical system. This gives the model a special relevance as it might bring in changes in both the analytical theory and a modeller's knowledge. Changes in a modeller's knowledge can involve changes of particular beliefs about details of the unfolding of events in the empirical system, insights about more fundamental theoretical aspects of the theory in the model, and more general conclusions about the empirical system.

8.2.7 Enveloping Outputs in Simulations of Chaotic Systems

The idea is to discuss how the notion of envelope might be used in simulations of chaotic systems. It might, for instance, be of interest to study closely the behaviour of a certain output around an attractor. A first envelope could be generated after specifying some constraints over the model's parameters to delimit that area of the dependent (controllable) variables where the attractor is supposed to be generated. This first approximation would probably not be very precise because, e.g., either the space of parameters is too wide or the

aimed envelope has proved to be too coarse. After this first envelope has been generated, some experience is won, 'tighter' envelopes may be attempted, and further investigation of the tendency can be carried out. This may be put into practice by redefining either the specification of the attempted envelope (the theorem to be proved in the case studied in this thesis) or the constraints over the space of parameters.

This interactive enveloping process would allow a modeller to learn and to collect data from the experimentation in the computer to attempt theoretically valid conclusions. Ultimatelly it would assist a modeller in characterising appropriately (the modeller will decide when to stop experimenting according to his satisfaction with the results) that region of parameters and outputs where the behaviour of interest happens. The modeller will have a set of maps: range of parameters → envelope of outputs.

8.3 Implications for the Modelling of Complex Systems

A hierarchy of modelling architectures, such as that proposed in Chapter 6, gives a modeller a battery of tools with which he can obtain complementary information. The automatic implementation of this architectures in a common platform would help a modeller to deal better with the complexity of a target system (and its model) than if using a single architecture. The different and complementary views given by the various architectural levels can assist modellers in improving their understanding and in being better informed about the simulated system. A characteristic of the proposed hierarchy is that, the lower the level of programming, the more difficult it is for a modeller to understand a simulation, but the wider will be the fragment of the simulation theory the program will be able to investigate and the more general will be the conclusions the modeller can draw from the simulation outputs.

In this thesis a MAS is proposed as the higher level of programming for modelling a target system that can be well described as the interaction of components. In many cases MAS are supposed to express the dynamics of a simulation more flexibly than traditional network-based languages (e.g., SLAM; Pritsker, 1995). MAS allow for the representation of the evolution (e.g., certain structural changes) of key elements of a simulation. For example, the evolution of functions (e.g., that function given the dynamics of a system) can be easily simulated in a declarative MAS by means of evolving rules. A rule can be seen as the finest 'evolving function' (a function-particle) and a set of evolving rules as an evolving function. In addition, the possibility of introducing and eliminating agents seems

convenient for modelling systems components of the simulation system commonly enter and leave.

On the other hand, ideas from MAS could be adapted to model event-driven simulations more flexible than traditional languages (e.g., SLAM). Both the network where an entity carries out its cycle of life (this sort of simulation is explained in Zeigler, 1976) and the entity itself, in an event-driven simulation, might be represented as agents. The net might be modelled as a composite agent, which defines the overall system of the simulation, while the components of the net (e.g., lists, gates, resources) might be represented as sub-agents. Evolving rules might be used for functions that give the dynamics of the simulation, such as control of the future event list (FEL) and event's consequences (e.g., transition rules implementing change in the state of the system and further scheduling of events).

A hierarchy of architectures would also help a modeller in elaborating alternative simplifications of a model (valid under different experimental frames). For example, using the (logical) model constraint-based architecture proposed in Chapter 6, different tendencies might be proved, after which each of them could be used for making a different simplification. Each simplified model would accurately represent a different aspect of the original simulation model. Linked to each simplification would be a different experimental frame, a different output of interest for a modeller, and a different homomorphism function. Another way of generating alternative simplifications of a simulation model is by changing the fragment of the simulation theory explored (see Figure 8.1).

8.4 Implications for the Social Simulation Community

Scientists in the social simulation community are especially interested in investigating what they have called emergent tendencies in social systems. However, the notions they use appear to be different from modeller to modeller. This introduces ambiguity, which seems to be due to the difficulties in understanding social behaviour. In turn these difficulties appear to be due in part to the complexity of a social system and to a modeller's bounded rationality. A more transparent conception of emergence, along with more appropriate simulation tools, might help a social modeller to deal with the complexity of these systems and to understand emergent tendencies better.

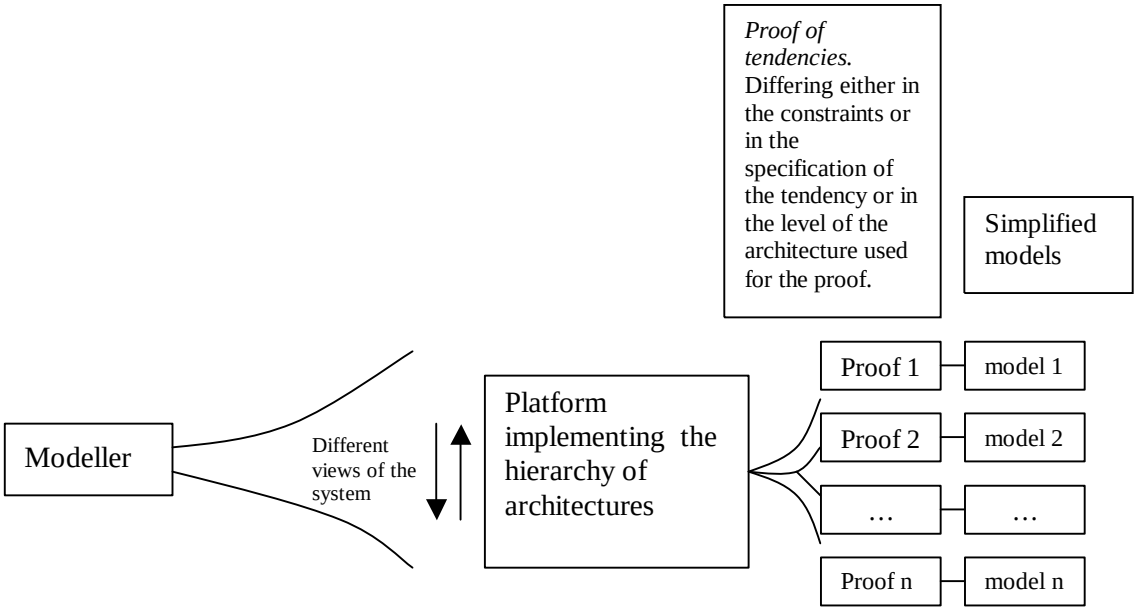


Figure 8.1. The proposed platform for a hierarchy of simulation architectures that can be used for different purposes, e.g., for generating alternative simplified models

More concretely, among the achievements of this thesis that might be useful to the social simulation community, there are: the conception of emergent tendencies as relative to a subject's notion of satisfiability and grounded in a trade-off between objective and subjective aspects, and the proposal of an automatic platform for translation simulation models among different programming architectures. The proposed relativistic notion of emergence of tendencies helps modellers to understand and accept the relativity of their individual conceptions about the emergence of tendencies, tells them about the sources of this relativity and may facilitate agreement within the scientific community. In addition, this might help them decrease the incidence of polemical discussions. Recognising such relativity would facilitate modellers' agreement in discussions about social processes, even when they disagree about whether particular tendencies observed in those processes are emergent.

On the other hand, the possibility of proving restricted homomorphic transformations between models facilitates access to a variety of procedures for validation, alignment, simplification, and integration of models, using stronger criteria than those currently used. The idea is to provide modellers with a methodology allowing the application of the strong notion of morphism given by Zeigler over fragments of the simulation theory, rather than the weak notion of approximation as used, for example, in Monte Carlo techniques. This methodology might be useful, for example: in drawing more general conclusions about

how roles, norms, and organisational design place bounds on social cognition (in research like that of Carley *et al.*, 1998); or increasing confidence about detecting gaps, inconsistencies, or errors in organisational theories (in studies like those of Moss *et al.*, 1998b; Axtell, 1996).

8.5 Implications for the MAS Community

Providing MAS programmers and researchers with a hierarchy of architectures might help them to deal better with their programs. For instance, a lower level than the MAS itself would be very helpful for a programmer in debugging and verifying programs. Moreover, providing automatic translation into and from other architectures may widen the applicability of MAS approaches, as more researchers will find them supportive.

It would be convenient to build platforms allowing the automatic translation of MAS models into other architectures. Different architectures might be enabled according to a modeller's interest and needs.

It is the intention of people working on SDML to provide these kinds of facilities for the social simulation community. In principle, SDML has been designed specifically for social simulation. At present, it is intended to provide additional facilities to allow a modeller to prove tendencies in social processes. The experience gained in developing this thesis is part of a permanent effort towards adding into SDML more appropriate tools in accord with a social modeller's needs. The idea of using MAS architectures for studying emergent tendencies in a simulation appears to be a novel one. Among other approaches for using architectural transformation in MAS are:

- DeLoach *et al.* (2000) suggest building a MAS by going through a hierarchy of architectural transformations. A methodology, 'Multiagent Systems Engineering (MaSE)', and a tool to support it, 'agentTool', are proposed. Their idea is to build a 'more concrete' sequence of models in a 'top-down' hierarchy of architectural levels, starting from the level of goals. There, the abstract specification of a MAS, the higher level in the hierarchy, is captured by a set of goals. This methodology is aimed to help with MAS verification, facilitating its building in accordance with some abstract specification, but not to assist in the understanding of the dynamics of a MAS.
- Shapiro *et al.* (2000) suggest using a language, 'Cognitive Agents Specification Language (CASL)', for specifying and verifying a MAS. They claim that this language provides a formal specification allowing abstraction over an agent's preferences and over agents' interaction (messaging) mechanism. This formal

representation is also meant to support a modeller when checking aspects of the agent's design but not to help in the understanding of the behaviour of a MAS.

- Koen *et al.* (2000) use contextual information for adding flexibility into the behaviour of agents' using preference models. In particular, they propose building agents able to 'adapt' their plans in an uncertain environment with soft deadlines by using a context-sensitive planning. They claim these agents have more 'realistic' preference models than those commonly used in other approaches. Their idea of a context-sensitive planning is comparable to the notion of context-driven exploration of simulation trajectories proposed in this thesis as a second level of architectural transformations.
- The study of Riley *et al.* (2000) is related to understanding MAS and observing aspects of their dynamics; in this sense it is related to the subjects of interest of this thesis. Concretely, they propose a 'layered disclosure by which autonomous agents have included in their architecture the foundations necessary to allow them to display upon request the specific reasons for their actions'. In fact, this mechanism permits a modeller to check the state of the internal model of an agent a at certain simulation time. This sort of mechanism is programmable in SDML by writing the specific rules for required reports, or stopping the simulation and querying the agents' databases. In addition, SDML allows a modeller to return to previous states in the simulation. The analysis of the dynamics of a simulation they propose is quite simple and not so useful for understanding aspects of the simulation theory. They do not address the more fundamental aspect of analysing tendencies (and in this way the simulated processes), but rather aspects at certain isolated simulation instants.

8.6 Bringing Ideas from other Areas of Research: The Conception of Emergent Tendencies

In order to help in *clarifying* the concept of *emergent tendencies* in fields such as that of the social simulation community (where there are discrepancies about this term), ideas from different areas of knowledge have been introduced and synthesised to produce the definitions used in this thesis. For this, I have found the following particularly valuable: a conception of complexity as relative to a subject's language (Edmonds, 1998), a conception of complexity as grounded in the complexity of the object itself (complexity categorised according to certain levels of phenomena given by Heylighen), and, finally, basic Kantian ideas about how human beings gather knowledge. The first two viewpoints

were reviewed in Chapter 4. The third, the Kantian view of the learning process in human beings (which seems to be the starting point for more elaborate models of knowledge and has been useful in different areas of research), will be briefly discussed below.

Kant recommended to humans (as subjects) the following *plan* for gathering knowledge (following the interpretation of Solomon (1996)):

1. There are rules via which a subject interprets its experience as true – necessarily true.

These are rules, facts, and processes taken as axioms; among them are:

- Causality
- Relativity of a human (subject)'s knowledge
- Induction

He also claimed that a subject's reasoning is always about his own mental models, built in part by using perception of phenomena from the empirical world, rather than about attributes existing in the empirical world, since empirical world features cannot be perfectly seized by the cognitive abilities of a human being.

2. Any other axiom contradicting the previous ones is false.

This point assures consistency of the chosen set of valid axioms while allowing alternative sets of valid axioms. It will permit the elaboration of different cognitive representations of systems of knowledge.

3. Other knowledge is either contingently true or contingently false.

These three first points in Kant's plan sound like to a process of elaboration of a logical system; e.g., this point allows the setting of axioms which are then used to check which knowledge is contingently true or contingently false via inference rules. In fact, it seems that a further development of Kant's ideas gave birth to the first logical formalisations (see Solomon, 1996) and, further on, they may have served as basic notions for elaborating cognitive models of agents in artificial intelligence.

Kant's plan situates knowledge as *relative* and contingent to a subject's worldview, as finally it is the subject who decides which axioms he takes as valid. Different subjects might disagree about which axioms to take as valid.

A *social environment* seems to have mechanisms for influencing individuals' beliefs. Agents' beliefs and actions are in part shaped by the set of values and norms taken as 'normal' in some society at a certain time. Different theories have attempted to explain such influence by different mechanisms. Some of them are found in philosophy, for

example in constructivism. These areas could be helpful in given additional ideas for a further elaboration of a concept of emergent tendencies.

In the following, it will be argued that *knowledge of systems at different levels of complexity* (see Chapter 4) can be categorised according to these three first points in the Kantian plan. To eliminate bias in the discussion, it will be assumed that the subject is a scientist specialised in the area of research and engaged in studying the phenomena alluded to.

First, consider a *physicist observing unanimated matter*, e.g., phenomena at level of complexity one. There is a vast mass of models and data that are commonly agreed upon, since individuals in that community agree about much of their knowledge. Hence, a lot of their knowledge is placed at levels one and two of this plan. However, a good deal of controversy exists in areas such as fundamental particle physics (these fields would be located at level three of the Kantian plan).

Now consider knowledge at the following higher level (e.g., at the level of *living organisms).* In this case, the *subject is a biologist.* Phenomena can still be discerned using these three first steps of Kant's plan. But there seems to exist less agreement about many observed aspects in the target system than in the case of a physicist observing an unanimated system (systems at this level undergo new phenomena in addition to that in the lower level). There will be more controversial issues of knowledge, i.e., more topics of knowledge at level three. As a result, it seems biologists' target systems are at a higher level of complexity than those of physicists. Emergent tendencies are more likely to appear in a biologist's model of his target systems than in physicists' models of their target systems.

Assume now that the object is *social phenomena* and that the *subject is a sociologist*. It would be even less easy for a social scientist than for a biologist to find consistent and more general explanations about tendencies in his target systems, as these are at a higher level of complexity, since a social scientist places a higher proportion of his knowledge at a higher level of complexity than does a biologist or a physicist. A higher proportion of his knowledge will be controversial than in the case of physicists'.

Finally, it seems that there are phenomena that are unreachable by human beings' understanding, for example, phenomena linked to theological and philosophical questions. Part of such knowledge would be the content of answers to questions about the existence of a god, or, the less soft answers to the question whether there exists an 'absolutely' correct set of valid axioms to be taken at level one of the Kantian plan. Questions like the

former of theses (that about the existence of a god) are not considered as answerable through human beings' reasoning, as topics in this area are not practically discernable via experience. This sort of knowledge is considered in conformity to the next Kantian policy. On the other hand, questions like the second one seem to tempt humans studying related subjects into endless discussions.

4. Phenomena not decidable via experience are not topics of knowledge.

Exploring the meaning of debatable terms (in this case related to studying systems at a high level of complexity) by using notions from thinkers working in foundational areas of knowledge (e.g., philosophy) can be useful for clarifying such terms. Hopefully, this sort of activity can be helpful in decreasing controversy within a research community and in facilitating communication among different research communities.

8.7 Implications for Policy Analysis

The aspect of the results achieved in this thesis that is more relevant for policy analysis is the 'educative' orientation of the proposed platform to assisting the user by allowing translation among different modelling architectures. In particular, the proposed constraint-based architecture enables a modeller to establish more general conclusions (as a wider fragment of the simulation is explored) than does a traditional MAS exploration. On the other hand, a MAS helps a modeller to improve his understanding about more specific aspects of a simulation. It is helpful for obtaining detailed information about the dynamics of single simulation trajectories.

In an organisation (e.g., a corporation) such a platform can be used for filtering and providing information relevant to users according to their role in the organisation in order to improve the behaviour of the whole corporation, as a complement to the sort of support given by, e.g., traditional information systems, scenario analysis, and Monte Carlo techniques.

The technique analyses the envelope of tendencies rather than central measures as in Monte Carlo techniques. Unlike central measures analysed via Monte Carlo techniques, this confers prominence to tendencies found in the worst investigated cases. An envelope expresses the outside bounds of a tendency, so that it may give managers or policy analysts greater confidence for acting in the system they are modelling compared to an analysis of central tendencies as in Monte Carlo techniques, which provides only probabilistic conclusions.

8.8 Proving Tendencies in MAS-based Models and Constraint Logic-programming

Usually languages used by modellers working in soft systems such as simulation of social systems have been different from those used by people working on problems related to harder systems such as applications in logic-programming and theorem-proving. Nevertheless, in the last years some groups working in logic-programming have moved towards using more flexible programming tools. They seek to go from the more traditional syntactic and backward-chaining mechanisms (e.g., Prolog and OTTER) towards more flexible tools exploiting better the semantics in a computational model. An example is that of people working in constraint forward-chaining-based programming (Frühwirth, *et al.*, 1992).

A move by the social simulation community towards more formal tools such as declarative programming also seems convenient. One of the main advantages of declarative programming is its potential for implementing formal proofs about the behaviour of a simulation that is difficult in traditional imperative programming (see Figure 8.2 and the final discussion in Chapters 7 and 8). This has been the intention of people working at the Centre for Policy Modelling, Manchester Metropolitan University, when adding the simulation language SDML facilities for theorem-proving (see Chapter 6). In fact, the experience given by this thesis is aimed at helping in furthering changes in such a direction.

Thus, me see some convergence between the theorem-proving and social simulation communities, a convergence that could hopefully be beneficial to both. This thesis can be seen as a step in this convergence.

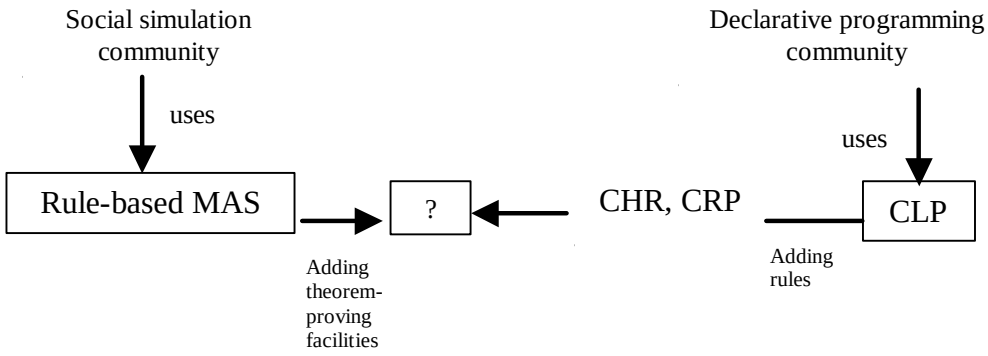


Figure 8.2. Declarative programming and social simulation communities' tools, and recent moves towards common platforms

9 Conclusion

In this thesis, a hierarchy of architectural transformations for exploring the dynamics of a simulation, a methodology for proving emergent tendencies in fragments of a simulation theory, a conception of emergent tendencies aimed at helping in practical modelling applications, and an envelope of tendencies as an alternative to statistical summaries for characterising simulation outputs are proposed. They are useful in the modelling process in terms of their potential to help a modeller in better understanding the dynamics of a simulation.

For example, in social simulation, modelling can help to better understand observed tendencies or to discover new tendencies in a social system. In this case, a hierarchy of programming architectures (such as that proposed) may provide a modeller with complementary information about behaviour in a simulation, especially about emergent tendencies, supporting modelling in both of these tasks. In addition, a definition of the concept of emergence of tendencies helps strengthen the support of this hierarchy of architectures for the social simulation community, as this notion is very important for understanding the processes a social system undergoes, by helping in clarifying this concept.

It has been argued that the emergence of tendencies is grounded in a trade-off between objective and subjective aspects. Objective aspects are linked to an object's potential for objective complexity, which in turn depends on the level of complexity of the observed system (as described in Chapter 4). Conversely, the subjective aspects are related to the difficulties a subject faces when understanding the system, which are finally linked to a subject's language. Moreover, it is recognised that the characterisation of a tendency as emergent in the end depends on a subject's judgement. The subject's decision depends on his satisfiability with an explanation a model offers about how the tendency arises. It was argued that the higher the potential for objective complexity in an object (that system where the tendency has been perceived by the subject) and the less appropriate for modelling the observed system the subject's language is, the higher is the likelihood that the subject characterises a tendency observed in such an object as emergent.

The hierarchy of architectures of programming that has been presented is aimed at helping a modeller increase his knowledge about the target system. The idea is to provide a modeller (conceived as a subject) with complementary information sources. The higher the level of programming, the more detail about the simulation the architecture will give, but

the smaller will be the fragment of the simulation theory such architecture can explore. In this particular study the hierarchy of architectures consists of: a MAS proposed as the highest level, a (logical) model constraint-based exploration of trajectories at an intermediate level, and a further system with a syntactic constraint-based inference mechanism as the lowest level of programming. Architectures at high levels would be better for exploratory analysis in a search for tendencies in the simulation, while architectures at low levels would be more valuable for making restricted proofs of tendencies in the simulation. In this sense, the last two architectures in the proposed hierarchy are intended to prove tendencies in a fragment of the simulation theory defined by a range of parameters of the simulation model and choices of the agents, and the logic of the simulation language.

A technique and an application were given in Chapter 7 for the intermediate architectural level. This enabled the investigation of all simulation trajectories for a range of parameters of the model and choices of the agents, and a finite number of time levels. This architecture allowed a restricted proof of tendencies in the case it was applied to.

Similar notions to the proposed hierarchy of architectures of programming aimed at informing and 'educating' a user can be found in computer science applications, e.g., in databases and information systems. For example, in some corporations different (information system) architectures are defined by the different accessibility criteria applied to users at different levels in the organisational hierarchy for accessing corporative databases. Thus a user at a low level of the corporative hierarchy of employees would access very limited data, probably only his personal record, while a manager in a high position would be much less restricted in accessing the database and would be able to obtain filtered (useful) information relative to the character of the decisions he has to take.

The envelope of tendencies in a simulation is an alternative to traditional statistical measures. It is valuable when exhaustive investigation of the dynamics of a simulation is pretended, as in theorem-proving. Moreover, it will hopefully be an option for studying complex systems overcoming drawbacks of traditional methods such as that pointed out by Crutchfield: 'fluctuations dominate behaviour and averages need not be centred in around the most likely value of its behaviour. This occurs for high complexity processes…since they have the requisite internal computational capacity to cause the convergence of observable statistics to deviate from the Law of Large Numbers' (Crutchfield, 1992, p. 35).

Among the issues discussed in Chapter 8, is the trade-off between the computational complexity and the usefulness of these techniques in some areas of application.

We have thus satisfied the *goals* of this thesis, namely:

- *To develop a methodological approach for exploring and proving emergent tendencies in computational models.*
- *To develop some techniques for implementing such a methodology.*
- *To investigate the trade-off between the computational complexity and the usefulness of these techniques.*
- *To develop some case studies.*
- *To review the conception of the emergence of tendencies.*

9.1 Further Work

Further work aimed at providing architectural transformations for better understanding complex systems includes:

- Implementing the third architecture of programming proposed in Chapter 7, namely a syntactic constraint-based architecture for exploring and proving tendencies of behaviour in a simulation.
- Building platforms allowing a modeller to implement automatic transformations of a single simulation model among different architectures.
- Making up a more elaborate theory of emergent tendencies guided by those ideas presented in Chapter 4 and in section 8.6.
- Investigating appropriate architectures of programming for different modelling applications.

10 Appendix 1 – Code of the Programmes

10.1 OTTER Code

Rules are listed as given by the ‘otter’ option ‘pretty_print’.

In otter, comments are indicated by written before the symbol ‘%’. This convention will be kept below.

Important points are:
- Rules in otter became clause in ‘usable_list’ (see Chapter 2)
- Initial data (initial state of the system) is place in the list SOS
- Hyper resolution is used (this is indicated in the first set used below)

```
set(hyper_res).
set(prolog_style_variables).
set(really_delete_clauses).
set(pretty_print).
set(split_when_given).
clear(print_new_demod).
clear(print_kept).
clear(print_back_demod).
clear(print_back_sub).
clear(no_fanl).
clear(no_fapl).
clear(print_given).
assign(demod_limit,400000).
make_evaluable(_-_,$DIFF(_,_)).
make_evaluable(_+_,$SUM(_,_)).
make_evaluable(_==_,$EQ(_,_)).
make_evaluable(_/_,$DIV(_,_)).
make_evaluable(_*_,$PROD(_,_)).
make_evaluable(_<=_,$LE(_,_)).
make_evaluable(_>=_,$GE(_,_)).
make_evaluable(_<_,$LT(_,_)).
make_evaluable(_>_,$GT(_,_)).

list(usable).
% Here will be placed clauses corresponding to what in SDML’s model
%initially  were rules. Also, the theorem to be proved will be in this
%list

1 [] -Factory(IFactory) | -Delay(Idelay) | delayFactory(IFactory,Idelay).
%gives the delay for a factory's production (it can also be seen as delay
%in a %trader's purchase

2 [] -Day(‘0’) | initialPrice(‘0.4’).
% gives initial price for one of the producers. The other producers'
%price will
% be a proportion of this value (0.4)

% the next three clauses give producer's price for day 1 (it is an
%initial price):
3 [] -Producer(IProd) | -$ID(IProd,’Producer-1’) |-initialPrice(Iprice) |
price(IProd,Iprice,’1’,[]).
4 []
-Producer(IProd) |-$ID(IProd,’Producer-2’) |-initialPrice(Iprice) |
price(IProd,fprod(Iprice,’1.2’),’1’,[]).
5 []
-Producer(IProd) | -$ID(IProd,’Producer-3’) | -initialPrice(Iprice) |
```

```
price(IProd,fprod(Iprice,'1.7'),'1',[]).

% Once price is given for 'Producer-1' at a day ('It'), and in a
%simulation path whose assumptions are taken in the variable 'Ia', next
%rule generates
% producer's choices for the following day (given as the sum
% of the value in 'It' and 1. The last parameter in the clause
% 'listSelProDay' gives the assumption for that choice. This assumption
% will be concatenated with the previous assumptions in 'Ia' for
% identifying data in the new branches.
6 [] -price('Producer-1',Iprice,It,Ia) |
listSelProDay('Producer-2','Producer-1','Producer-1',$FSUM(It,'1'),1) |
listSelProDay('Producer-2','Producer-1','Producer-2',$FSUM(It,'1'),2) |
listSelProDay('Producer-2','Producer-3','Producer-1',$FSUM(It,'1'),3) |
listSelProDay('Producer-2','Producer-3','Producer-2',$FSUM(It,'1'),4) |
listSelProDay('Producer-3','Producer-1','Producer-1',$FSUM(It,'1'),5) |
listSelProDay('Producer-3','Producer-1','Producer-2',$FSUM(It,'1'),6) |
listSelProDay('Producer-3','Producer-3','Producer-1',$FSUM(It,'1'),7) |
listSelProDay('Producer-3','Producer-3','Producer-2',$FSUM(It,'1'),8).

% Next clause calculates price. Notice that data used in the precedent
% (having the symbol '-'  preceding the clause) is checked to be sure
% they have the same assumption, expect listSelProDay which bring the new
% assumption ('Newa'). In the consequent the new price has as the
% assumption the concatenation of the old assumptions
%('Ia') and the new one 'Newa'. This price corresponds to a new iteration
%than that of the data used in the precedent. The new assumption for the
%new branch is brought in the variable 'Nt' of the clause
%'listSelProDay'. The final manipulations to calculate price are left to
% a demodulator 'demprice'.
% Demodulators work in a similar way than backward-chaining rules in
% SDML: their intermediate results are not written in the database-only
% the final result is returned.
7 []
-Producer(Iprod) |
-price(Iprod,Ioldprice,It,Ia) |
-maxDays(ImaxDays) |
-saleProd(Iprod,Mysales,It,Ia) |
-$FLE(It,ImaxDays) |
-saletot(Totsales,It,Ia) |
-price('Producer-1',Ioldprice1,It,Ia) |
-price('Producer-2',Ioldprice2,It,Ia) |
-price('Producer-3',Ioldprice3,It,Ia) |
-listSelProDay(Isel1,Isel2,Isel3,Nt,Newa) |
-$FEQ(Nt,$FSUM(It,'1')) |
-price(Isel1,Icomprice1,It,Ia) |
-price(Isel2,Icomprice2,It,Ia) |
-price(Isel3,Icomprice3,It,Ia) |
-pp1(Ip1,Ip2) |
price(
    Iprod,
    demprice(
        Iprod,
        Ioldprice,
        Ioldprice1,
        Ioldprice2,
        Ioldprice3,
        Icomprice1,
        Icomprice2,
        Icomprice3,
        Mysales,
```

```
        Totsales,
        Ip1,
        Ip2,
        '0.5'
    ),
    Nt,
    [Newa|Ia]
).

% This rule is responsible for given the initial production of a factory,
% e.g., % production at day 1. Notice that the list of assumptions (last
% field in the predicate 'productionDay') is empty.
8 []
-Producer(IProd) |
-storeOwner(IStore,IProd) |
-factoryOwner(IFact,IProd) |
-maxCapacity(IFact,Max) |
productionDay(IFact,Max,'1',[]).

%Next rule is responsible for setting production of a Producers'
% factories or setting traders' purchases). It can be seen that the value
% at present (iteration 'Nt') depends of the value of such a production
% at the previous iteration 'Ia'.  This rule is a translation into a
% clause in OTTER of the corresponding rule in SDML. A demodulator is
% used -to calculate production
9 []
-Producer(IProd) |
-storeOwner(IStore,IProd) |
-factoryOwner(IFact,IProd) |
-maxCapacity(IFact,Max) |
-level(IStore,Ilevelstore,IDay,Ia) |
-$FGE(IDay,'1') |
-orderprod(IProd,ITot,Nt,[Newa|Ia]) |
-$FEQ($FSUM(IDay,'1'),Nt) |
-delayFactory(IFact,Idelay) |
-capacityStore(IStore,Icapacity) |
productionDay(
    IFact,
    DemProd(Max,Icapacity,Ilevelstore,ITot,Idelay),
    Nt,
    [Newa|Ia]
).

% Clauses 10 and 11 are responsible for setting store's level. Clause 10
% sets initial level at time 1, while clause 11 updates level as time
% goes on. Some built in demodulators OTTER offers are used to calculated
% a sum and a difference.
10 []
-Producer(IProd) |
-storeOwner(IStore,IProd) |
-factoryOwner(IFact,IProd) |
-productionDay(IFact,Iproductionday,'1',Ia) |
level(IStore,Iproductionday,'1',Ia).
11 []
-storeOwner(IStore,IProd) |
-factoryOwner(IFact,IProd) |
-productionDay(IFact,Iproductionday,Nt,[Newa|Ia]) |
-maxDays(IMaxDays) |
-level(IStore,Iyesterdaylevel,It,Ia) |
-$FGE(IMaxDays,Nt) |
-$FEQ(Nt,$FSUM(It,'1')) |
```

```
-saleProd(IProd,Isaleprod,It,Ia) |
level(
    IStore,
    $FDIFF($FSUM(Iyesterdaylevel,Iproductionday),Isaleprod),
    Nt,
    [Newa|Ia]
).

%Clause 12 is responsible for setting producers' (traders') sales at
% iteration Ia.
12 []
-order(Icons,Iprod,Itotcons,Nt,Ia) |
-level(Istore,Ilevel,Nt,Ia) |
-storeOwner(Istore,Iprod) |
-orderprod(Iprod,Itotal,Nt,Ia) |
sale(
    Iprod,
    Icons,
    $IF(
        $FGE(Ilevel,Itotal),
        Itotcons,
        $IF(
            $FEQ(Itotal,'0'),
            '0',
            TruncPos($FPROD(Ilevel,$FDIV(Itotcons,Itotal)))
        )
    ),
    Nt,
    Ia
).

% Clauses 13 and 14 give consumers (distributors)' demand. Clause 13
% gives the initial demand at iteration 1, and clause 14 is responsible
% for updating this value at each state transition
13 []
-Consumer(Iconsumer) |
-Producer(Iproducer) |
-maxDays(InoD) |
-demandRange(Imin,Imax) |
-selProd(Iprodsel,Nt,Ia) |
-$FEQ(Nt,'1') |
demand(Iconsumer,demnD(Nt,InoD,Imin,Imax),'1',[]).
14 []
-Consumer(Iconsumer) |
-Producer(Iproducer) |
-maxDays(InoD) |
-demandRange(Imin,Imax) |
-demand(Iconsumer,Ioldlevel,It,Ia) |
-order(Iconsumer,Iproducer,Ioldorder,It,Ia) |
-salecons(Iconsumer,Isalecons,It,Ia) |
-sale(Iproducer,Iconsumer,Isale,It,Ia) |
-selProd(Iprodsel,Nt,[Newa|Ia]) |
-$FGT(Nt,'1') |
-$FEQ(It,$FDIFF(Nt,'1')) |
demand(
    Iconsumer,
    $FDIFF($FSUM(demnD(Nt,InoD,Imin,Imax),Ioldlevel),Isalecons),
    Nt,
    [Newa|Ia]
).
```

```
% Clause 15 gives the initial consumer (distributor) order and clause 16
% updates order for each state transition
15 []
-Consumer(Iconsumer) |
-Producer(Iproducer) |
-maxDays(InoD) |
-demandRange(Imin,Imax) |
-selProd(Iprodsel,Nt,Ia) |
-$FEQ(Nt,'1') |
order(
    Iconsumer,
    Iproducer,
    $IF($ID(Iprodsel,Iproducer),demnD(Nt,InoD,Imin,Imax),'0'),
    '1',
    Ia
).
16 []
-Consumer(Iconsumer) |
-Producer(Iproducer) |
-maxDays(InoD) |
-demandRange(Imin,Imax) |
-demand(Iconsumer,Ioldlevel,It,Ia) |
-order(Iconsumer,Iproducer,Ioldorder,It,Ia) |
-salecons(Iconsumer,Isalecons,It,Ia) |
-sale(Iproducer,Iconsumer,Isale,It,Ia) |
-selProd(Iprodsel,Nt,[Newa|Ia]) |
-$FGT(Nt,'1') |
-$FEQ(It,$FDIFF(Nt,'1')) |
order(
    Iconsumer,
    Iproducer,
    $FDIFF(
        $FSUM(
            $IF($ID(Iprodsel,Iproducer),demnD(Nt,InoD,Imin,Imax),'0'),
            Ioldorder
        ),
        Isale
    ),
    Nt,
    [Newa|Ia]
).

% This clause (17) is used to find that producer with the lowest price at
% iteration Ia. The prices at that iteration are the input to the
% demodulator 'demSelProd' responsible for returning the required
% producer.
17 []
-price('Producer-1',Iprice1,Nt,Ia) |
-price('Producer-2',Iprice2,Nt,Ia) |
-price('Producer-3',Iprice3,Nt,Ia) |
selProd(demSelProd(Iprice1,Iprice2,Iprice3),Nt,Ia).

% Next clauses, from 18 to 21, are used for calculating total orders and
% total sales by producer (trader) and consumer (distributor).
18 []
-order('Consumer-1',Iproducer,Iorder1,It,Ia) |
-order('Consumer-2',Iproducer,Iorder2,It,Ia) |
-order('Consumer-3',Iproducer,Iorder3,It,Ia) |
orderprod(Iproducer,$FSUM($FSUM(Iorder1,Iorder2),Iorder3),It,Ia).
19 []
```

```
-sale(Iproducer,'Consumer-1',Isale1,It,Ia) |
-sale(Iproducer,'Consumer-2',Isale2,It,Ia) |
-sale(Iproducer,'Consumer-3',Isale3,It,Ia) |
saleProd(Iproducer,$FSUM($FSUM(Isale1,Isale2),Isale3),It,Ia).
20 []
-sale('Producer-1',Iconsumer,Isale1,It,Ia) |
-sale('Producer-2',Iconsumer,Isale2,It,Ia) |
-sale('Producer-3',Iconsumer,Isale3,It,Ia) |
salecons(Iconsumer,$FSUM($FSUM(Isale1,Isale2),Isale3),It,Ia).
21 []
-salecons('Consumer-1',Isale1,It,Ia) |
-salecons('Consumer-2',Isale2,It,Ia) |
-salecons('Consumer-3',Isale3,It,Ia) |
saletot($FSUM($FSUM(Isale1,Isale2),Isale3),It,Ia).

% Next rule is the theorem. First, the data is brought in: Producer's
% prices at time Time1, time2 and time3, whose values are iterationg 1, 4
% and 7 as seen in the 10th clause. Afterwards, it is checked if the size
% of the intervals for prices is decreasing. There, two OTTER's built in
% facilities are used: $FGE, which means grater or equal than, and '$AND'
% used to evaluate disjunctions.
% The size of the interval of prices is calculated by the demodulator
% 'Interval'. In case the theorem is true the clause '-Contradition(1)'
% is generated creating a contradiction as the clause 'Contradiction(1)
% has been already put in the database (this clause is produced in the
% list 'sos', see below). This theorem not necessarily is the one
% reported in the thesis – this one checks only for not increase in the
% interval of prices for days 1, 4 and 7.
22 []
-price('Producer-1',Iprice11,Time1,Ia1) |
-price('Producer-2',Iprice12,Time1,Ia1) |
-price('Producer-3',Iprice13,Time1,Ia1) |
-price('Producer-1',Iprice21,Time2,Ia2) |
-price('Producer-2',Iprice22,Time2,Ia2) |
-price('Producer-3',Iprice23,Time2,Ia2) |
-price('Producer-1',Iprice31,Time3,Ia3) |
-price('Producer-2',Iprice32,Time3,Ia3) |
-price('Producer-3',Iprice33,Time3,Ia3) |
-$AND($AND($FEQ(Time1,'1'),$FEQ(Time2,'4')),$FEQ(Time3,'7')) |
-
$FGE(Interval(Iprice11,Iprice12,Iprice13),Interval(Iprice21,Iprice22,Ipri
ce23)) |
-
$FGE(Interval(Iprice21,Iprice22,Iprice23),Interval(Iprice31,Iprice32,Ipri
ce33)) |
-Contradiction(1).
end_of_list.


% Next list define OTTER demodulators used above for calculations. They
% work as backward-chaining rules in SDML list(demodulators).
% This demodulator calculates traders (producers)' prices
23 []
=(
    demprice(
        Iprod,
        Ioldprice,
        Ioldprice1,
        Ioldprice2,
        Ioldprice3,
        Icomprice1,
```

```
        Icomprice2,
        Icomprice3,
        Mysales,
        Totsales,
        Ip1,
        Ip2,
        ‘0.5’
    ),
    $IF(
        $FGT(comp(Mysales,Totsales),’0’),
        $FSUM(
            Ioldprice,
            $FPROD(
                ‘0.5’,
                $FPROD(
                    Ip1,
                    add(
                        Iprod,
                        Ioldprice1,
                        Ioldprice2,
                        Ioldprice3,
                        Icomprice1,
                        Icomprice2,
                        Icomprice3
                    )
                )
            )
        ),
        $IF(
            $AND(
                $FLT(comp(Mysales,Totsales),’0’),
                $FGE(
                    $FDIFF(
                        Ioldprice,
                        $FPROD(
                            ‘0.5’,
                            $FPROD(
                                Ip1,
                                add(
                                    Iprod,
                                    Ioldprice1,
                                    Ioldprice2,
                                    Ioldprice3,
                                    Icomprice1,
                                    Icomprice2,
                                    Icomprice3
                                )
                            )
                        )
                    ),
                    ‘0.05’
                )
            ),
            $FDIFF(
                Ioldprice,
                $FPROD(
                    ‘0.5’,
                    $FPROD(
                        Ip1,
                        add(
                            Iprod,
```

```
                            Ioldprice1,
                            Ioldprice2,
                            Ioldprice3,
                            Icomprice1,
                            Icomprice2,
                            Icomprice3
                        )
                    )
                )
            ),
            Ioldprice
        )
    )
).

% Next demodulator calculates the difference between a producer's sales
% and the mean of producer's sales. It is used by the demodulator
% 'demprice'.
24 []
comp(Mysales,Totsales)=$FDIFF(Mysales,$FDIV(Totsales,'3')).

%This demodulator gives the biggest difference between any pair of three
%numbers. It is used for calculating the size of the interval of prices
% at %certain day.
25 []
=(
    Interval(Iprice11,Iprice12,Iprice13),

$FDIFF(fmax3(Iprice11,Iprice12,Iprice13),fmin3(Iprice11,Iprice12,Iprice13
))
).

% Next demodulator is used for calculating producers' demand
26 []
=(
    DemProd(Max,Icapacity,Ilevelstore,Itot,Idelay),
    fmax2(
        TruncPos($FDIV(Max,'4')),
        fmin2(
            Max,

fmin2($FDIFF(Icapacity,Ilevelstore),TruncPos($FDIV(Itot,Idelay)))
        )
    )
).

% Next demodulator is used for determining the producer with the lowest
% price.
% It uses as auxiliary demodulators 'demSelProdi', i = 1,2, …7; e.g.,
% demodultores from 28 to 34.
27 []
=(
    demSelProd(Iprice1,Iprice2,Iprice3),
    $IF(
        $AND($FLT(Iprice1,Iprice2),$FLT(Iprice1,Iprice3)),
        'Producer-1',
        demSelProd1(Iprice1,Iprice2,Iprice3)
    )
).
28 []
=(
```

```
    demSelProd1(Iprice1,Iprice2,Iprice3),
    $IF(
        $AND($FEQ(Iprice1,Iprice2),$FLT(Iprice1,Iprice3)),
        ‘Producer-1’,
        demSelProd2(Iprice1,Iprice2,Iprice3)
    )
).
29 []
=(
    demSelProd2(Iprice1,Iprice2,Iprice3),
    $IF(
        $AND($FLT(Iprice1,Iprice2),$FEQ(Iprice1,Iprice3)),
        ‘Producer-1’,
        demSelProd3(Iprice1,Iprice2,Iprice3)
    )
).
30 []
=(
    demSelProd3(Iprice1,Iprice2,Iprice3),
    $IF(
        $AND($FLT(Iprice1,Iprice2),$FEQ(Iprice2,Iprice3)),
        ‘Producer-1’,
        demSelProd4(Iprice1,Iprice2,Iprice3)
    )
).
31 []
=(
    demSelProd4(Iprice1,Iprice2,Iprice3),
    $IF(
        $AND($FLT(Iprice2,Iprice1),$FLT(Iprice2,Iprice3)),
        ‘Producer-2’,
        demSelProd5(Iprice1,Iprice2,Iprice3)
    )
).
32 []
=(
    demSelProd5(Iprice1,Iprice2,Iprice3),
    $IF(
        $AND($FLT(Iprice2,Iprice1),$FEQ(Iprice1,Iprice3)),
        ‘Producer-2’,
        demSelProd6(Iprice1,Iprice2,Iprice3)
    )
).
33 []
=(
    demSelProd6(Iprice1,Iprice2,Iprice3),
    $IF(
        $AND($FLT(Iprice2,Iprice1),$FEQ(Iprice2,Iprice3)),
        ‘Producer-2’,
        demSelProd7(Iprice1,Iprice2,Iprice3)
    )
).
34 []
=(
    demSelProd7(Iprice1,Iprice2,Iprice3),
    $IF(
        $AND($FEQ(Iprice1,Iprice2),$FEQ(Iprice1,Iprice3)),
        ‘Producer-2’,
        ‘Producer-3’
    )
).
```

```
% This demodulator (35) is used to calculate consumers' new demand
35 []
=(
    demnD(Nt,InoD,Imin,Imax),
    TruncPos(
        $FPROD(
            '0.8',
            $FPROD(
                fmedia(Imin,Imax),

$FSUM('1',$FPROD('0.5',$FDIV($FDIFF(Nt,'1'),$FDIFF(InoD,'1'))))
            )
        )
    )
).

% The following demodulator (35) is used to trunk a non-negative 'real'
% (float point number) 'A'. First, the demodulator checks if it is
% between 0 and 1, in such a case then the answer is 0, in other case
% demodulator 'Trunck1' (demodulator 36) is called which recursively will
% check if the number is between B and B+1 starting with B = 1. This
% demodulator is used when calculating sales.
36 [] TruncPos(A)=$IF($AND($FLE('0',A),$FGT('1',A)),'0',Trunc1(A,'1')).
37 []
Trunc1(A,B)=$IF($AND($FLE(B,A),$FGT($FSUM(B,'1'),A)),B,Trunc1(A,$FSUM(B,'
1'))).

% The following demodulator returns the absolute value of the difference
% between a producer's price (Ioldpricei in case of producer-i, i =
% 1,2,3) and the price of the selected (chosen) producer for comparing
% prices (value kept in the variable Icompricei, i =1,2,3). This
% demodulator is called from the demodulator used for price-setting
% ('demprice').
38 []
=(
    add(
        Iprod,
        Ioldprice1,
        Ioldprice2,
        Ioldprice3,
        Icomprice1,
        Icomprice2,
        Icomprice3
    ),
    $IF(
        $ID(Iprod,'Producer-1'),
        fabsdif(Ioldprice1,Icomprice1),
        $IF(
            $ID(Iprod,'Producer-2'),
            fabsdif(Ioldprice2,Icomprice2),
            fabsdif(Ioldprice3,Icomprice3)
        )
    )
).

% Next demodulator is used in add to calculate the absolute value of the
% difference between A and B
39 [] fabsdif(A,B)=$IF($FGE($FDIFF(A,B),'0'),$FDIFF(A,B),$FDIFF(B,A)).

% clause 40 calculates mean value between two integers Min and Max and
```

```
% clause 41 calculates the mean of two float point numbers.
40 [] media(Min,Max)=sum(Min,Max)/2.
41 [] fmedia(Min,Max)=$FDIV($FSUM(Min,Max),'2').

42 [] dif(A,B)=A-B. % calculates the difference between A and B
(integers)
43 [] div(A,B)=A/B. % calculates the division between A and B (integers)
44 [] sum(A,B)=$SUM(A,B). % calculate the sum between A and B (integers)

% Next demodulators are responsible for trivial operations deducible from
% the demodulator itself and so no additional comment will be placed.
45 [] proddiv(A,B,C)=prod(A,B)/C.
46 [] demod1(A,B)=dif(A,B)*3.
47 [] demod2(A,B)=demod1(A,B)/2.
48 [] prod(A,B)=A*B.
49 [] fprod(A,B)=$FPROD(A,B).
50 [] min2(A,B)=$IF(A<=B,A,B).
51 [] max2(A,B)=$IF(A>=B,A,B).
52 [] fmin2(A,B)=$IF($FLE(A,B),A,B).
53 [] fmin3(A,B,C)=$IF($AND($FLE(A,B),$FLE(A,C)),A,fmin2(B,C)).
54 [] fmax2(A,B)=$IF($FGE(A,B),A,B).
55 [] fmax3(A,B,C)=$IF($AND($FGE(A,B),$FGE(A,C)),A,fmax2(B,C)).
56 [] total([])=0.
57 [] total([X|Z])=total(Z)+X.

end_of_list.

list(sos).

% This list places in the rulebase initial data and parameters.

% Next predicate is used to check the theorem. If the theorem is true the
% predicate '-Contradiction(1)' is generated creating a contradiction in
% the search path. If that happens OTTER backtracks to search into
% another branch
Contradiction(1).

Day('0'). %auxiliary clause used when generating the initial price

% Below other predicates like those indicating capacity of store, maximum
% capacity of a factory, names of factories, stores, producer owner of
% factories and stores and producers and consumers' names are given

capacityStore('Store-1','750').
capacityStore('Store-2','750').
capacityStore('Store-3','750').
listProd('Producer-1','Producer-2','Producer-3').
maxCapacity('Factory-1','750').
maxCapacity('Factory-2','750').
maxCapacity('Factory-3','750').
maxDays('8').
Factory('Factory-1').
Factory('Factory-2').
Factory('Factory-3').
Delay('4').
Store('Store-1').
Store('Store-2').
Store('Store-3').
factoryOwner('Factory-1','Producer-1').
storeOwner('Store-1','Producer-1').
factoryOwner('Factory-2','Producer-2').
```

```
storeOwner('Store-2','Producer-2').
factoryOwner('Factory-3','Producer-3').
storeOwner('Store-3','Producer-3').
Consumer('Consumer-1').
Producer('Producer-1').
Consumer('Consumer-2').
Producer('Producer-2').
Consumer('Consumer-3').
Producer('Producer-3').

% Other initialisations are given below (e.g., number of factories in the
% simulation and the predicates 'orderprod' and 'salecons' at time '0',
% which do not have significance in the model, but are used as fictitious
% data to make some rules more general):pp1('1','1').
demandRange('200','300').
noOfConsumers(1).
noOfProducers(1).
noOfFactories(1).
noOfStores(1).
salecons('Consumer-1','0','0',[]).
salecons('Consumer-2','0','0',[]).
salecons('Consumer-3','0','0',[]).
orderprod('Producer-3','0','0',[]).
orderprod('Producer-1','0','0',[]).
orderprod('Producer-2','0','0',[]).
saleProd('Producer-1','0','0',[]).
saleProd('Producer-2','0','0',[]).
saleProd('Producer-3','0','0',[]).
saletot('0','0',[]).
order('Consumer-1','Producer-1','0','0',[]).
order('Consumer-2','Producer-1','0','0',[]).
order('Consumer-3','Producer-1','0','0',[]).
order('Consumer-1','Producer-2','0','0',[]).
order('Consumer-2','Producer-2','0','0',[]).
order('Consumer-3','Producer-2','0','0',[]).
order('Consumer-1','Producer-3','0','0',[]).
order('Consumer-2','Producer-3','0','0',[]).
order('Consumer-3','Producer-3','0','0',[]).
sale('Producer-1','Consumer-1','0','0',[]).
sale('Producer-2','Consumer-1','0','0',[]).
sale('Producer-3','Consumer-1','0','0',[]).
sale('Producer-1','Consumer-2','0','0',[]).
sale('Producer-2','Consumer-2','0','0',[]).
sale('Producer-3','Consumer-2','0','0',[]).
sale('Producer-1','Consumer-3','0','0',[]).
sale('Producer-2','Consumer-3','0','0',[]).
sale('Producer-3','Consumer-3','0','0',[]).
```

end_of_list.

10.2 SDML Code, after Unwrapping Rules

10.2.1 Module Model

```
The forward-chaining rules driven the search are listed first. Then, at
the end, of this list, the list of backward-chaining rules will be given.

Rulebase: proving (n)>ModelAgent (content)

---------------------------------------------------------------------
```

```
Rule: alternatives for delay delivering (parameters)

Antecedents:
or
      is ?p2 4\
      is ?p2 10

Consequents:
p2 ?p2

Comment:
Gives delay for producers' purchasing. In fact only one alternative (4)
will be used

-------------------------------------------------------------------

Rule: create consumers (create)

Antecedents:
and
      namedInstance ?cons1 Consumer 'consumer-1'\
      namedInstance ?cons2 Consumer 'consumer-2'\
      namedInstance ?cons3 Consumer 'consumer-3'\
      = ?list [?cons1 ?cons2 ?cons3]

Consequents:
and
      consumer ?cons1\
      consumer ?cons2\
      consumer ?cons3\
      listCons ?list

Commment:
Create objects consumers (distributors) and a list with pointers to them
at the beginning of the simulation
-------------------------------------------------------------------

Rule: create producers (create)

Antecedents:
and
      namedInstance ?prod1 Producer 'producer-1'\
      namedInstance ?prod2 Producer 'producer-2'\
      namedInstance ?prod3 Producer 'producer-3'\
      = ?list [?prod1 ?prod2 ?prod3]

Consequents:
and
      listProd ?list\
      producer ?prod1\
      producer ?prod2\
      producer ?prod3
Commment:
Create objects producers (traders) and a list with pointers to them at
the beginning of the simulation
-------------------------------------------------------------------

Rule: create stores and factories (create)

Antecedents:
and
```

```
      producer ?producer\
      noOfFactories ?no1\
      noOfStores ?no3\
      inInterval ?i 1 ?no1\
      inInterval ?k 1 ?no3\
      generatedInstance ?factory Factory [?i ?producer]\
      generatedInstance ?store Store [?k ?producer]

Consequents:
and
      factory ?factory\
      store ?store\
      factoryOwner ?factory ?producer\
      storeOwner ?store ?producer

Commment:
Create factory and stores. The additional clauses indexes them to the
owner producer.
-------------------------------------------------------------------------

Rule: delay and maxcapacity values ... (parameters)

Antecedents:
and
      factory ?factory\
      demandRange ?min ?max\
      is ?maxcap ?max - ?min\
      truncated ?tmaxcap ?maxcap\
      is ?prom (?max + ?min) / 2\
      truncated ?promt ?prom\
      is ?maxprod 3 * ?promt\
      is ?cap 3 * ?max\
      randomChoice ?p2r ?p2 1
            p2 ?p2

Consequents:
and
      delay ?factory ?p2r\
      maxCapacity ?factory ?maxprod

Comment:
This rule gives: first the delay trader's get their purchase. It is take
as a random choice, but the simulation was run only for one.
The second value in the consequent is the maximum (upper bound) amount a
factory of producer can produce (factory production corresponds to a
trader’s purchase).


-------------------------------------------------------------------------

Rule: demand range (parameters)

Antecedents:
and
      = ?min 200\
      = ?max 300

Consequents:
demandRange ?min ?max

Comment:
```

```
Rule given what initially was thought as the range of demand … two values
used when calculating demand.

-------------------------------------------------------------------------

Rule: end simulation (simulation, start, end)

Antecedents:
and
      time day ?dayar\
      greater ?dayar 1

Consequents:
and
      final day

Comment:
This rules is used to stop the simulation.
The module 'model' keeps its internal time at 1 while the proof is going
on. The time of the model (which is changing) is manipulated explicitly
as it can be seen in the clauses.

As soon as the proof is successful and there is not possible backtracking
at time 1 of the module it goes to time 2 and then this rule stops the
simulation.

-------------------------------------------------------------------------

Rule: error computer (simulation, start, end)

Antecedents:
and
      = ?errorFactor 100000000\
      is ?roundError 1 / ?errorFactor

Consequents:
and
      errorFactor 100000000\
      roundError ?roundError

Comment:
This rule is used to deal with round error in the simulation. Two values
whose difference is too small might be considered equal. The computer
round error might be greater than the difference between the numbers. The
idea is that when comparing two numbers if their difference is smaller
than this error (given above as ?roundError) then the numbers are equal
for practical purposes. This idea is used when checking the theorem for
comparing the monotonic behaviour of the amplitude of the interval of
prices. If the difference between the amplitude of two intervals is
smaller than this value, then the intervals are assumed to be of similar
size.

-------------------------------------------------------------------------

Rule: factor for minimum price, factor for delta price (parameters)

Antecedents:
true

Consequents:
and
```

```
      factorDeltaPrice 0.5\
      factorMinimumPrice 0.3
Commment:
Obsolete?…

Gives a parameter factorMinimumPrice to be used to set lower bound for
price, and a parameter (factorDeltaPrice) used when changing prices.

-----------------------------------------------------------------------

Rule: initialice demand, order  (initializing)

Antecedents:
and
      producer ?prod\
      consumer ?cons\
      calcDemand ?idem and [1]\
      listSelProd1 [?prodsel ?other1 ?other2]\
      calcNewOrder ?prod ?prodsel ?idem and ?ordervalue

Consequents:
and
      demand1 ?cons ?idem and\
      order1 ?cons ?prod ?ordervalue

Comment:
Sets the demand for each consumer, where the new demand for each day
is calculated according to a linear increasing function. The total
demand of the consumer is the addition of the demand of the day and the
previous accumulated demand (or demand of the last day) minus the sales
at present day.
Each consumer places an order to each producer, among which only one is
different from zero. This is placed at that producers with the lowest
price.

noOfDays is used to determine the slop of the function. It is assumed
that
the demand at the end of the simulation period is twice the initial
demand.

In this case this rule is very specific and only given the initial values
at time 1.
-----------------------------------------------------------------------

Rule: initialice OrderCons, OrderProd (initializing)

Antecedents:
and
      listCons ?listCons\
      = ?listCons [?cons1 ?cons2 ?cons3]\
      listProd ?listProd\
      = ?listProd [?prod1 ?prod2 ?prod3]\
      order1 ?cons1 ?prod1 ?s1\
      order1 ?cons2 ?prod1 ?s2\
      order1 ?cons3 ?prod1 ?s3\
      order1 ?cons1 ?prod2 ?s4\
      order1 ?cons2 ?prod2 ?s5\
      order1 ?cons3 ?prod2 ?s6\
      order1 ?cons1 ?prod3 ?s7\
      order1 ?cons2 ?prod3 ?s8\
      order1 ?cons3 ?prod3 ?s9\
```

```
      is ?order1 ?s1 + ?s4 + ?s7\
      is ?order2 ?s2 + ?s5 + ?s8\
      is ?order3 ?s3 + ?s6 + ?s9\
      is ?orderp1 ?s1 + ?s2 + ?s3\
      is ?orderp2 ?s4 + ?s5 + ?s6\
      is ?orderp3 ?s7 + ?s8 + ?s9

Consequents:
and
      orderCons1 ?cons1 ?order1\
      orderCons1 ?cons2 ?order2\
      orderCons1 ?cons3 ?order3\
      orderProd1 ?prod1 ?orderp1\
      orderProd1 ?prod2 ?orderp2\
      orderProd1 ?prod3 ?orderp3

Comment:
Gives the total orders by producers and consumers at the initial
simulation time.

---------------------------------------------------------------------

Rule: initialice price (initializing)

Antecedents:
and
      iPrice ?iPrice\
      factoriPrice1 ?fiP1\
      factoriPrice2 ?fiP2\
      listProd ?listProd\
      = ?listProd [?prod1 ?prod2 ?prod3]\
      = ?iprice1 ?iPrice\
      is ?iprice2 ?iPrice * ?fiP1\
      is ?iprice3 ?iPrice * ?fiP2

Consequents:
and
      price1 ?prod1 ?iprice1\
      price1 ?prod2 ?iprice2\
      price1 ?prod3 ?iprice3

Comment:
Rule for initialising prices at time 1. Producer's 1 prices is as given
in ?iPrice in the beginning of the simulation an the other slightly
different according to the two factors given in ?fiP1, fiP2. In the
original model (before splitting) this factors were introduced directly
in the rule.

---------------------------------------------------------------------

Rule: initialice price param. (initializing)

Antecedents:
true

Consequents:
and
      iPrice 0.4\
      factoriPrice1 1.2\
      factoriPrice2 1.7
```

```
Comment:
Rule given parameters for prices.
Initial price will be set at 0.4 for a producer, and for the others, it
will be set as these values multiplied by factoriPrice1 and by
factoriPrice2.

-----------------------------------------------------------------------

Rule: initialise production day and level (initialising)

Antecedents:
and
      store ?store\
      factory ?factory\
      demandRange ?min ?max\
      is ?maxcap ?max - ?min\
      truncated ?tmaxcap ?maxcap\
      is ?prom (?max + ?min) / 2\
      truncated ?promt ?prom\
      is ?maxprod 3 * ?promt\
      is ?cap 3 * ?promt

Consequents:
and
      productionDay1 ?factory ?cap\
      level1 ?store ?cap\
      capacityStore ?store ?cap

Comment:
Rule to initialise production of factory (equal to its capacity) and
level of store at day 1. Also max. capacity of store is set ot ?cap.


-----------------------------------------------------------------------

Rule: initialice sales (initializing)

Antecedents:
and
      consumer ?consumer\
      producer ?producer\
      level1 ?store ?levelStore\
      storeOwner ?store ?producer\
      order1 ?consumer ?producer ?orderConsProd\
      level1 ?store ?levelStore\
      orderProd1 ?producer ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale1 ?producer ?consumer ?saleValue

Comment:
Determines a producer's (initial) sales for a consumer at day 1.

-----------------------------------------------------------------------

Rule: initialice SalesCons, SalesProd (initializing)

Antecedents:
and
      listCons ?listCons\
```

```
      = ?listCons [?cons1 ?cons2 ?cons3]\
      listProd ?listProd\
      = ?listProd [?prod1 ?prod2 ?prod3]\
      sale1 ?prod1 ?cons1 ?s1\
      sale1 ?prod1 ?cons2 ?s2\
      sale1 ?prod1 ?cons3 ?s3\
      sale1 ?prod2 ?cons1 ?s4\
      sale1 ?prod2 ?cons2 ?s5\
      sale1 ?prod2 ?cons3 ?s6\
      sale1 ?prod3 ?cons1 ?s7\
      sale1 ?prod3 ?cons2 ?s8\
      sale1 ?prod3 ?cons3 ?s9\
      is ?salec1 ?s1 + ?s4 + ?s7\
      is ?salec2 ?s2 + ?s5 + ?s8\
      is ?salec3 ?s3 + ?s6 + ?s9\
      is ?salep1 ?s1 + ?s2 + ?s3\
      is ?salep2 ?s4 + ?s5 + ?s6\
      is ?salep3 ?s7 + ?s8 + ?s9\
      is ?totalSales ?salec1 + ?salec2 + ?salec3

Consequents:
and
      saleCons1 ?cons1 ?salec1\
      saleCons1 ?cons2 ?salec2\
      saleCons1 ?cons3 ?salec3\
      saleProd1 ?prod1 ?salep1\
      saleProd1 ?prod2 ?salep2\
      saleProd1 ?prod3 ?salep3\
      totalSales1 ?totalSales

Comment:
Calculates the total sales by producer and by consumer.

------------------------------------------------------------------------

Rule: initialise select prod for first day (initialising)

Antecedents:
and
      backSelProd price1 ?listSel\
      clauseList ?cons listSelProd1 [?listSel]

Consequents:
?cons

Comment:
Writes in the clause listSelProd1 the list of producers ordered in
accordance to the value of their prices at time 1. This will be used by a
consumer to pick up the producer it will order from and for determining
the amplitude of the interval for prices.

------------------------------------------------------------------------

Rule: initializing choices1 (initializing)

Antecedents:
and
      listProd ?list\
      = ?list [?p1 ?p2 ?p3]\
      (or
            = ?choice [?p2 ?p3 ?p1]\
```

```
            = ?choice [?p2 ?p3 ?p2]\
            = ?choice [?p2 ?p1 ?p1]\
            = ?choice [?p2 ?p1 ?p2]\
            = ?choice [?p3 ?p3 ?p1]\
            = ?choice [?p3 ?p3 ?p2]\
            = ?choice [?p3 ?p1 ?p1]\
            = ?choice [?p3 ?p1 ?p2])

Consequents:
and
      choices1 ?choice

Comment:
Places the different alternatives for the different random choices of
producers for price-setting.  (those are not the choices at time 1)
----------------------------------------------------------------------

Rule: initialSelProd (initializing)

Antecedents:
listProd ?listProd

Consequents:
listChoiceProd1 ?listProd


Comment:
Gives the list of producers ordered in accordance to their price at time
1

----------------------------------------------------------------------

Rule: list NAMES prices, dependencies among prices ... (initializing)

Antecedents:
and
      = ?listNameTotalSales [totalSales1 totalSales2 totalSales3
totalSales4 totalSales5 totalSales6 totalSales7 totalSales8 totalSales9
totalSales10]\
      = ?listNamePrices [price1 price2 price3 price4 price5 price6 price7
price8 price9 price10]\
      = ?listNameListSelProd [listSelProd1 listSelProd2 listSelProd3
listSelProd4 listSelProd5 listSelProd6 listSelProd7 listSelProd8
listSelProd9 listSelProd10]\
      = ?listNameLevel [level1 level2 level3 level4 level5 level6 level7
level8 level9 level10]\
      = ?listNameOrder [order1 order2 order3 order4 order5 order6 order7
order8 order9 order10]\
      = ?listNameOrderProd [orderProd1 orderProd2 orderProd3 orderProd4
orderProd5 orderProd6 orderProd7 orderProd8 orderProd9 orderProd10]\
      = ?listNameOrderCons [orderCons1 orderCons2 orderCons3 orderCons4
orderCons5 orderCons6 orderCons7 orderCons8 orderCons9 orderCons10]\
      = ?listNameDemand [demand1 demand2 demand3 demand4 demand5 demand6
demand7 demand8 demand9 demand10]\
      = ?listNameSales [sale1 sale2 sale3 sale4 sale5 sale6 sale7 sale8
sale9 sale10]\
      = ?listNameSalesProd [saleProd1 saleProd2 saleProd3 saleProd4
saleProd5 saleProd6 saleProd7 saleProd8 saleProd9 saleProd10]\
      = ?listNameSalesCons [saleCons1 saleCons2 saleCons3 saleCons4
saleCons5 saleCons6 saleCons7 saleCons8 saleCons9 saleCons10]\
```

```
      = ?listNameProductionDay [productionDay1 productionDay2
productionDay3 productionDay4 productionDay5 productionDay6
productionDay7 productionDay8 productionDay9 productionDay10]\
      = ?listNameListChoice [listChoiceProd1 listChoiceProd2
listChoiceProd3 listChoiceProd4 listChoiceProd5 listChoiceProd6
listChoiceProd7 listChoiceProd8 listChoiceProd9 listChoiceProd10]

Consequents:
and
      listNameTotalSales ?listNameTotalSales\
      listNameSalesCons ?listNameSalesCons\
      listNameSalesProd ?listNameSalesProd\
      listNamePrices ?listNamePrices\
      listNameListSelProd ?listNameListSelProd\
      listNameLevel ?listNameLevel\
      listNameOrder ?listNameOrder\
      listNameOrderCons ?listNameOrderCons\
      listNameOrderProd ?listNameOrderProd\
      listNameDemand ?listNameDemand\
      listNameSales ?listNameSales\
      listNameProductionDay ?listNameProductionDay\
      listNameListChoice ?listNameListChoice

Comment:
This rule is very useful for meta, as it gives a list of the predicates
to be used (to instantiate as explicitly data like prices, sales, orders
…
In this model rather that having the predicate ‘price’ only and
indicating the time (1, 2, …) as a field in the predicate, it is
aggregate to the name of the predicate, and so will be more specific.

---------------------------------------------------------------------

Rule: minimumPrice (parameters)

Antecedents:
and
      iPrice ?initialPrice\
      factorMinimumPrice ?fMP\
      is ?minimumPrice ?initialPrice * ?fMP

Consequents:
minimumPrice ?minimumPrice

Consumer:
MinimumPrice will be the minimum price any producer can sell its good. It
can be said that minimum price is the production cost of the good.

---------------------------------------------------------------------

Rule: number of consumers, etc. (parameters)

Antecedents:
true

Consequents:
and
      noOfConsumers 3\
      noOfProducers 3\
      noOfStores 1\
      noOfFactories 1
```

Commment:
Indicates the number of consumers and Producer to be created in the model.
Also it indicate how many factories and Stores a producer has.

Rule: numbers of simulation days (simulation, start, end)

Antecedents:
true

Consequents:
noOfDays 6

Comment:
This rule indicates the number of iterations (called days) the simulation will be run

List of Backward-chaining Rules:

In SDML a clause can be defined as backward-chaining. A backward-chaining clause works as a ‘database’ with several several backward-chaining rules to generate the output of the clause. This is seen below. In the next list of backward-chaining clauses, the name of the clause appears after the word ‘definition’. The category of the clause is placed in parenthesis.

All backward-chaining rules but that responsible for building the theorem in meta have only one rule.

```
-------------------------------------------------------------------------
Rulebase: definition backSelProd (Calculations)

Rule: rule

Antecedents:
and
      listProd ?listProd\
      = ?listProd [?prod1 ?prod2 ?prod3]\
      clauseList ?NP1 ?nameprice [?prod1 ?price1]\
      clauseList ?NP2 ?nameprice [?prod2 ?price2]\
      clauseList ?NP3 ?nameprice [?prod3 ?price3]\
      ?NP1\
      ?NP2\
      ?NP3\
      (or
            (and
                  less ?price1 ?price2\
                  less ?price1 ?price3\
                  = ?prodsel ?prod1\
                  (or
                        (and
                              less ?price2 ?price3\
                              = ?prodsel1 ?prod2\
                              = ?prodsel2 ?prod3)\
                        (and
                              (or
                                    less ?price3 ?price2\
                                    = ?price3 ?price2)\
                              = ?prodsel1 ?prod3\
                              = ?prodsel2 ?prod2)))\
            (and
                  less ?price2 ?price1\
                  less ?price2 ?price3\
                  = ?prodsel ?prod2\
                  (or
                        (and
                              less ?price1 ?price3\
                              = ?prodsel1 ?prod1\
                              = ?prodsel2 ?prod3)\
                        (and
                              (or
                                    less ?price3 ?price1\
                                    = ?price3 ?price1)\
                              = ?prodsel1 ?prod3\
                              = ?prodsel2 ?prod1)))\
            (and
                  less ?price3 ?price2\
                  less ?price3 ?price1\
                  = ?prodsel ?prod3\
```

```
                  (or
                        (and
                              less ?price1 ?price2\
                              = ?prodsel1 ?prod1\
                              = ?prodsel2 ?prod2)\
                        (and
                              (or
                                    less ?price2 ?price1\
                                    = ?price2 ?price1)\
                              = ?prodsel1 ?prod2\
                              = ?prodsel2 ?prod1)))\
            (and
                  = ?price1 ?price2\
                  less ?price1 ?price3\
                  = ?prodsel ?prod1\
                  = ?prodsel1 ?prod2\
                  = ?prodsel2 ?prod3)\
            (and
                  = ?price1 ?price3\
                  less ?price1 ?price2\
                  = ?prodsel ?prod1\
                  = ?prodsel1 ?prod3\
                  = ?prodsel2 ?prod2)\
            (and
                  = ?price2 ?price3\
                  less ?price2 ?price1\
                  = ?prodsel ?prod2\
                  = ?prodsel1 ?prod3\
                  = ?prodsel2 ?prod1)\
            (and
                  = ?price2 ?price3\
                  = ?price2 ?price1\
                  = ?prodsel ?prod2\
                  = ?prodsel1 ?prod1\
                  = ?prodsel2 ?prod3))\
      = ?listSel [?prodsel ?prodsel1 ?prodsel2]

Consequents:
backSelProd ?nameprice ?listSel

Comment:
This rule gives the name predicate names of producers' in the list
?listSel in order of value for certain iteration. It gets as input the
names of predicate of prices corresponding to that iteration in the
variable ?nameprice. For example, if the variable has 'price3', then
prices to be compared are the prices for iteration 3. This name is used
to build the clause to instantiate prices. E.g.; clauseList ?NP1
?nameprice [?prod1 ?price1]\
would build the clause to instantiate price3 of producer 1 and the value
would be placed in ?price1.
The returned list: ?listSel is used by consumers for order-setting (they
order form producer with the lowest price) and for calculating the size
of the interval for prices in theorem-checking.



------------------------------------------------------------------------
Rulebase: definition calcDemand (Calculations)

Rule: rule
```

```
Antecedents:
and
      noOfDays ?noD\
      = ?list [?nt]\
      demandRange ?min ?max\
      is ?media ?min + (?max - ?min) / 2\
      is ?newD (?media + ?media * 0.5 * ((?nt - 1) / (?noD - 1))) * 0.8\
      truncated ?nD ?newD

Consequents:
calcDemand ?nD ?list

Comment:
This rule does some numerical manipulations when calculating demand.
This particular way of calculating demand is not relevant for the
methodology.

----------------------------------------------------------------------
Rulebase: definition calcNewOrder (Calculations)

Rule: rule

Antecedents:
or
      (and
            = ?producer ?prodsel\
            = ?neworder ?demandDay)\
      (and
            notInferred
                  = ?producer ?prodsel\
            = ?neworder 0)

Consequents:
calcNewOrder ?producer ?prodsel ?demandDay ?neworder


Comment:
This rule performs some numerical manipulation for calculating consumers'
order. Orders are set to zero expect that to producer with the lowest
price (?prodsel)

----------------------------------------------------------------------
Rulebase: definition calcprice (Calculations)

Rule: calculating delta price

Antecedents:
and
      = ?listdata [?oldprice ?otherprice ?mysales ?totalSales]\
      is ?mediasales ?totalSales / 3\
      is ?difsales ?mysales - ?mediasales\
      factorDeltaPrice ?factorDeltaPrice\
      is ?difprices (?otherprice - ?oldprice) * ?factorDeltaPrice\
      absoluteValue ?absdifprice ?difprices\
      (or
            (and
                  greater ?difsales 0\
                  is ?pprice ?oldprice + ?absdifprice)\
            (and
                  less ?difsales 0\
                  is ?pprice ?oldprice - ?absdifprice)\
```

```
            (and
                  = ?difsales 0\
                  is ?pprice ?oldprice))\
      minimumPrice ?mP\
      (or
            (and
                  (or
                        = ?pprice ?mP\
                        less ?pprice ?mP)\
                  = ?price ?mP)\
            (and
                  greater ?pprice ?mP\
                  = ?price ?pprice))\
      errorFactor ?a\
      is ?milllionprice ?price * ?a\
      rounded ?millionpureprice ?milllionprice\
      is ?cleanedprice ?millionpureprice / ?a

Consequents:
calcprice ?cleanedprice ?listdata

Comment:
This clause is responsible for calculating price. The list input given in
the variable ?lisdata contain the prices and sales necessary for this
task. Then price is returned in ?cleanedprice. Round error is controlled
rounding the lest significant digits of the float point number in the
variable ?price. That is made using the inverse of the small number
considered as cero kept in the clause 'errorFactor'.

------------------------------------------------------------------------
Rulebase: definition calcProductionDay (Calculations)


Rule: rule

Antecedents:
and
      is ?prod2 ?orderProdToday / ?delay\
      truncated ?prod1 ?prod2\
      is ?limitStore ?capacity - ?levelInValue\
      is ?min1 ?maxCapacity / 4\
      truncated ?min ?min1\
      min ?prodDay1 ?prod1 ?maxCapacity ?limitStore\
      max ?productionDayOutValue ?min ?prodDay1

Consequents:
calcProductionDay ?orderProdToday ?delay ?capacity ?levelInValue
?maxCapacity ?productionDayOutValue

Comment:
This rule does numerical manipulations necessary for Production setting
are placed here.


------------------------------------------------------------------------
Rulebase: definition calcSale (Calculations)

Rule: rule

Antecedents:
and
```

```
      (or
            (and
                  = ?orderProd 0\
                  = ?per 0)\
            (and
                  greater ?orderProd 0\
                  is ?per ?orderConsProd / ?orderProd))\
      (or
            (and
                  greater ?levelStore ?orderProd\
                  = ?sale ?orderConsProd)\
            (and
                  (or
                        less ?levelStore ?orderProd\
                        = ?levelStore ?orderProd)\
                  is ?apsale ?levelStore * ?per\
                  truncated ?sale ?apsale))

Consequents:
calcSale ?levelStore ?orderProd ?orderConsProd ?sale

Comment:
This rule does numerical manipulations necessary for sale setting are
done using this rule.

-------------------------------------------------------------------------
Rulebase: definition otherPrice (Calculations)


Rule: taking the other price

Antecedents:
and
      listProd ?listProd\
      index ?listProd ?index ?myself\
      index ?choiceProd ?index ?mychoice\
      clauseList ?priceInsC ?priceIn [?mychoice ?otherprice]\
      ?priceInsC

Consequents:
otherPrice ?otherprice ?choiceProd ?priceIn ?myself

Comment:
This rule is used to get the price of a chosen producer. Two list are
used: the original list of producers ?listProd, and the list of chosen
producers ?choiceProd, for certain iteration. The producer in position
(index) i of the second list (?choiceProd) is the chosen producer by
producer i (that in position i in the list of ?listProd). This is
indicate by using the variable ?index. The position of a producer in
?listProd is taken in this index and then this value is used to take the
corresponding element from the list ?choiceProd. The correspondent clause
for instantiating price is built and then used. The searched price is
output in the variable ?otherprice.


-------------------------------------------------------------------------
Rulebase: definition getClauseName (Calculations)

Rule: rule

Antecedents:
```

```
and
      clauseList ?knowlistC ?knowlistname [?knowlist]\
      ?knowlistC\
      index ?knowlist ?index ?know\
      clauseList ?unknownlistC ?unknownlistname [?unknownlist]\
      ?unknownlistC\
      index ?unknownlist ?index ?unknown

Consequents:
getClauseName ?know ?knowlistname ?unknown ?unknownlistname

Comment:
As list of predicate names are built in accordance to the iteration the
data will be valid. E.g., considering lists for prices and sales: [price1
price2 …] and [sale1 sale2 ..], elements in position i in different lists
correspond to data valid for the same iteration. Given a predicate name
we could find its position in the list it has been place and then other
predicate names corresponding to data valid at the same iteration can be
found. That is the task of this rule. Given the predicate ?know in the
list of predicates ?knowlistname, then the predicate in the same position
(?index) in the list ?unknownlistname is found and placed in returned in
the variable ?unknown.
This rule is used by rule in the backward-chaining clause amIntPrice
(listed below).


---------------------------------------------------------------------

Rulebase: definition amIntPrice (prover)

Rule: back

Antecedents:
and
      getClauseName ?nameprice listNamePrices ?nameListSelProd
listNameListSelProd\
      clauseList ?listSelProdC ?nameListSelProd [?listSelProd]\
      ?listSelProdC\
      = ?listSelProd [?prodsel1 ?prodsel2 ?prodsel3]\
      clauseList ?priceC1 ?nameprice [?prodsel1 ?price1]\
      ?priceC1\
      clauseList ?priceC3 ?nameprice [?prodsel3 ?price3]\
      ?priceC3\
      is ?dif ?price3 - ?price1

Consequents:
amIntPrice ?nameprice ?dif

Comment:
This rule is responsible for calculating the amplitude of the interval of
prices for certain day (iteration). The input is the name of the
predicate of prices, input in the variable ?nameprice. E. g., if the
predicate ‘price3’ is given as input (in ?nameprice) then clauses to
instantiate the higher and the lowest price are build using information
from the list ?listSelProd, which contains the producer name predicates
(producer-1, ..) ordered in accordance to the value of price. Clause to
instantiate the required prices are build and then used. The value of the
difference is returned in the variable ?dif.
```

10.2.2 Module Meta

These are the rules to write automatically at prover the transition rules.

Rulebase: proving (n)>MetaProver (content)

Rule: writing checkTheorem (building transition rules)

Antecedents:
and
listProd ?list\
= ?list [?prod1 ?prod2 ?prod3]\
noOfDays ?noDays\
listNamePrices ?listNamePrices\
subList ?actualListPrices ?listNamePrices 1 ?noDays\
backBuiltTheorem ?actualListPrices ?antList\
clauseList ?antecedent and ?antList\
namedInstance ?ruletheorem RuleName ‘checkTheorem’

Consequents:
rule ?ruletheorem ?antecedent false

Comment:
Rule for writing the theorem at ‘prover’. It first collects necessary information from the database to build the theorem:
1. data collection:
Producer's names in the variables: ?prod1, ?prod2, ?prod3

Total number of simulation days in: ?noDays

The list of price predicate names ?listNamePrices (e.g, it contains, price1, price2, ...pricen, where n is the number of days the simulation is run). As this list could be kept fixed while noDays is changed in different experiments, the sublist of predicate names relevant in accordance to noDays in a experiment is taken in the variable ?actualListPries:

subList ?actualListPrices ?listNamePrices 1 ?noDays\

Then a backward-chaining rule is called to build the antecedent of the theorem:
backBuiltTheorem ?actualListPrices ?antList\
This rule gets as input the list of price predicates names and outputs a list of clause to build in the antecedent of the rule in: ?antList

2. building clauses to make up the rule:
Then the antecedent is built, using the clause ‘clauseList’, as the disjunction of the clause elements of the list ‘?antList’ (this is indicated by the second element of the clause: ‘and’). The antecedent is called ‘antecedent’.

clauseList ?antecedent and ?antList\

The name of the rules is generated using namedInstance:
namedInstance ?ruletheorem RuleName ‘checkTheorem’
The instance created is called ‘checkTheorem’ is of type RuleName andhas been placed in the variable ?ruletheorem

3. Finally the rule is created in the consequent:

```
rule ?ruletheorem ?antecedent false/
```

The name of the clause used to write the rule is ‘rule’. There can be seen as the elements this clause uses: name of the theorem, the antecedent of rule and the consequent of the rule (false).

Rule: writing Choices-Iter (building transition rules)

Antecedents:

```
and
      noOfDays ?nDays\
      listNameListChoice ?listNameListChoice\
      inInterval ?index 2 ?nDays\
      index ?listNameListChoice ?index ?choiceName\
      is ?previous ?index - 1\
      index ?listNameListChoice ?previous ?choiceNamePrevious\
      clauseList ?previousChoiceC ?choiceNamePrevious [?previousChoice]\
      clauseList ?choicesC choices1 [?allchoices]\
      clauseList ?rChC randomChoice [?randchoice ?allchoices ?index
?choicesC]\
      clauseList ?ant and [?previousChoiceC ?rChC]\
      clauseList ?cons ?choiceName [?randchoice]\
      generatedInstance ?ruleChoices RuleName ‘Choices-Iter’ ?index
```

Consequents:

```
rule ?ruleChoices ?ant ?cons
```

Comment:

This rule is responsible for splitting and writing at prover rules for making producer (traders)'s choices for price-setting.
To figure out how meta write rules at prover, this rule in particular will be explained in more detail:

1. First data is brought from the rulebase and auxiliary variables are generated:

a) Number of simulation days: noOfDays ?nDays\

b) Choices predicate names for each day, e.g., choice1, choice2, ... which are in a list whose clause is called listNameListChoice are brought:

```
listNameListChoice ?listNameListChoice\
```

c) Then ?index is create as a variable keeping the integers from 2 to the number of simulation days: inInterval ?index 2 ?nDays\

An auxiliary index is defined as index minus one:

```
is ?previous ?index - 1\
```

These two indexes are used as pointers to the names of choices in the list kept in the variable ?listNamListChoice. There will be a rule for each name in the list, e.g., there will be a rule for each iteration (day) from one to number of days.

The rules written at prover will generate choices one at a time (so that the program backtracks for one at a time) -a different assumption is will be set for each choice. For this, the rule to make the choice for day-i, to be written in module ‘prove’, will instantiate the choice for the previous day-(i-1) (there will be dependency among every two rules).

While the variable ?index is used to take the name of the predicate of choice for the day the rule will give the choice (e.g., choice-i; if ?index is at i), the variable ?previous is used to take the name of the choice predicate for the previous day and will be used to instantiate it.

index ?listNameListChoice ?index ?choiceName\
Takes in ?choiceName the name of the predicate with index ?index (2, ...noOfDays)(e.g, choice3)

index ?listNameListChoice ?previous ?choiceNamePrevious\
Takes in ?choiceNamePrevious the name of the previous choice (e.g., choice 2)

2. Then clause are written .. two to be used in the antecedent of the rule directly (?previousChoiceC and ?rChC) and other (?choiceC) to be used inside other clause (?rChC).

a) clauseList ?previousChoiceC ?choiceNamePrevious [?previousChoice]\
This clause builds a clause, in the varible ?previousChoiceC, for instantiating the previous choice.

b) clauseList ?choicesC choices1 [?allchoices]\
This build a chause to instantiate choices of producers which name ?choiceC (the list of choice of producers are in the dababase under the clause ‘choices1’, so the variable ?allchoices will instantiate the choices when the clause is used.

c) clauseList ?rChC randomChoice [?randchoice ?allchoices ?index ?choicesC]\
This build a clasue for doing the random choice (notice that the choice will be taken from those lists instantiates in ?allchoices, which is in the clause ?choicesC. The name of the variable containing the clause is ?rChC. The choice will be placed in the variable ?randchoice.

d) clauseList ?ant and [?previousChoiceC ?rChC]\
The antecedent of the rules is built as the disjunction of two clasues built above: those given by the variables ?previousChoiceC and ?rChC. The consequent clause has name ?ant

e) clauseList ?cons ?choiceName [?randchoice]\
The consequent of the rules is built. It is given as the choice given in ?randChoice and the name of the predicate is written is in the variable ?choiceName

f) generatedInstance ?ruleChoices RuleName ‘Choices-Iter’ ?index
Finally the name of the rules is build with ‘sub-index’ determined by ?index

3) Finally, the rules are written (by the consequent) with name in the varible ?ruleChoices, antecedent in ?and and consequent in ?cons:

rule ?ruleChoices ?ant ?cons

Rule: writing Data-OrderProd-OrderCons (building transition rules)

Antecedents:
and
      listCons ?listCons\

```
      = ?listCons [?cons1 ?cons2 ?cons3]\
      listProd ?listProd\
      = ?listProd [?prod1 ?prod2 ?prod3]\
      noOfDays ?nDays\
      inInterval ?index 2 ?nDays\
      listNameOrder ?listNameOrder\
      listNameOrderProd ?listNameOrderProd\
      listNameOrderCons ?listNameOrderCons\
      index ?listNameOrder ?index ?orderName\
      index ?listNameOrderProd ?index ?orderProdName\
      index ?listNameOrderCons ?index ?orderConsName\
      clauseList ?order11 ?orderName [?cons1 ?prod1 ?s1]\
      clauseList ?order21 ?orderName [?cons2 ?prod1 ?s2]\
      clauseList ?order31 ?orderName [?cons3 ?prod1 ?s3]\
      clauseList ?order12 ?orderName [?cons1 ?prod2 ?s4]\
      clauseList ?order22 ?orderName [?cons2 ?prod2 ?s5]\
      clauseList ?order32 ?orderName [?cons3 ?prod2 ?s6]\
      clauseList ?order13 ?orderName [?cons1 ?prod3 ?s7]\
      clauseList ?order23 ?orderName [?cons2 ?prod3 ?s8]\
      clauseList ?order33 ?orderName [?cons3 ?prod3 ?s9]\
      clauseList ?is1 is [?order1 ?s1 + ?s4 + ?s7]\
      clauseList ?is2 is [?order2 ?s2 + ?s5 + ?s8]\
      clauseList ?is3 is [?order3 ?s3 + ?s6 + ?s9]\
      clauseList ?is4 is [?orderp1 ?s1 + ?s2 + ?s3]\
      clauseList ?is5 is [?orderp2 ?s4 + ?s5 + ?s6]\
      clauseList ?is6 is [?orderp3 ?s7 + ?s8 + ?s9]\
      clauseList ?antecedents and [?order11 ?order21 ?order31 ?order12
?order22 ?order32 ?order13 ?order23 ?order33 ?is1 ?is2 ?is3 ?is4 ?is5
?is6]\
      clauseList ?conseq1 ?orderConsName [?cons1 ?order1]\
      clauseList ?conseq2 ?orderConsName [?cons2 ?order2]\
      clauseList ?conseq3 ?orderConsName [?cons3 ?order3]\
      clauseList ?conseq4 ?orderProdName [?prod1 ?orderp1]\
      clauseList ?conseq5 ?orderProdName [?prod2 ?orderp2]\
      clauseList ?conseq6 ?orderProdName [?prod3 ?orderp3]\
      clauseList ?consequents and [?conseq1 ?conseq2 ?conseq3 ?conseq4
?conseq5 ?conseq6]\
      generatedInstance ?rulename RuleName 'Data-OrderProd-OrderCons'
?index

Consequents:
rule ?rulename ?antecedents ?consequents

Comment:
This rule split and write the rules for calculating consumer
(distributors) orders and total of orders by consumer and producer.

-------------------------------------------------------------------------

Rule: writing Data-SaleProd-OrderCons (building transition rules)

Antecedents:
and
      listCons ?listCons\
      = ?listCons [?cons1 ?cons2 ?cons3]\
      listProd ?listProd\
      = ?listProd [?prod1 ?prod2 ?prod3]\
      noOfDays ?nDays\
      inInterval ?index 2 ?nDays\
      listNameSales ?listNameSales\
      listNameSalesProd ?listNameSalesProd\
```

```
      listNameSalesCons ?listNameSalesCons\
      listNameTotalSales ?listNameTotalSales\
      index ?listNameSales ?index ?salesName\
      index ?listNameSalesProd ?index ?salesProdName\
      index ?listNameSalesCons ?index ?salesConsName\
      index ?listNameTotalSales ?index ?nameTotalSales\
      clauseList ?sales11 ?salesName [?prod1 ?cons1 ?s1]\
      clauseList ?sales21 ?salesName [?prod1 ?cons2 ?s2]\
      clauseList ?sales31 ?salesName [?prod1 ?cons3 ?s3]\
      clauseList ?sales12 ?salesName [?prod2 ?cons1 ?s4]\
      clauseList ?sales22 ?salesName [?prod2 ?cons2 ?s5]\
      clauseList ?sales32 ?salesName [?prod2 ?cons3 ?s6]\
      clauseList ?sales13 ?salesName [?prod3 ?cons1 ?s7]\
      clauseList ?sales23 ?salesName [?prod3 ?cons2 ?s8]\
      clauseList ?sales33 ?salesName [?prod3 ?cons3 ?s9]\
      clauseList ?is1 is [?salesc1 ?s1 + ?s4 + ?s7]\
      clauseList ?is2 is [?salesc2 ?s2 + ?s5 + ?s8]\
      clauseList ?is3 is [?salesc3 ?s3 + ?s6 + ?s9]\
      clauseList ?is4 is [?salesp1 ?s1 + ?s2 + ?s3]\
      clauseList ?is5 is [?salesp2 ?s4 + ?s5 + ?s6]\
      clauseList ?is6 is [?salesp3 ?s7 + ?s8 + ?s9]\
      clauseList ?is7 is [?totalSales ?salesc1 + ?salesc2 + ?salesc3]\
      clauseList ?antecedents and [?sales11 ?sales21 ?sales31 ?sales12
?sales22 ?sales32 ?sales13 ?sales23 ?sales33 ?is1 ?is2 ?is3 ?is4 ?is5
?is6 ?is7]\
      clauseList ?conseq1 ?salesConsName [?cons1 ?salesc1]\
      clauseList ?conseq2 ?salesConsName [?cons2 ?salesc2]\
      clauseList ?conseq3 ?salesConsName [?cons3 ?salesc3]\
      clauseList ?conseq4 ?salesProdName [?prod1 ?salesp1]\
      clauseList ?conseq5 ?salesProdName [?prod2 ?salesp2]\
      clauseList ?conseq6 ?salesProdName [?prod3 ?salesp3]\
      clauseList ?conseq7 ?nameTotalSales [?totalSales]\
      clauseList ?consequents and [?conseq1 ?conseq2 ?conseq3 ?conseq4
?conseq5 ?conseq6 ?conseq7]\
      generatedInstance ?rulename RuleName 'Data-SalesProd-OrderCons'
?index

Consequents:
rule ?rulename ?antecedents ?consequents

Comment:
This rule is responsible for splitting and writing rules for calculating
sales, and total sales by consumer and producer.

----------------------------------------------------------------------

Rule: writing Data-SelProd (building transition rules)

Antecedents:
and
      listProd ?list\
      = ?list [?prod1 ?prod2 ?prod3]\
      listNamePrices ?listpr\
      noOfDays ?noDays\
      inInterval ?index 2 ?noDays\
      index ?listpr ?index ?nameprice\
      listNameListSelProd ?listNameListSelProd\
      index ?listNameListSelProd ?index ?namesel\
      clauseList ?getSelProdC backSelProd [?nameprice ?listSel]\
      clauseList ?cons ?namesel [?listSel]\
      generatedInstance ?ruleSelProd RuleName 'Data-SelProd' ?nameprice
```

```
Consequents:
rule ?ruleSelProd ?getSelProdC ?cons

Comment:
This rule split and write the rule to order producers in accordance to
their price .. the result of these rules will be useful for determining
size of the interval of prices at each day and that producer with the
lowest prices which is chosen by consumers to place their orders at.

-------------------------------------------------------------------------

Rule: writing TR-Demand-and-Order (building transition rules)

Antecedents:
and
      noOfDays ?nDays\
      listNameDemand ?listNameDemands\
      inInterval ?index 2 ?nDays\
      index ?listNameDemands ?index ?demandOut\
      is ?previous ?index - 1\
      index ?listNameDemands ?previous ?demandIn\
      calcDemand ?demandDay [?index]\
      listNameOrder ?listNameOrder\
      listNameSales ?listNameSales\
      listNameSalesCons ?listNameSalesCons\
      listNameListSelProd ?listNameListSelProd\
      index ?listNameOrder ?previous ?orderIn\
      index ?listNameOrder ?index ?orderOut\
      index ?listNameSales ?previous ?namesale\
      index ?listNameSalesCons ?previous ?namesalescons\
      index ?listNameListSelProd ?index ?namelistselprod\
      clauseList ?dataDemandIn ?demandIn [?cons ?olddemand]\
      clauseList ?oldSaleC ?namesale [?prod ?cons ?oldsalevalue]\
      clauseList ?oldSaleConsC ?namesalescons [?cons ?oldsaleconsvalue]\
      clauseList ?firstprodC ?namelistselprod [[?prodsel ?other1
?other2]]\
      clauseList ?oldOrderC ?orderIn [?cons ?prod ?oldordervalue]\
      clauseList ?isC is [?newdemand ?olddemand + ?demandDay -
?oldsaleconsvalue]\
      clauseList ?neworderC calcNewOrder [?prod ?prodsel ?demandDay
?newordervalue]\
      clauseList ?isC1 is [?neworder ?oldordervalue + ?newordervalue -
?oldsalevalue]\
      clauseList ?antecedents and [?dataDemandIn ?oldSaleC ?oldSaleConsC
?firstprodC ?oldOrderC ?isC ?neworderC ?isC1]\
      clauseList ?cons1 ?demandOut [?cons ?newdemand]\
      clauseList ?cons2 ?orderOut [?cons ?prod ?neworder]\
      clauseList ?consequents and [?cons1 ?cons2]\
      generatedInstance ?rulename RuleName 'TR-Demand-and-Order'
?demandIn

Consequents:
rule ?rulename ?antecedents ?consequents
Comment:

This rule split and write at prover the rule for calculating demand and
order for each iteration. If the data the rule needs from previous
iterations is well known then the rule can also be split other variables,
e.g., the producer or consumer. E.g., when calculating a new level of
demand for consumer-i, that level depends on consumer-i's demand in the
```

```
previous day. So a rule could be written more specifically for each
consumer.
-----------------------------------------------------------------------

Rule: writing TR-Price (building transition rules)

Antecedents:
and
      noOfDays ?nDays\
      inInterval ?index 2 ?nDays\
      is ?previous ?index - 1\
      listNamePrices ?listNamePrices\
      listNameListChoice ?listNameListChoice\
      index ?listNamePrices ?index ?priceOut\
      index ?listNamePrices ?previous ?priceIn\
      index ?listNameListChoice ?index ?choicesNameList\
      listNameSalesProd ?listNameSalesProd\
      index ?listNameSalesProd ?previous ?nameSalesProd\
      listNameTotalSales ?listNameTotalSales\
      index ?listNameTotalSales ?previous ?nameTotalSales\
      clauseList ?clauseSalesProd ?nameSalesProd [?prod ?mysales]\
      clauseList ?clauseTotalSales ?nameTotalSales [?totalSales]\
      clauseList ?dataPriceIn ?priceIn [?prod ?oldprice]\
      clauseList ?choices1C ?choicesNameList [?choices]\
      clauseList ?eq1C = [?choices [?sel1 ?sel2 ?sel3]]\
      clauseList ?dataPriceIn ?priceIn [?prod ?oldprice]\
      clauseList ?otherpriceC otherPrice [?otherprice ?choices ?priceIn
?prod]\
      clauseList ?calcPriceC calcprice [?newprice [?oldprice ?otherprice
?mysales ?totalSales]]\
      clauseList ?ant and [?clauseSalesProd ?clauseTotalSales
?dataPriceIn ?choices1C ?eq1C ?otherpriceC ?calcPriceC]\
      clauseList ?cons ?priceOut [?prod ?newprice]\
      generatedInstance ?rulename RuleName 'TR-Price' ?priceIn

Consequents:
rule ?rulename ?ant ?cons

Comment:
Splits and writes transition rules for calculating prices.

-----------------------------------------------------------------------

Rule: writing TR-ProductionDay-and-Level (building transition rules)

Antecedents:
and
      producer ?prod\
      store ?store\
      factory ?factory\
      storeOwner ?store ?prod\
      capacityStore ?store ?capacity\
      factoryOwner ?factory ?prod\
      delay ?factory ?delay\
      noOfDays ?nDays\
      inInterval ?index 2 ?nDays\
      is ?previous ?index - 1\
      maxCapacity ?factory ?maxCapacity\
      listNameLevel ?listNameLevel\
      listNameProductionDay ?listNameProductionDay\
      listNameSalesProd ?listNameSalesProd\
```

```
      listNameOrderProd ?listNameOrderProd\
      index ?listNameLevel ?index ?levelOut\
      index ?listNameProductionDay ?index ?productionDayOut\
      index ?listNameLevel ?previous ?levelIn\
      index ?listNameProductionDay ?previous ?productionDayIn\
      index ?listNameSalesProd ?previous ?salesProdIn\
      index ?listNameOrderProd ?index ?orderProd\
      clauseList ?levelInC ?levelIn [?store ?levelInValue]\
      clauseList ?salesProdInC ?salesProdIn [?prod ?salesProdInValue]\
      clauseList ?orderProdC ?orderProd [?prod ?orderProdToday]\
      clauseList ?productionDayC calcProductionDay [?orderProdToday
?delay ?capacity ?levelInValue ?maxCapacity ?productionDayOutValue]\
      clauseList ?isLevel is [?levelOutValue ?levelInValue +
?productionDayOutValue - ?salesProdInValue]\
      clauseList ?antecedents and [?levelInC ?salesProdInC ?orderProdC
?productionDayC ?isLevel]\
      clauseList ?consLevel ?levelOut [?store ?levelOutValue]\
      clauseList ?consProductionDay ?productionDayOut [?factory
?productionDayOutValue]\
      clauseList ?consequents and [?consLevel ?consProductionDay]\
      generatedInstance ?rule RuleName 'TR-ProductionDay-and-Level'
?previous

Consequents:
rule ?rule ?antecedents ?consequents

Comment:
Split the transition rules for calculating a producer's production at a
factory and its level of good at a store

----------------------------------------------------------------------

Rule: writing TR-Sales (building transition rules)

Antecedents:
and
      storeOwner ?store ?prod\
      consumer ?cons\
      producer ?prod\
      noOfDays ?nDays\
      listNameSales ?listNameSales\
      listNameLevel ?listNameLevel\
      listNameOrder ?listNameOrder\
      listNameOrderProd ?listNameOrderProd\
      inInterval ?index 2 ?nDays\
      index ?listNameSales ?index ?saleOut\
      index ?listNameLevel ?index ?levelOut\
      index ?listNameOrder ?index ?orderOut\
      index ?listNameOrderProd ?index ?orderProdOut\
      clauseList ?levelC ?levelOut [?store ?levelStore]\
      clauseList ?orderC ?orderOut [?cons ?prod ?orderConsProd]\
      clauseList ?orderProdC ?orderProdOut [?prod ?orderProd]\
      clauseList ?calSaleC calcSale [?levelStore ?orderProd
?orderConsProd ?saleValue]\
      clauseList ?antecedent and [?levelC ?orderC ?orderProdC ?calSaleC]\
      clauseList ?consequent ?saleOut [?prod ?cons ?saleValue]\
      generatedInstance ?rule RuleName 'TR-Sales' ?index

Consequents:
rule ?rule ?antecedent ?consequent
```

```
Comment:
This rule is responsible for splitting and writing the transition rules
for calculating producer's sales.


================================================================
================================================================
Backward-chaining rule to write up the theorem.

First the rule for the general case, back-rule, is listed. Then the rule
for the basic case (to implement the last recursive call), base, will be
listed.

Rulebase: definition backBuiltTheorem (theorem)

----------------------------------------------------------------------

Rule: back-rule

Antecedents:
and
      clauseList ?amIntPrice1 amIntPrice [?nprice1 ?dif1]\
      clauseList ?amIntPrice2 amIntPrice [?nprice2 ?dif2]\
      clauseList ?greaterC greater [?dif2 ?dif1]\
      clauseList ?notInferredC notInferred [?greaterC]\
      roundError ?roundError\
      clauseList ?isC is [?dif ?dif1 - ?dif2]\
      clauseList ?aVC absoluteValue [?absdif ?dif]\
      clauseList ?greaterE greater [?absdif ?roundError]\
      clauseList ?notInferredE notInferred [?greaterE]\
      clauseList ?notInferred or [?notInferredC ?notInferredE]\
      backBuiltTheorem [?nprice2 | ?rest] ?antListPost\

Consequents:
backBuiltTheorem [?nprice1 ?nprice2 | ?rest] [?amIntPrice1 ?amIntPrice2
?isC ?aVC ?notInferred | ?antListPost]\

Comment:
This rules calls itself recursively. Its is called to build the theorem,
in this form:
backBuiltTheorem ?actualListPrices ?antList\

A list of names of price predicates (price1, ...) is sent in
?actualListPrices and a list of clauses to build the antecedent of the
rules is returned in ?antList. For each recursive call of the rule a list
of clauses to build part of the rule like the following must be sent (see
the theorem in the rules of prover):
      amIntPrice price1 ?_-1\
      amIntPrice price3 ?_-2\
      is ?_-3 ?_-1 - ?_-2\
      absoluteValue ?_-4 ?_-3\
      (or
            notInferred
                  greater ?_-2 ?_-1\
            notInferred
                  greater ?_-4 1.0e-8)\

This clauses are built using the clause ‘clauseList’ and the three first
names of prices listed in the input (called ?nprice1 ?nprice2 and
?nprice3] (see the first list in the consequent of the rule). The clauses
built are placed in the list to be returned (second list in the
```

```
predicate) and concatenated to the results got from the recursive call
made at the end of the list of antecedents. The input to the recursive
call of the rule are the two second price predicate names already used
(?price2 ?price3]. The answer from this recursive call is got in the
variable ?antListPost and concatenated, in the output list (see second
list in the consequent)

-------------------------------------------------------------------------

Rule: base

Antecedents:
and
      clauseList ?amIntPrice1 amIntPrice [?nprice1 ?dif1]\
      clauseList ?amIntPrice2 amIntPrice [?nprice2 ?dif2]\
      clauseList ?greaterC greater [?dif2 ?dif1]\
      clauseList ?notInferredC notInferred [?greaterC]\
      roundError ?roundError\
      clauseList ?isC is [?dif ?dif1 - ?dif2]\
      clauseList ?aVC absoluteValue [?absdif ?dif]\
      clauseList ?greaterE greater [?absdif ?roundError]\
      clauseList ?notInferredE notInferred [?greaterE]\
      clauseList ?notInferred or [?notInferredC ?notInferredE]\
      = ?listPost [?amIntPrice1 ?amIntPrice2 ?isC ?aVC ?notInferred]\

Consequents:
backBuiltTheorem [?nprice1 ?nprice2] ?listPost\

Commment:
This rule implements the last recursive call to the rule building the
theorem.
It will fire in case the input list (first list in the call of the rule,
backBuiltTheorem, e.g., the list with name of price predicates) has only
two elements, e.g., there is not 'rest'.
```

10.2.3 Module Prover

```
Rulebase: prover@model.universe (day: 1)

-------------------------------------------------------------------------

Rule: checkTheorem (meta-accessible)

Antecedents:

and
      amIntPrice price1 ?_-1\
      amIntPrice price2 ?_-2\
      is ?_-3 ?_-1 - ?_-2\
      absoluteValue ?_-4 ?_-3\
      (or
            notInferred
                  greater ?_-2 ?_-1\
            notInferred
                  greater ?_-4 1.0e-8)\
      amIntPrice price2 ?_-5\
      amIntPrice price3 ?_-6\
      is ?_-7 ?_-5 - ?_-6\
      absoluteValue ?_-8 ?_-7\
      (or
            notInferred
```

```
                  greater ?_-6 ?_-5\
            notInferred
                  greater ?_-8 1.0e-8)\
      amIntPrice price3 ?_-9\
      amIntPrice price4 ?_-10\
      is ?_-11 ?_-9 - ?_-10\
      absoluteValue ?_-12 ?_-11\
      (or
            notInferred
                  greater ?_-10 ?_-9\
            notInferred
                  greater ?_-12 1.0e-8)\
      amIntPrice price4 ?_-13\
      amIntPrice price5 ?_-14\
      is ?_-15 ?_-13 - ?_-14\
      absoluteValue ?_-16 ?_-15\
      (or
            notInferred
                  greater ?_-14 ?_-13\
            notInferred
                  greater ?_-16 1.0e-8)\
      amIntPrice price5 ?_-17\
      amIntPrice price6 ?_-18\
      is ?_-19 ?_-17 - ?_-18\
      absoluteValue ?_-20 ?_-19\
      (or
            notInferred
                  greater ?_-18 ?_-17\
            notInferred
                  greater ?_-20 1.0e-8)\


Consequents:
false

----------------------------------------------------------------------

Rule: Choices-Iter-1 (meta-accessible)

Antecedents:
and
      listChoiceProd1 ?previousChoice\
      randomChoice ?randchoice ?allchoices 2
            choices1 ?allchoices

Consequents:
listChoiceProd2 ?randchoice

----------------------------------------------------------------------

Rule: Choices-Iter-2 (meta-accessible)

Antecedents:
and
      listChoiceProd2 ?previousChoice\
      randomChoice ?randchoice ?allchoices 3
            choices1 ?allchoices

Consequents:
listChoiceProd3 ?randchoice
```

```
-------------------------------------------------------------------------

Rule: Choices-Iter-3 (meta-accessible)

Antecedents:
and
      listChoiceProd3 ?previousChoice\
      randomChoice ?randchoice ?allchoices 4
            choices1 ?allchoices

Consequents:
listChoiceProd4 ?randchoice

-------------------------------------------------------------------------

Rule: Choices-Iter-4 (meta-accessible)

Antecedents:
and
      listChoiceProd4 ?previousChoice\
      randomChoice ?randchoice ?allchoices 5
            choices1 ?allchoices

Consequents:
listChoiceProd5 ?randchoice

-------------------------------------------------------------------------

Rule: Choices-Iter-5 (meta-accessible)

Antecedents:
and
      listChoiceProd5 ?previousChoice\
      randomChoice ?randchoice ?allchoices 6
            choices1 ?allchoices

Consequents:
listChoiceProd6 ?randchoice

-------------------------------------------------------------------------

Rule: Data-OrderProd-OrderCons-1 (meta-accessible)

Antecedents:
and
      order2 'consumer-1'@simulation 'producer-1'@simulation ?s1\
      order2 'consumer-2'@simulation 'producer-1'@simulation ?s2\
      order2 'consumer-3'@simulation 'producer-1'@simulation ?s3\
      order2 'consumer-1'@simulation 'producer-2'@simulation ?s4\
      order2 'consumer-2'@simulation 'producer-2'@simulation ?s5\
      order2 'consumer-3'@simulation 'producer-2'@simulation ?s6\
      order2 'consumer-1'@simulation 'producer-3'@simulation ?s7\
      order2 'consumer-2'@simulation 'producer-3'@simulation ?s8\
      order2 'consumer-3'@simulation 'producer-3'@simulation ?s9\
      is ?order1 ?s1 + ?s4 + ?s7\
      is ?order2 ?s2 + ?s5 + ?s8\
      is ?order3 ?s3 + ?s6 + ?s9\
      is ?orderp1 ?s1 + ?s2 + ?s3\
      is ?orderp2 ?s4 + ?s5 + ?s6\
      is ?orderp3 ?s7 + ?s8 + ?s9
```

```
Consequents:
and
      orderCons2 'consumer-1'@simulation ?order1\
      orderCons2 'consumer-2'@simulation ?order2\
      orderCons2 'consumer-3'@simulation ?order3\
      orderProd2 'producer-1'@simulation ?orderp1\
      orderProd2 'producer-2'@simulation ?orderp2\
      orderProd2 'producer-3'@simulation ?orderp3

------------------------------------------------------------------------

Rule: Data-OrderProd-OrderCons-2 (meta-accessible)

Antecedents:
and
      order3 'consumer-1'@simulation 'producer-1'@simulation ?s1\
      order3 'consumer-2'@simulation 'producer-1'@simulation ?s2\
      order3 'consumer-3'@simulation 'producer-1'@simulation ?s3\
      order3 'consumer-1'@simulation 'producer-2'@simulation ?s4\
      order3 'consumer-2'@simulation 'producer-2'@simulation ?s5\
      order3 'consumer-3'@simulation 'producer-2'@simulation ?s6\
      order3 'consumer-1'@simulation 'producer-3'@simulation ?s7\
      order3 'consumer-2'@simulation 'producer-3'@simulation ?s8\
      order3 'consumer-3'@simulation 'producer-3'@simulation ?s9\
      is ?order1 ?s1 + ?s4 + ?s7\
      is ?order2 ?s2 + ?s5 + ?s8\
      is ?order3 ?s3 + ?s6 + ?s9\
      is ?orderp1 ?s1 + ?s2 + ?s3\
      is ?orderp2 ?s4 + ?s5 + ?s6\
      is ?orderp3 ?s7 + ?s8 + ?s9

Consequents:
and
      orderCons3 'consumer-1'@simulation ?order1\
      orderCons3 'consumer-2'@simulation ?order2\
      orderCons3 'consumer-3'@simulation ?order3\
      orderProd3 'producer-1'@simulation ?orderp1\
      orderProd3 'producer-2'@simulation ?orderp2\
      orderProd3 'producer-3'@simulation ?orderp3

------------------------------------------------------------------------

Rule: Data-OrderProd-OrderCons-3 (meta-accessible)

Antecedents:
and
      order4 'consumer-1'@simulation 'producer-1'@simulation ?s1\
      order4 'consumer-2'@simulation 'producer-1'@simulation ?s2\
      order4 'consumer-3'@simulation 'producer-1'@simulation ?s3\
      order4 'consumer-1'@simulation 'producer-2'@simulation ?s4\
      order4 'consumer-2'@simulation 'producer-2'@simulation ?s5\
      order4 'consumer-3'@simulation 'producer-2'@simulation ?s6\
      order4 'consumer-1'@simulation 'producer-3'@simulation ?s7\
      order4 'consumer-2'@simulation 'producer-3'@simulation ?s8\
      order4 'consumer-3'@simulation 'producer-3'@simulation ?s9\
      is ?order1 ?s1 + ?s4 + ?s7\
      is ?order2 ?s2 + ?s5 + ?s8\
      is ?order3 ?s3 + ?s6 + ?s9\
      is ?orderp1 ?s1 + ?s2 + ?s3\
      is ?orderp2 ?s4 + ?s5 + ?s6\
```

```
      is ?orderp3 ?s7 + ?s8 + ?s9

Consequents:
and
      orderCons4 'consumer-1'@simulation ?order1\
      orderCons4 'consumer-2'@simulation ?order2\
      orderCons4 'consumer-3'@simulation ?order3\
      orderProd4 'producer-1'@simulation ?orderp1\
      orderProd4 'producer-2'@simulation ?orderp2\
      orderProd4 'producer-3'@simulation ?orderp3

------------------------------------------------------------------------

Rule: Data-OrderProd-OrderCons-4 (meta-accessible)

Antecedents:
and
      order5 'consumer-1'@simulation 'producer-1'@simulation ?s1\
      order5 'consumer-2'@simulation 'producer-1'@simulation ?s2\
      order5 'consumer-3'@simulation 'producer-1'@simulation ?s3\
      order5 'consumer-1'@simulation 'producer-2'@simulation ?s4\
      order5 'consumer-2'@simulation 'producer-2'@simulation ?s5\
      order5 'consumer-3'@simulation 'producer-2'@simulation ?s6\
      order5 'consumer-1'@simulation 'producer-3'@simulation ?s7\
      order5 'consumer-2'@simulation 'producer-3'@simulation ?s8\
      order5 'consumer-3'@simulation 'producer-3'@simulation ?s9\
      is ?order1 ?s1 + ?s4 + ?s7\
      is ?order2 ?s2 + ?s5 + ?s8\
      is ?order3 ?s3 + ?s6 + ?s9\
      is ?orderp1 ?s1 + ?s2 + ?s3\
      is ?orderp2 ?s4 + ?s5 + ?s6\
      is ?orderp3 ?s7 + ?s8 + ?s9

Consequents:
and
      orderCons5 'consumer-1'@simulation ?order1\
      orderCons5 'consumer-2'@simulation ?order2\
      orderCons5 'consumer-3'@simulation ?order3\
      orderProd5 'producer-1'@simulation ?orderp1\
      orderProd5 'producer-2'@simulation ?orderp2\
      orderProd5 'producer-3'@simulation ?orderp3

------------------------------------------------------------------------

Rule: Data-OrderProd-OrderCons-5 (meta-accessible)

Antecedents:
and
      order6 'consumer-1'@simulation 'producer-1'@simulation ?s1\
      order6 'consumer-2'@simulation 'producer-1'@simulation ?s2\
      order6 'consumer-3'@simulation 'producer-1'@simulation ?s3\
      order6 'consumer-1'@simulation 'producer-2'@simulation ?s4\
      order6 'consumer-2'@simulation 'producer-2'@simulation ?s5\
      order6 'consumer-3'@simulation 'producer-2'@simulation ?s6\
      order6 'consumer-1'@simulation 'producer-3'@simulation ?s7\
      order6 'consumer-2'@simulation 'producer-3'@simulation ?s8\
      order6 'consumer-3'@simulation 'producer-3'@simulation ?s9\
      is ?order1 ?s1 + ?s4 + ?s7\
      is ?order2 ?s2 + ?s5 + ?s8\
      is ?order3 ?s3 + ?s6 + ?s9\
      is ?orderp1 ?s1 + ?s2 + ?s3\
```

```
      is ?orderp2 ?s4 + ?s5 + ?s6\
      is ?orderp3 ?s7 + ?s8 + ?s9

Consequents:
and
      orderCons6 'consumer-1'@simulation ?order1\
      orderCons6 'consumer-2'@simulation ?order2\
      orderCons6 'consumer-3'@simulation ?order3\
      orderProd6 'producer-1'@simulation ?orderp1\
      orderProd6 'producer-2'@simulation ?orderp2\
      orderProd6 'producer-3'@simulation ?orderp3

------------------------------------------------------------------------

Rule: Data-SalesProd-OrderCons-1 (meta-accessible)

Antecedents:
and
      sale2 'producer-1'@simulation 'consumer-1'@simulation ?s1\
      sale2 'producer-1'@simulation 'consumer-2'@simulation ?s2\
      sale2 'producer-1'@simulation 'consumer-3'@simulation ?s3\
      sale2 'producer-2'@simulation 'consumer-1'@simulation ?s4\
      sale2 'producer-2'@simulation 'consumer-2'@simulation ?s5\
      sale2 'producer-2'@simulation 'consumer-3'@simulation ?s6\
      sale2 'producer-3'@simulation 'consumer-1'@simulation ?s7\
      sale2 'producer-3'@simulation 'consumer-2'@simulation ?s8\
      sale2 'producer-3'@simulation 'consumer-3'@simulation ?s9\
      is ?salesc1 ?s1 + ?s4 + ?s7\
      is ?salesc2 ?s2 + ?s5 + ?s8\
      is ?salesc3 ?s3 + ?s6 + ?s9\
      is ?salesp1 ?s1 + ?s2 + ?s3\
      is ?salesp2 ?s4 + ?s5 + ?s6\
      is ?salesp3 ?s7 + ?s8 + ?s9\
      is ?totalSales ?salesc1 + ?salesc2 + ?salesc3

Consequents:
and
      saleCons2 'consumer-1'@simulation ?salesc1\
      saleCons2 'consumer-2'@simulation ?salesc2\
      saleCons2 'consumer-3'@simulation ?salesc3\
      saleProd2 'producer-1'@simulation ?salesp1\
      saleProd2 'producer-2'@simulation ?salesp2\
      saleProd2 'producer-3'@simulation ?salesp3\
      totalSales2 ?totalSales

------------------------------------------------------------------------

Rule: Data-SalesProd-OrderCons-2 (meta-accessible)

Antecedents:
and
      sale3 'producer-1'@simulation 'consumer-1'@simulation ?s1\
      sale3 'producer-1'@simulation 'consumer-2'@simulation ?s2\
      sale3 'producer-1'@simulation 'consumer-3'@simulation ?s3\
      sale3 'producer-2'@simulation 'consumer-1'@simulation ?s4\
      sale3 'producer-2'@simulation 'consumer-2'@simulation ?s5\
      sale3 'producer-2'@simulation 'consumer-3'@simulation ?s6\
      sale3 'producer-3'@simulation 'consumer-1'@simulation ?s7\
      sale3 'producer-3'@simulation 'consumer-2'@simulation ?s8\
      sale3 'producer-3'@simulation 'consumer-3'@simulation ?s9\
      is ?salesc1 ?s1 + ?s4 + ?s7\
```

```
      is ?salesc2 ?s2 + ?s5 + ?s8\
      is ?salesc3 ?s3 + ?s6 + ?s9\
      is ?salesp1 ?s1 + ?s2 + ?s3\
      is ?salesp2 ?s4 + ?s5 + ?s6\
      is ?salesp3 ?s7 + ?s8 + ?s9\
      is ?totalSales ?salesc1 + ?salesc2 + ?salesc3

Consequents:
and
      saleCons3 'consumer-1'@simulation ?salesc1\
      saleCons3 'consumer-2'@simulation ?salesc2\
      saleCons3 'consumer-3'@simulation ?salesc3\
      saleProd3 'producer-1'@simulation ?salesp1\
      saleProd3 'producer-2'@simulation ?salesp2\
      saleProd3 'producer-3'@simulation ?salesp3\
      totalSales3 ?totalSales

-------------------------------------------------------------------------

Rule: Data-SalesProd-OrderCons-3 (meta-accessible)

Antecedents:
and
      sale4 'producer-1'@simulation 'consumer-1'@simulation ?s1\
      sale4 'producer-1'@simulation 'consumer-2'@simulation ?s2\
      sale4 'producer-1'@simulation 'consumer-3'@simulation ?s3\
      sale4 'producer-2'@simulation 'consumer-1'@simulation ?s4\
      sale4 'producer-2'@simulation 'consumer-2'@simulation ?s5\
      sale4 'producer-2'@simulation 'consumer-3'@simulation ?s6\
      sale4 'producer-3'@simulation 'consumer-1'@simulation ?s7\
      sale4 'producer-3'@simulation 'consumer-2'@simulation ?s8\
      sale4 'producer-3'@simulation 'consumer-3'@simulation ?s9\
      is ?salesc1 ?s1 + ?s4 + ?s7\
      is ?salesc2 ?s2 + ?s5 + ?s8\
      is ?salesc3 ?s3 + ?s6 + ?s9\
      is ?salesp1 ?s1 + ?s2 + ?s3\
      is ?salesp2 ?s4 + ?s5 + ?s6\
      is ?salesp3 ?s7 + ?s8 + ?s9\
      is ?totalSales ?salesc1 + ?salesc2 + ?salesc3

Consequents:
and
      saleCons4 'consumer-1'@simulation ?salesc1\
      saleCons4 'consumer-2'@simulation ?salesc2\
      saleCons4 'consumer-3'@simulation ?salesc3\
      saleProd4 'producer-1'@simulation ?salesp1\
      saleProd4 'producer-2'@simulation ?salesp2\
      saleProd4 'producer-3'@simulation ?salesp3\
      totalSales4 ?totalSales

-------------------------------------------------------------------------

Rule: Data-SalesProd-OrderCons-4 (meta-accessible)

Antecedents:
and
      sale5 'producer-1'@simulation 'consumer-1'@simulation ?s1\
      sale5 'producer-1'@simulation 'consumer-2'@simulation ?s2\
      sale5 'producer-1'@simulation 'consumer-3'@simulation ?s3\
      sale5 'producer-2'@simulation 'consumer-1'@simulation ?s4\
      sale5 'producer-2'@simulation 'consumer-2'@simulation ?s5\
```

```
      sale5 'producer-2'@simulation 'consumer-3'@simulation ?s6\
      sale5 'producer-3'@simulation 'consumer-1'@simulation ?s7\
      sale5 'producer-3'@simulation 'consumer-2'@simulation ?s8\
      sale5 'producer-3'@simulation 'consumer-3'@simulation ?s9\
      is ?salesc1 ?s1 + ?s4 + ?s7\
      is ?salesc2 ?s2 + ?s5 + ?s8\
      is ?salesc3 ?s3 + ?s6 + ?s9\
      is ?salesp1 ?s1 + ?s2 + ?s3\
      is ?salesp2 ?s4 + ?s5 + ?s6\
      is ?salesp3 ?s7 + ?s8 + ?s9\
      is ?totalSales ?salesc1 + ?salesc2 + ?salesc3

Consequents:
and
      saleCons5 'consumer-1'@simulation ?salesc1\
      saleCons5 'consumer-2'@simulation ?salesc2\
      saleCons5 'consumer-3'@simulation ?salesc3\
      saleProd5 'producer-1'@simulation ?salesp1\
      saleProd5 'producer-2'@simulation ?salesp2\
      saleProd5 'producer-3'@simulation ?salesp3\
      totalSales5 ?totalSales

------------------------------------------------------------------------

Rule: Data-SalesProd-OrderCons-5 (meta-accessible)

Antecedents:
and
      sale6 'producer-1'@simulation 'consumer-1'@simulation ?s1\
      sale6 'producer-1'@simulation 'consumer-2'@simulation ?s2\
      sale6 'producer-1'@simulation 'consumer-3'@simulation ?s3\
      sale6 'producer-2'@simulation 'consumer-1'@simulation ?s4\
      sale6 'producer-2'@simulation 'consumer-2'@simulation ?s5\
      sale6 'producer-2'@simulation 'consumer-3'@simulation ?s6\
      sale6 'producer-3'@simulation 'consumer-1'@simulation ?s7\
      sale6 'producer-3'@simulation 'consumer-2'@simulation ?s8\
      sale6 'producer-3'@simulation 'consumer-3'@simulation ?s9\
      is ?salesc1 ?s1 + ?s4 + ?s7\
      is ?salesc2 ?s2 + ?s5 + ?s8\
      is ?salesc3 ?s3 + ?s6 + ?s9\
      is ?salesp1 ?s1 + ?s2 + ?s3\
      is ?salesp2 ?s4 + ?s5 + ?s6\
      is ?salesp3 ?s7 + ?s8 + ?s9\
      is ?totalSales ?salesc1 + ?salesc2 + ?salesc3

Consequents:
and
      saleCons6 'consumer-1'@simulation ?salesc1\
      saleCons6 'consumer-2'@simulation ?salesc2\
      saleCons6 'consumer-3'@simulation ?salesc3\
      saleProd6 'producer-1'@simulation ?salesp1\
      saleProd6 'producer-2'@simulation ?salesp2\
      saleProd6 'producer-3'@simulation ?salesp3\
      totalSales6 ?totalSales

------------------------------------------------------------------------

Rule: Data-SelProd-1 (meta-accessible)

Antecedents:
backSelProd price2 ?listSel
```

```
Consequents:
listSelProd2 ?listSel

---------------------------------------------------------------------

Rule: Data-SelProd-2 (meta-accessible)

Antecedents:
backSelProd price3 ?listSel

Consequents:
listSelProd3 ?listSel

---------------------------------------------------------------------

Rule: Data-SelProd-3 (meta-accessible)

Antecedents:
backSelProd price4 ?listSel

Consequents:
listSelProd4 ?listSel

---------------------------------------------------------------------

Rule: Data-SelProd-4 (meta-accessible)

Antecedents:
backSelProd price5 ?listSel

Consequents:
listSelProd5 ?listSel

---------------------------------------------------------------------

Rule: Data-SelProd-5 (meta-accessible)

Antecedents:
backSelProd price6 ?listSel

Consequents:
listSelProd6 ?listSel

---------------------------------------------------------------------

Rule: TR-Demand-and-Order-1 (meta-accessible)

Antecedents:
and
      demand1 ?cons ?olddemand\
      sale1 ?prod ?cons ?oldsalevalue\
      saleCons1 ?cons ?oldsaleconsvalue\
      listSelProd2 [?prodsel ?other1 ?other2]\
      order1 ?cons ?prod ?oldordervalue\
      is ?newdemand ?olddemand + 216 - ?oldsaleconsvalue\
      calcNewOrder ?prod ?prodsel 216 ?neworderva1ue\
      is ?neworder ?oldordervalue + ?neworderva1ue - ?oldsalevalue

Consequents:
and
```

```
      demand2 ?cons ?newdemand\
      order2 ?cons ?prod ?neworder

-------------------------------------------------------------------

Rule: TR-Demand-and-Order-2 (meta-accessible)

Antecedents:
and
      demand2 ?cons ?olddemand\
      sale2 ?prod ?cons ?oldsalevalue\
      saleCons2 ?cons ?oldsaleconsvalue\
      listSelProd3 [?prodsel ?other1 ?other2]\
      order2 ?cons ?prod ?oldordervalue\
      is ?newdemand ?olddemand + 233 - ?oldsaleconsvalue\
      calcNewOrder ?prod ?prodsel 233 ?newordervalue\
      is ?neworder ?oldordervalue + ?newordervalue - ?oldsalevalue

Consequents:
and
      demand3 ?cons ?newdemand\
      order3 ?cons ?prod ?neworder

-------------------------------------------------------------------

Rule: TR-Demand-and-Order-3 (meta-accessible)

Antecedents:
and
      demand3 ?cons ?olddemand\
      sale3 ?prod ?cons ?oldsalevalue\
      saleCons3 ?cons ?oldsaleconsvalue\
      listSelProd4 [?prodsel ?other1 ?other2]\
      order3 ?cons ?prod ?oldordervalue\
      is ?newdemand ?olddemand + 250 - ?oldsaleconsvalue\
      calcNewOrder ?prod ?prodsel 250 ?newordervalue\
      is ?neworder ?oldordervalue + ?newordervalue - ?oldsalevalue

Consequents:
and
      demand4 ?cons ?newdemand\
      order4 ?cons ?prod ?neworder

-------------------------------------------------------------------

Rule: TR-Demand-and-Order-4 (meta-accessible)

Antecedents:
and
      demand4 ?cons ?olddemand\
      sale4 ?prod ?cons ?oldsalevalue\
      saleCons4 ?cons ?oldsaleconsvalue\
      listSelProd5 [?prodsel ?other1 ?other2]\
      order4 ?cons ?prod ?oldordervalue\
      is ?newdemand ?olddemand + 266 - ?oldsaleconsvalue\
      calcNewOrder ?prod ?prodsel 266 ?newordervalue\
      is ?neworder ?oldordervalue + ?newordervalue - ?oldsalevalue

Consequents:
and
      demand5 ?cons ?newdemand\
```

```
      order5 ?cons ?prod ?neworder

-------------------------------------------------------------------------

Rule: TR-Demand-and-Order-5 (meta-accessible)

Antecedents:
and
      demand5 ?cons ?olddemand\
      sale5 ?prod ?cons ?oldsalevalue\
      saleCons5 ?cons ?oldsaleconsvalue\
      listSelProd6 [?prodsel ?other1 ?other2]\
      order5 ?cons ?prod ?oldordervalue\
      is ?newdemand ?olddemand + 283 - ?oldsaleconsvalue\
      calcNewOrder ?prod ?prodsel 283 ?newordervalue\
      is ?neworder ?oldordervalue + ?newordervalue - ?oldsalevalue

Consequents:
and
      demand6 ?cons ?newdemand\
      order6 ?cons ?prod ?neworder

-------------------------------------------------------------------------
Rule: TR-Price-1 (meta-accessible)

Antecedents:
and
      saleProd1 ?prod ?mysales\
      totalSales1 ?totalSales\
      price1 ?prod ?oldprice\
      listChoiceProd2 ?choices\
      = ?choices [?sel1 ?sel2 ?sel3]\
      otherPrice ?otherprice ?choices price1 ?prod\
      calcprice ?newprice [?oldprice ?otherprice ?mysales ?totalSales]

Consequents:
price2 ?prod ?newprice

-------------------------------------------------------------------------
Rule: TR-Price-2 (meta-accessible)

Antecedents:
and
      saleProd2 ?prod ?mysales\
      totalSales2 ?totalSales\
      price2 ?prod ?oldprice\
      listChoiceProd3 ?choices\
      = ?choices [?sel1 ?sel2 ?sel3]\
      otherPrice ?otherprice ?choices price2 ?prod\
      calcprice ?newprice [?oldprice ?otherprice ?mysales ?totalSales]

Consequents:
price3 ?prod ?newprice

-------------------------------------------------------------------------

Rule: TR-Price-3 (meta-accessible)

Antecedents:
and
      saleProd3 ?prod ?mysales\
```

```
      totalSales3 ?totalSales\
      price3 ?prod ?oldprice\
      listChoiceProd4 ?choices\
      = ?choices [?sel1 ?sel2 ?sel3]\
      otherPrice ?otherprice ?choices price3 ?prod\
      calcprice ?newprice [?oldprice ?otherprice ?mysales ?totalSales]

Consequents:
price4 ?prod ?newprice

----------------------------------------------------------------------

Rule: TR-Price-4 (meta-accessible)

Antecedents:
and
      saleProd4 ?prod ?mysales\
      totalSales4 ?totalSales\
      price4 ?prod ?oldprice\
      listChoiceProd5 ?choices\
      = ?choices [?sel1 ?sel2 ?sel3]\
      otherPrice ?otherprice ?choices price4 ?prod\
      calcprice ?newprice [?oldprice ?otherprice ?mysales ?totalSales]

Consequents:
price5 ?prod ?newprice

----------------------------------------------------------------------

Rule: TR-Price-5 (meta-accessible)

Antecedents:
and
      saleProd5 ?prod ?mysales\
      totalSales5 ?totalSales\
      price5 ?prod ?oldprice\
      listChoiceProd6 ?choices\
      = ?choices [?sel1 ?sel2 ?sel3]\
      otherPrice ?otherprice ?choices price5 ?prod\
      calcprice ?newprice [?oldprice ?otherprice ?mysales ?totalSales]

Consequents:
price6 ?prod ?newprice

----------------------------------------------------------------------
Rule: TR-ProductionDay-and-Level-1 (meta-accessible)

Antecedents:
and
      level1 'store-2'@simulation ?levelInValue\
      saleProd1 'producer-2'@simulation ?salesProdInValue\
      orderProd2 'producer-2'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level2 'store-2'@simulation ?levelOutValue\
      productionDay2 'factory-2'@simulation ?productionDayOutValue
```

```
-------------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-1 (meta-accessible)

Antecedents:
and
      level1 'store-3'@simulation ?levelInValue\
      saleProd1 'producer-1'@simulation ?salesProdInValue\
      orderProd2 'producer-1'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level2 'store-3'@simulation ?levelOutValue\
      productionDay2 'factory-3'@simulation ?productionDayOutValue

-------------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-1 (meta-accessible)

Antecedents:
and
      level1 'store-1'@simulation ?levelInValue\
      saleProd1 'producer-3'@simulation ?salesProdInValue\
      orderProd2 'producer-3'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level2 'store-1'@simulation ?levelOutValue\
      productionDay2 'factory-1'@simulation ?productionDayOutValue

-------------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-2 (meta-accessible)

Antecedents:
and
      level2 'store-2'@simulation ?levelInValue\
      saleProd2 'producer-2'@simulation ?salesProdInValue\
      orderProd3 'producer-2'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level3 'store-2'@simulation ?levelOutValue\
      productionDay3 'factory-2'@simulation ?productionDayOutValue

-------------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-2 (meta-accessible)
```

```
Antecedents:
and
      level2 'store-1'@simulation ?levelInValue\
      saleProd2 'producer-3'@simulation ?salesProdInValue\
      orderProd3 'producer-3'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level3 'store-1'@simulation ?levelOutValue\
      productionDay3 'factory-1'@simulation ?productionDayOutValue

-----------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-2 (meta-accessible)

Antecedents:
and
      level2 'store-3'@simulation ?levelInValue\
      saleProd2 'producer-1'@simulation ?salesProdInValue\
      orderProd3 'producer-1'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level3 'store-3'@simulation ?levelOutValue\
      productionDay3 'factory-3'@simulation ?productionDayOutValue

-----------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-3 (meta-accessible)

Antecedents:
and
      level3 'store-1'@simulation ?levelInValue\
      saleProd3 'producer-3'@simulation ?salesProdInValue\
      orderProd4 'producer-3'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level4 'store-1'@simulation ?levelOutValue\
      productionDay4 'factory-1'@simulation ?productionDayOutValue

-----------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-3 (meta-accessible)

Antecedents:
and
      level3 'store-2'@simulation ?levelInValue\
```

```
      saleProd3 'producer-2'@simulation ?salesProdInValue\
      orderProd4 'producer-2'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level4 'store-2'@simulation ?levelOutValue\
      productionDay4 'factory-2'@simulation ?productionDayOutValue

----------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-3 (meta-accessible)

Antecedents:
and
      level3 'store-3'@simulation ?levelInValue\
      saleProd3 'producer-1'@simulation ?salesProdInValue\
      orderProd4 'producer-1'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level4 'store-3'@simulation ?levelOutValue\
      productionDay4 'factory-3'@simulation ?productionDayOutValue

----------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-4 (meta-accessible)

Antecedents:
and
      level4 'store-2'@simulation ?levelInValue\
      saleProd4 'producer-2'@simulation ?salesProdInValue\
      orderProd5 'producer-2'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level5 'store-2'@simulation ?levelOutValue\
      productionDay5 'factory-2'@simulation ?productionDayOutValue

----------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-4 (meta-accessible)

Antecedents:
and
      level4 'store-3'@simulation ?levelInValue\
      saleProd4 'producer-1'@simulation ?salesProdInValue\
      orderProd5 'producer-1'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
```

```
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level5 'store-3'@simulation ?levelOutValue\
      productionDay5 'factory-3'@simulation ?productionDayOutValue

----------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-4 (meta-accessible)

Antecedents:
and
      level4 'store-1'@simulation ?levelInValue\
      saleProd4 'producer-3'@simulation ?salesProdInValue\
      orderProd5 'producer-3'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level5 'store-1'@simulation ?levelOutValue\
      productionDay5 'factory-1'@simulation ?productionDayOutValue

----------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-5 (meta-accessible)

Antecedents:
and
      level5 'store-3'@simulation ?levelInValue\
      saleProd5 'producer-1'@simulation ?salesProdInValue\
      orderProd6 'producer-1'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level6 'store-3'@simulation ?levelOutValue\
      productionDay6 'factory-3'@simulation ?productionDayOutValue

----------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-5 (meta-accessible)

Antecedents:
and
      level5 'store-2'@simulation ?levelInValue\
      saleProd5 'producer-2'@simulation ?salesProdInValue\
      orderProd6 'producer-2'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
```

```
and
      level6 'store-2'@simulation ?levelOutValue\
      productionDay6 'factory-2'@simulation ?productionDayOutValue

------------------------------------------------------------------------

Rule: TR-ProductionDay-and-Level-5 (meta-accessible)

Antecedents:
and
      level5 'store-1'@simulation ?levelInValue\
      saleProd5 'producer-3'@simulation ?salesProdInValue\
      orderProd6 'producer-3'@simulation ?orderProdToday\
      calcProductionDay ?orderProdToday 10 750 ?levelInValue 750
?productionDayOutValue\
      is ?levelOutValue ?levelInValue + ?productionDayOutValue -
?salesProdInValue

Consequents:
and
      level6 'store-1'@simulation ?levelOutValue\
      productionDay6 'factory-1'@simulation ?productionDayOutValue

------------------------------------------------------------------------

Rule: TR-Sales-1 (meta-accessible)

Antecedents:
and
      level2 'store-3'@simulation ?levelStore\
      order2 'consumer-2'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd2 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale2 'producer-1'@simulation 'consumer-2'@simulation ?saleValue

------------------------------------------------------------------------

Rule: TR-Sales-1 (meta-accessible)

Antecedents:
and
      level2 'store-1'@simulation ?levelStore\
      order2 'consumer-2'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd2 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale2 'producer-3'@simulation 'consumer-2'@simulation ?saleValue

------------------------------------------------------------------------

Rule: TR-Sales-1 (meta-accessible)

Antecedents:
and
      level2 'store-2'@simulation ?levelStore\
```

```
      order2 'consumer-1'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd2 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale2 'producer-2'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-1 (meta-accessible)

Antecedents:
and
      level2 'store-2'@simulation ?levelStore\
      order2 'consumer-2'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd2 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale2 'producer-2'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-1 (meta-accessible)

Antecedents:
and
      level2 'store-2'@simulation ?levelStore\
      order2 'consumer-3'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd2 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale2 'producer-2'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-1 (meta-accessible)

Antecedents:
and
      level2 'store-3'@simulation ?levelStore\
      order2 'consumer-3'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd2 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale2 'producer-1'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-1 (meta-accessible)

Antecedents:
and
      level2 'store-1'@simulation ?levelStore\
```

```
      order2 'consumer-3'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd2 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale2 'producer-3'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-1 (meta-accessible)

Antecedents:
and
      level2 'store-3'@simulation ?levelStore\
      order2 'consumer-1'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd2 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale2 'producer-1'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-1 (meta-accessible)

Antecedents:
and
      level2 'store-1'@simulation ?levelStore\
      order2 'consumer-1'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd2 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale2 'producer-3'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-2 (meta-accessible)

Antecedents:
and
      level3 'store-1'@simulation ?levelStore\
      order3 'consumer-1'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd3 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale3 'producer-3'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-2 (meta-accessible)

Antecedents:
and
      level3 'store-1'@simulation ?levelStore\
```

```
      order3 'consumer-2'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd3 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale3 'producer-3'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------

Rule: TR-Sales-2 (meta-accessible)

Antecedents:
and
      level3 'store-3'@simulation ?levelStore\
      order3 'consumer-3'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd3 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale3 'producer-1'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------

Rule: TR-Sales-2 (meta-accessible)

Antecedents:
and
      level3 'store-3'@simulation ?levelStore\
      order3 'consumer-2'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd3 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale3 'producer-1'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------

Rule: TR-Sales-2 (meta-accessible)

Antecedents:
and
      level3 'store-1'@simulation ?levelStore\
      order3 'consumer-3'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd3 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale3 'producer-3'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------

Rule: TR-Sales-2 (meta-accessible)

Antecedents:
and
      level3 'store-2'@simulation ?levelStore\
```

```
      order3 'consumer-1'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd3 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale3 'producer-2'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-2 (meta-accessible)

Antecedents:
and
      level3 'store-2'@simulation ?levelStore\
      order3 'consumer-2'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd3 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale3 'producer-2'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-2 (meta-accessible)

Antecedents:
and
      level3 'store-2'@simulation ?levelStore\
      order3 'consumer-3'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd3 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale3 'producer-2'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-2 (meta-accessible)

Antecedents:
and
      level3 'store-3'@simulation ?levelStore\
      order3 'consumer-1'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd3 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale3 'producer-1'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-3 (meta-accessible)

Antecedents:
and
      level4 'store-3'@simulation ?levelStore\
```

```
      order4 'consumer-2'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd4 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale4 'producer-1'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-3 (meta-accessible)

Antecedents:
and
      level4 'store-1'@simulation ?levelStore\
      order4 'consumer-1'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd4 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale4 'producer-3'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-3 (meta-accessible)

Antecedents:
and
      level4 'store-1'@simulation ?levelStore\
      order4 'consumer-2'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd4 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale4 'producer-3'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-3 (meta-accessible)

Antecedents:
and
      level4 'store-1'@simulation ?levelStore\
      order4 'consumer-3'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd4 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale4 'producer-3'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-3 (meta-accessible)

Antecedents:
and
      level4 'store-2'@simulation ?levelStore\
```

```
      order4 'consumer-1'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd4 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale4 'producer-2'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-3 (meta-accessible)

Antecedents:
and
      level4 'store-3'@simulation ?levelStore\
      order4 'consumer-3'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd4 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale4 'producer-1'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-3 (meta-accessible)

Antecedents:
and
      level4 'store-2'@simulation ?levelStore\
      order4 'consumer-2'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd4 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale4 'producer-2'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-3 (meta-accessible)

Antecedents:
and
      level4 'store-2'@simulation ?levelStore\
      order4 'consumer-3'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd4 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale4 'producer-2'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-3 (meta-accessible)

Antecedents:
and
      level4 'store-3'@simulation ?levelStore\
```

```
      order4 'consumer-1'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd4 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale4 'producer-1'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-4 (meta-accessible)

Antecedents:
and
      level5 'store-3'@simulation ?levelStore\
      order5 'consumer-3'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd5 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale5 'producer-1'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-4 (meta-accessible)

Antecedents:
and
      level5 'store-1'@simulation ?levelStore\
      order5 'consumer-1'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd5 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale5 'producer-3'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-4 (meta-accessible)

Antecedents:
and
      level5 'store-2'@simulation ?levelStore\
      order5 'consumer-3'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd5 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale5 'producer-2'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-4 (meta-accessible)

Antecedents:
and
      level5 'store-3'@simulation ?levelStore\
```

```
      order5 'consumer-2'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd5 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale5 'producer-1'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-4 (meta-accessible)

Antecedents:
and
      level5 'store-2'@simulation ?levelStore\
      order5 'consumer-2'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd5 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale5 'producer-2'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-4 (meta-accessible)

Antecedents:
and
      level5 'store-2'@simulation ?levelStore\
      order5 'consumer-1'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd5 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale5 'producer-2'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-4 (meta-accessible)

Antecedents:
and
      level5 'store-1'@simulation ?levelStore\
      order5 'consumer-3'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd5 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale5 'producer-3'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-4 (meta-accessible)

Antecedents:
and
      level5 'store-3'@simulation ?levelStore\
```

```
      order5 'consumer-1'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd5 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale5 'producer-1'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-4 (meta-accessible)

Antecedents:
and
      level5 'store-1'@simulation ?levelStore\
      order5 'consumer-2'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd5 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale5 'producer-3'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-5 (meta-accessible)

Antecedents:
and
      level6 'store-1'@simulation ?levelStore\
      order6 'consumer-2'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd6 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale6 'producer-3'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-5 (meta-accessible)

Antecedents:
and
      level6 'store-1'@simulation ?levelStore\
      order6 'consumer-3'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd6 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale6 'producer-3'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-5 (meta-accessible)

Antecedents:
and
      level6 'store-3'@simulation ?levelStore\
```

```
      order6 'consumer-3'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd6 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale6 'producer-1'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-5 (meta-accessible)

Antecedents:
and
      level6 'store-3'@simulation ?levelStore\
      order6 'consumer-2'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd6 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale6 'producer-1'@simulation 'consumer-2'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-5 (meta-accessible)

Antecedents:
and
      level6 'store-2'@simulation ?levelStore\
      order6 'consumer-3'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd6 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale6 'producer-2'@simulation 'consumer-3'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-5 (meta-accessible)

Antecedents:
and
      level6 'store-2'@simulation ?levelStore\
      order6 'consumer-1'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd6 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale6 'producer-2'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-5 (meta-accessible)

Antecedents:
and
      level6 'store-1'@simulation ?levelStore\
```

```
      order6 'consumer-1'@simulation 'producer-3'@simulation
?orderConsProd\
      orderProd6 'producer-3'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale6 'producer-3'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-5 (meta-accessible)

Antecedents:
and
      level6 'store-3'@simulation ?levelStore\
      order6 'consumer-1'@simulation 'producer-1'@simulation
?orderConsProd\
      orderProd6 'producer-1'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale6 'producer-1'@simulation 'consumer-1'@simulation ?saleValue

----------------------------------------------------------------------

Rule: TR-Sales-5 (meta-accessible)

Antecedents:
and
      level6 'store-2'@simulation ?levelStore\
      order6 'consumer-2'@simulation 'producer-2'@simulation
?orderConsProd\
      orderProd6 'producer-2'@simulation ?orderProd\
      calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale6 'producer-2'@simulation 'consumer-2'@simulation ?saleValue
```

11 Appendix 2 - Dependency Graphs

11.1 For Module Model (after Splitting)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Creator:
VisualWorks(R)
Preview:
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with a preview included in it.
Comment:
This EPS picture will print to a
PostScript printer, but not to
other types of printers.


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with a preview included in it.
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This EPS picture will print to a
PostScript printer, but not to
other types of printers.

11.2 For Module Meta (after Splitting)

11.3 For Module Prover (after Splitting)

(the number of rules is smaller than the number reported in the previous appendix, as the model suffered some improvements – in this version of the model some of the transition rules do not write data for a final step)


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VisualWorks(R)
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with a preview included in it.
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PostScript printer, but not to
other types of printers.


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PostScript printer, but not to
other types of printers.


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other types of printers.


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PostScript printer, but not to
other types of printers.


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VisualWorks(R)
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with a preview included in it.
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PostScript printer, but not to
other types of printers.

11.4 For the whole Model before Splitting


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Creator:
VisualWorks(R)
Preview:
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with a preview included in it.
Comment:
This EPS picture will print to a
PostScript printer, but not to
other types of printers.


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VisualWorks(R)
Preview:
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with a preview included in it.
Comment:
This EPS picture will print to a
PostScript printer, but not to
other types of printers.

12 Appendix 3 - Set of Rules before and after Splitting

There were eight 'types' of rules (see Table 12.1 involved in the application of the technique, those associated with generating the dynamics of the simulation. Excluding those rules for testing the theorem and setting up traders (producers)' choices, which did not suffer additional split, the other seven rules in the original simulation model where split into 85 rules in the efficient simulation model. All of them were split by transition step (TS) or iteration (five transitions); among these seven rules, two suffered additional split by producer, and one, among the last two, was also split by distributor (consumer). This gives: *(5 + (1 + (1 * 3)) * 3) * 5 = 85* rules in the new simulation model replacing the referred seven rules in the old simulation model (this corresponds to the last seven rules listed below and in Table 12.1). In the following the more relevant rules will be presented. In the list, first the original rule is named and second the names of the instances of the split rules as given in the module *prover* after are given. More detail about these rules can be seen in the module *prover* given in Appendix 1 section 10.2.3. Table 12.1 summarises facts about the performed splitting.

Checking theorem –this rule was not split.

```
Rule: checkTheorem (meta-accessible)
```

Producers' choice of a Producer for price-setting.

```
Rule: Choices-Iter-1 (meta-accessible)
Rule: Choices-Iter-2 (meta-accessible)
Rule: Choices-Iter-3 (meta-accessible)
Rule: Choices-Iter-4 (meta-accessible)
Rule: Choices-Iter-5 (meta-accessible)
```

Comment:

Split by iteration. This rule had already been written specifically for each iteration before splitting but without using a *meta* module. It was useful for improving SDML's efficiency when backtracking. In the MAS model all this rules are fired previously to the transition rules, e.g., they fire outside the partition of rules implementing the simulation transition steps.

Rule for calculating distributors (or consumer)' total order. It was split into:

```
Rule: Data-OrderProd-OrderCons-1 (meta-accessible)
Rule: Data-OrderProd-OrderCons-2 (meta-accessible)
Rule: Data-OrderProd-OrderCons-3 (meta-accessible)
Rule: Data-OrderProd-OrderCons-4 (meta-accessible)
Rule: Data-OrderProd-OrderCons-5 (meta-accessible)
```

Comment:

This rule was split only by iteration.

Rule for calculating traders (producers)' total sales. Split into:

```
Rule: Data-SalesProd-OrderCons-1 (meta-accessible)
Rule: Data-SalesProd-OrderCons-2 (meta-accessible)
```

```
Rule: Data-SalesProd-OrderCons-3 (meta-accessible)
Rule: Data-SalesProd-OrderCons-4 (meta-accessible)
Rule: Data-SalesProd-OrderCons-5 (meta-accessible)
```

Rule for distributors (consumers') demand and order-setting. It was split into:

```
Rule: TR-Demand-and-Order-1 (meta-accessible)
Rule: TR-Demand-and-Order-2 (meta-accessible)
Rule: TR-Demand-and-Order-3 (meta-accessible)
Rule: TR-Demand-and-Order-4 (meta-accessible)
Rule: TR-Demand-and-Order-5 (meta-accessible)
```

Rule for traders (producers)' price-setting, it was split into:

```
Rule: TR-Price-1 (meta-accessible)
Rule: TR-Price-2 (meta-accessible)
Rule: TR-Price-3 (meta-accessible)
Rule: TR-Price-4 (meta-accessible)
Rule: TR-Price-5 (meta-accessible)
```

Rule for traders (producer)'s sale-setting, it was split into:

```
Rule: TR-Sales-1 (meta-accessible)
Rule: TR-Sales-1 (meta-accessible)
Rule: TR-Sales-1 (meta-accessible)
Rule: TR-Sales-1 (meta-accessible)
Rule: TR-Sales-1 (meta-accessible)
Rule: TR-Sales-1 (meta-accessible)
Rule: TR-Sales-1 (meta-accessible)
Rule: TR-Sales-1 (meta-accessible)
Rule: TR-Sales-1 (meta-accessible)
Rule: TR-Sales-2 (meta-accessible)
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Rule: TR-Sales-2 (meta-accessible)
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Rule: TR-Sales-3 (meta-accessible)
Rule: TR-Sales-4 (meta-accessible)
Rule: TR-Sales-4 (meta-accessible)
Rule: TR-Sales-4 (meta-accessible)
Rule: TR-Sales-4 (meta-accessible)
Rule: TR-Sales-4 (meta-accessible)
Rule: TR-Sales-4 (meta-accessible)
Rule: TR-Sales-4 (meta-accessible)
Rule: TR-Sales-4 (meta-accessible)
Rule: TR-Sales-4 (meta-accessible)
Rule: TR-Sales-5 (meta-accessible)
Rule: TR-Sales-5 (meta-accessible)
Rule: TR-Sales-5 (meta-accessible)
```

```
Rule: TR-Sales-5 (meta-accessible)
Rule: TR-Sales-5 (meta-accessible)
Rule: TR-Sales-5 (meta-accessible)
Rule: TR-Sales-5 (meta-accessible)
Rule: TR-Sales-5 (meta-accessible)
Rule: TR-Sales-5 (meta-accessible)
```

Comment:
This rule was split not only by iteration but also by producer and by consumer. When setting sales, a producer (trader) can specify the data it is using by Producer (itself) and consumer (the one the sale is intended to). That is why there appear nine rules for iteration (the nine possible combination of three producers and three consumers). To make this clearer, consider two rules of iteration 1 given below. The first rule involves `producer-2` and `consumer-3`, and the second rule involves `producer-1` and `consumer-3`. Data involved in the precedent of these rule are: level in store of the involved producer, order the involved consumer has placed to the involved producer and total order placed to the involved producer. In the consequent the two involved instance and the resulting value of the sale are placed in a clause. Notice that the instances of producer, consumer and store are instances pertaining to the simulation domain, as they are generated during the simulation (this is indicated in SDML with the symbol `@`). As it is explained in Appendix 5, this specificity can be used not only for achieving efficiency making rules dependencies more explicitly instantiated but also for making assumptions and backtracking more specific, and branch exploration more efficient.

```
Rule: TR-Sales-1 (meta-accessible)
Antecedents:
and
   level2 'store-2'@simulation ?levelStore\
   order2 'consumer-3'@simulation 'producer-2'@simulation
?orderConsProd\
   orderProd2 'producer-2'@simulation ?orderProd\
   calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale2 'producer-2'@simulation 'consumer-3'@simulation ?saleValue

-----------------------------------------------------------------
Rule: TR-Sales-1 (meta-accessible)
Antecedents:
and
   level2 'store-3'@simulation ?levelStore\
   order2 'consumer-3'@simulation 'producer-1'@simulation
?orderConsProd\
   orderProd2 'producer-1'@simulation ?orderProd\
   calcSale ?levelStore ?orderProd ?orderConsProd ?saleValue

Consequents:
sale2 'producer-1'@simulation 'consumer-3'@simulation ?saleValue
```

Rule for trader's (producer)' production and level updating. It was split into:

```
Rule: TR-ProductionDay-and-Level-1 (meta-accessible)
Rule: TR-ProductionDay-and-Level-1 (meta-accessible)
Rule: TR-ProductionDay-and-Level-1 (meta-accessible)
Rule: TR-ProductionDay-and-Level-2 (meta-accessible)
Rule: TR-ProductionDay-and-Level-2 (meta-accessible)
Rule: TR-ProductionDay-and-Level-2 (meta-accessible)
Rule: TR-ProductionDay-and-Level-3 (meta-accessible)
Rule: TR-ProductionDay-and-Level-3 (meta-accessible)
```

```
Rule: TR-ProductionDay-and-Level-3 (meta-accessible)
Rule: TR-ProductionDay-and-Level-4 (meta-accessible)
 Rule: TR-ProductionDay-and-Level-4 (meta-accessible)
Rule: TR-ProductionDay-and-Level-4 (meta-accessible)
Rule: TR-ProductionDay-and-Level-5 (meta-accessible)
Rule: TR-ProductionDay-and-Level-5 (meta-accessible)
 Rule: TR-ProductionDay-and-Level-5 (meta-accessible)
```

Comment:

This rule was split not only by iteration but also by producer. This was possible because the updated producer (trader) variable only depends on its own value last iteration and Producer can be instantiated explicitly for each rule. In case there were some random choices or any other manipulation involving generation of assumptions, this split would permit to make them more specific and backtracking could be also implemented more particularly.

Rule for ordering producer (trader) in order of value of price

```
Rule: Data-SelProd-1 (meta-accessible)
Rule: Data-SelProd-2 (meta-accessible)
Rule: Data-SelProd-3 (meta-accessible)
Rule: Data-SelProd-4 (meta-accessible)
Rule: Data-SelProd-5 (meta-accessible)
```

Comment:

Data generated by this rule are useful for consumers' (distributors) order-setting and for calculating size of the interval of prices when checking theorem.

Description of the rule (rule for:)	No. of rules in the original model	Rule split by	No. of rules in the efficient model
Checking theorem	1	---	1
Traders 'choice of a trader for price-setting	5	it had already been split by iteration	5
Rule for calculating distributors (or consumer) 'total order	1	iteration	5
Rule for calculating traders 'total sales	1	iteration	5
Rule for distributors (consumers') demand and order-setting	1	iteration	5
Rule for traders 'price-setting	1	iteration	5
Rule for trader's sale-setting	1	iteration, trader and consumer	45
Rule for trader's supply and level updating	1	iteration and trader	15
Rule for ordering trader in order of value of price	1	iteration	5

Table 12.1. Comparing the number of rules in the MAS-based and in the constraint-based architectures

13 Appendix 4 - Runs/Result Tables

Aspect	*OTTER model (Run in a SUN)*	*SDML model after revealing dependencies (Run on a PC)*	*SDML Efficient Model after unwrapping dependencies (Run on a PC)*
Number of rules implementing the transition step	13	13	91
Time the simulation lasts using as unit of time the time required for one transition step in the efficient model. (see Appendix 5)	N (see note 4)[4]	N(N-1)/2	N
RAM Memory used	Typical: 3.328 MB[1]	256MB[2]	256 MB[2]
Hard Disk Memory used	- 0 in case the output file is deleted. - 100MB. Typical in case a minimal amount of data is sent to the output file[3].	0 But, 256 MB were needed if the SDML 'image' (file in RAM) was saved	0 But, 256 MB were needed if the image was saved
Number of backtrackings for proving over N iterations	8^N (N = 5 in the proof reported in Chapter 7)	8^N	8^N
Dependencies manager	User defined assumption tracking mechanism	Automatic (It is a SDML's facility)	Automatic (It is a SDML's facility)
Underlying logic of the language	Defined by the clauses of the sim. model and hyperresolution	SDML underlying logic is close to SGAL (Moss, *et al.*, 1997)	SDML underlying logic is close to SGAL (Moss, *et al.*, 1997)

Table 13.1. Runs/Result Tables

Notes:

1: Otter splits the search assigning, after a branch point, the search task of each possible branch to a different process. In a typical moment during the search there were five

processes (each one responsible for a different split, with each split corresponding to a transition step) each using the following amount of RAM memory: 544 KB, 608 KB, 672 KB, 736 KB, 768 KB. Total: 3,328 KB (or 3.328 MB).

2: The total RAM memory available in the PC at that moment (December, 1999) was 256 MB.

3: It depends on the required data to be sent to the output file. Default options controlling sending of data to the output file, e.g., sending ‘given clauses’ or ‘kept clauses’, can be cleared. In this case, after a proof, the file grew up to about 100MB. Even in this case the output file can be too big. This file can be deleted while the simulation is going on and then no hard disk memory will be used.

4: Comparing in terms of the manipulations but its speed is not comparable with the implementations on SDML as it was run in a different platform.

14 Appendix 5 - Estimation of Speeding-up Gained from Unwrapping Rules

The idea is to compare the two architectures built in SDML after 'revealing' dependencies: the first one, where only revealing of rule dependencies is implemented, and the efficient one, where also 'unwrapping' of rules is used. The comparison will be in terms of the amount of data each implementation searches into when generating a state transition. This would give a rough idea about the speeding-up gained through the technique, implemented in the second architecture and exposed in Chapter 7, with respect to the MAS model.

Details about the whole translation process from the original MAS model into the efficient constraint-based architecture and, in particular, about revealing dependencies and unwrapping of rules were given in Chapters 5 and 7 (see especially section 5.9.3). There, it can be seen how the space of searched data grows after revealing dependencies and how this problem is managed in the efficient implementation via unwrapping of rules.

In the efficient program rules instantiate data more specifically. Discrimination among time iterations speed up the simulation. Similarly, discrimination among agents and objects can be exploited to add efficiency in the exploration of trajectories; e.g., it can be used to drive the search more efficiently.

A 'unit of data' is defined as the data generated in a single iteration (which does not change significantly). As the efficient program searches into the data of one iteration for each state transition (e.g., it uses data from *iteration-i* to generate data for *iteration-(i+1);* see Chapters 5 and 7), it searches into a unit of data for state transition. If a simulation trajectory were generated per *N* iterations, then the size of the space of searched data would be *N-1* (there are *N-1* time transitions).

On the other hand, if the original implementation were used, the program would have to search into one unit of data in the first transition (when the transition is from *iteration-1* to *iteration-2*), two units in the second one, …, *k-1* units at the *transition-k,* and *N-1* units at the final *transition-(N-1).* So, in this case the amount of data searched in the whole simulation would be $1 + 2 + .. + N-1 = N(N-1)/2$ units. Finally, the factor of efficiency would be the proportion between these two results: $N/2$. So, the efficiency of the improved SDML-model is linearly increasing over time with respect to the efficiency of the original SDML-model.

Likewise, unwrapping of rules by agents and objects may be used for further improvements. It would help to exploit the semantics of the simulation in order to add efficiency during the search (though this was not implemented in the technique, it is a

strategy that will help in future applications). To explain this, it should be remembered that assumptions are linked to a rule, i.e., assumptions are generated when a rule is non-deterministic –as it can generate alternative consequents. In this case, the rule causes a branch point where each possible consequent has a different assumption and gives a different branch origin of a different simulation trajectory. If were possible to instantiate data explicitly (in fact, this is the case in the efficient implementation), then data given in the original rule under the same assumption could now be differentiated via assigning different assumptions, as they can be generated by different rules. The idea is to make the branching as fine as possible in order to make the backtracking also as specific as possible. To illustrate this, consider the predicates used to give agents' choices in Chapter 7:

Choice 1: listSelTrader(Trader-2, Trader-1, Trader-1, i);
Choice 2: listSelTrader(Trader-2, Trader-1, Trader-2, i);
Choice 3: listSelTrader(Trader-2, Trader-3, Trader-1, i);
Choice 4: listSelTrader(Trader-2, Trader-3, Trader-2, i);
Choice 5: listSelTrader(Trader-3, Trader-1, Trader-1, i);
Choice 6: listSelTrader(Trader-3, Trader-1, Trader-2, i);
Choice 7: listSelTrader(Trader-3, Trader-3, Trader-1, i);
Choice 8: listSelTrader(Trader-3, Trader-3, Trader-2, i);

For each choice there will be a different assumption. In the implemented simulations, when making a different choice, the computational system (SDML in this case) assumes all traders' choices change as they are linked to a single assumption and so all data linked to the same assumption has also to be updated when the assumption is changed. Should these choices be more specific, e.g., if there were a different assumption for each trader's choice, then there will also exist a different branch for each trader's choice. One choice can be changed while keeping the others and only the data in the simulation linked to that choice would be updated. Considering the choices listed above, assume choice 1 is left and choice 2 is taken, *Trader-3's* choice (given by the third component in the list) changes from *Trader-1* to *Trader-2* while the other *trader's* choices are kept unchanged.

In this case, where only one trader changes his choice, e.g., assumptions and branching of the simulation are more specific, backtracking might also be more specific as less data has to be modified and regenerated for each backtracking. This would be possible if, rather than writing a single rule for prices, this rule were split and each trader's choice could be given independently and linked to a different assumption.

15 Appendix 6 - Complexity of the Search

The *aim* of this appendix is to prove that the exploration of trajectories proposed in Chapter 5 and explained more specifically in section 5.5, applied over an infinite (theoretically) number of iterations, is *PSPACE-complete*. To make clearer the exposition, this problem will be called the *target problem*, and the case where it appeared (that exposed in Chapter 7) the *target example*. As is usual for this sort of proof, two steps will be followed:

First, it will be proved that the target problem is in PSPACE by expressing it as a binary tree of depth *n*. According to Papadimitriou, this is sufficient (Papadimitriou, 1994; see examples in pp. 455-462).

Second, it will be proved that the problem is also *PSPACE-complete* by translating another *PSPACE-complete* problem into the target problem. For this comparison, one of the problems Woolridge (2000) presents has been chosen, concretely that of agent-task-maintenance.

For the first part of the proof it must be possible to construct in polynomial space the game three, which is possible if the target problem is expressed in the form of a Boolean quantified expression (see examples 19.1 and 19.2 in Papadimitriou, *ibid*), as follows:

$$?x_1\ ?x_2 ?x_3\ ?x_4 ?x_5 \ldots Q_n\, x_n\ (F) \qquad (1)$$

where F is the formula to be evaluated over the variables $x_1 \ldots x_n$, and Q_n is the last quantifier, which will be $?$ in case of *n impair* and $?$ in case of *n even*.

The *impair* variables correspond to the environment's action. The *deterministic* part in the state transition of the simulation will be called *environment's actions*. In the target example, it corresponds to all those changes not associated with *agents' choices*. Consequently, there is only one alternative action for the *impair* variables. The *even* variables correspond to the agents' choices (which are going to be called *agents' actions*). In the target example, there are eight alternative agents' choices. So far, a state transition in a simulation has been divided into two parts: that *deterministic* part associated with the *existential* variables and that *non-deterministic* part associated with the *quantified* variables. A simulation path (a trajectory) is represented by a concatenation of branches, where each branch corresponds to an assignment of values to a variable x_i.

Finally, *F* will be the question: whether the searched *tendency* has occurred in a simulation trajectory, where that trajectory is associated with an assignment of values for the variables, x_i. The whole expression (1) is true if for all possible assignments of values to the variables the tendency is true (remember that there is only one choice for the existential variables). As each particular assignment of values to the whole set of quantified variables corresponds to a trajectory, the proof is successful if this expression is valid for all possible values the quantified variables can take? (e.g., for all possible agents' choices).

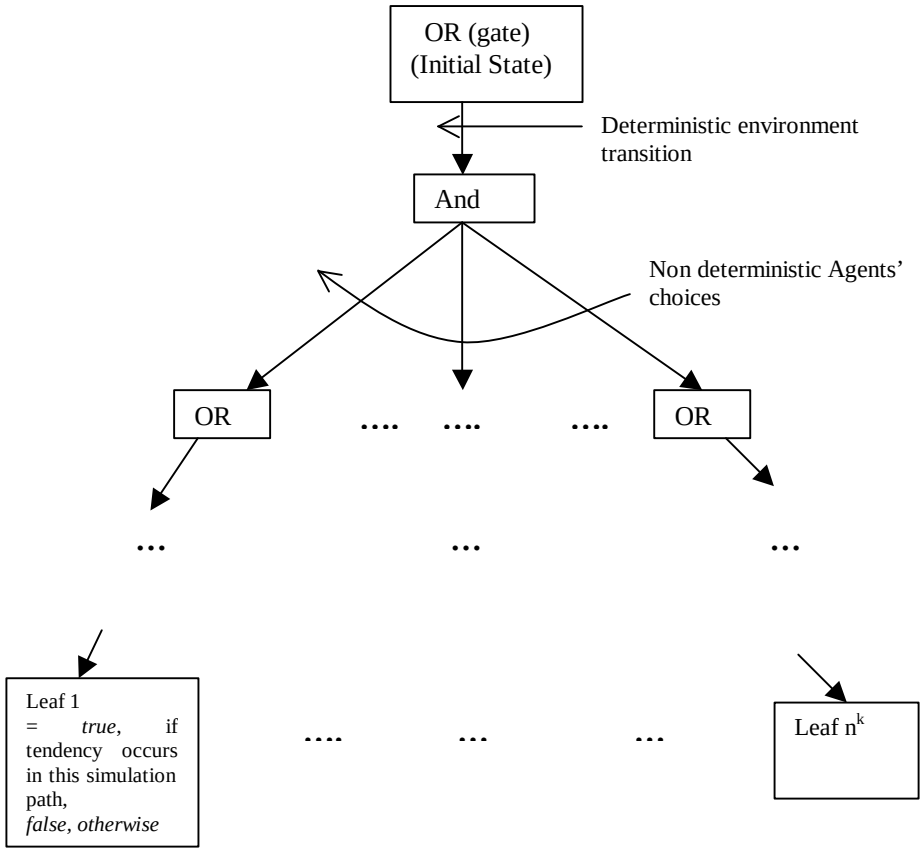

Figure 15.1. Boolean circuit for the target problem

To check if the proof is successful, a boolean circuit, where and *OR* gate stands for the *?* quantifier and an *AND* gate stands for the *?* quantifier, will be written. A leaf in this circuit is evaluated to *true* if the tendency is found in the corresponding simulation path and to *false* otherwise (see Figure 15.1). The whole circuit will be *true* if and only if the tendency appears in all simulation paths. Hence, the proof is successful if and only if the circuit is true (e.g., the tendency is found in *all* paths).

These two expressions of the problem are sufficient to prove that the target problem is PSPACE. The next task is to prove that the problem is *PSPACE-complete*.

Comparing with Woolridge (2000), an algorithm to check the proof might be written. This will bring the example close to the one he uses when considering *maintenance tasks*. Assuming a Turing machine *M* is called recursively at each branch point (at agents' choices) and that this machine is kept in use while actions are deterministic (environment's action), the algorithm for *M* will be:

Algorithm 15.1:

1. If the tendency appears, then the branch is evaluated to *true (success*);
2. If there are no allowable simulation actions, the branch is evaluated to *false* (*fail);*
3. Execute the deterministic aspects of the state transition (environment action), then for each agent's choice recursively call *M;*
4. If all recursive calls in 3, are *successful* (i.e., evaluated to *true*), then *M* is *true (success).*

To prove that the target problem is *PSPACE-complete*, consider the maintenance problem in Woolridge (2000). There, agents are chosen non-deterministically to act against the environment. Each agent's actions are deterministic, while environment's actions are non-deterministic. The idea is to check if there is any choice of agents' actions that is successful in bringing the environment into one in a set of states whatever the environment chooses. It is like a game where agents play against the environment. Woolridge proves that the agent-maintenance problem is in NPSPACE using the following *algorithm*:

Algorithm 15.2:

1. if r [the run until a branch point] ends with state $\in$ *G* [the set of goals], then *M* accepts;
2. if there are no allowable actions given r, then *M* rejects;
3. non-deterministically choose an action *a* from *Ac* (possible agents' actions, there is one per agent) and then for each $e \in ?$ (set of possible environment's states) recursively call M with the run r. *a. e;*
4. if all of these accept, then *M* accepts, otherwise *M* rejects.

In Woolridge's problem, rather than searching for a tendency, the idea is to bring the simulation into one among a set of environment states. If the environment is brought into one of these states, it is said that the selected agents have been successful in their game against the environment. In Woolridge's example, the agents' actions are deterministic; e.g., they have only one choice, but different agents can be selected. Selection of agents corresponds to the *OR* nodes in the circuit shown in the Figure 15.1, each branch

corresponding to the choice of a different agent. On the other hand, the environment has non-deterministic actions, and, correspondingly, their choices are associated with the *AND* gates in the circuit.

A first difference between *algorithm 15.1* and Woolridge's algorithm (e.g., *algorithm 15.2*) is that in the latter the recursive calls at step 3 are made for the environment choices, while in the former they correspond to the called environment actions. A major difference is that the deterministic action of the environment in step 3 in the former algorithm corresponds to the non-deterministic choice of agents in the latter. The translation of Woolridge's problem into the target problem in this presentation seems straightforward, but there is still a small difficulty: his case study is non-deterministic (owing to the non-deterministic choice of agents in step 3 in *algorithm 15.2*), while the target problem in this presentation is deterministic.

Woolridge's original problem is NPSPACE. Consider the deterministic version of this problem. Think about checking the successfulness of agents' actions in Woolridge's problem once an agent has been chosen in advance at each branch point. This is a deterministic problem. It is in PSPACE but still as hard as Woolridge's original one as NPSPACE = PSPACE (Papadimitriou, p. 150). These problems are both *PSPACE-complete*. Woolridge's algorithm for this deterministic version of the agent-maintenance tasks becomes:

Algorithm 15.3:

1. if r [the run until a branch point] ends with state $\in$ *G* [the set of goals], then *M* accepts;
2. if there are no allowable actions given r, then *M* rejects;
3. deterministically use the action *a given in advance* from *Ac* (possible agents' actions, there is one per agent) and then, for each $e \in ?$ (set of possible sates of the environment), recursively call M with the run r. *a. e;*
4. if all of these accept, then *M* accepts, otherwise *M* rejects.

The translation of the determinist version of Woolridge's problem into the target problem is straightforward from the *algorithms 15.1* and *15.3*. The deterministic action of the environment at step 3 of *algorithm 15.1* corresponds in Woolridge's algorithm (*algorithm 15.3*) to the deterministic action of the agent already chosen. The recursive calls of *M* made for agents' choices in *algorithm 15.1* correspond in *algorithm 15.3* to the environment's choices. With regard to the circuit shown in Figure 15.1, agents' choices (now deterministic) are placed at the *OR* gates and environment (non-deterministic)

choices are placed at the *AND* nodes. Therefore, the deterministic version of Woolridge's maintenance problem has been translated into the target problem, and, consequently, the target problem is also *PSPACE-complete*.

It has been proved that the target problem is *PSPACE-complete* for an infinite number of iterations, *i*. Using the experience accumulated so far in this proof for *i* infinite (particularly useful is the expression of the problem in the circuit given above), and theorems 17.8 (especially its corollary 2) and 17.10 in Papadimitriou (1994), it should be possible to prove that the problem is $?_i$ *P-complete* if the number of iterations, *i*, is finite.

16 Appendix 7 - Mapping the Envelope of Social Simulation Trajectories

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17 Appendix 8 - Determining the Envelope of Emergent Agent Behaviour via Architectural Transformation

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