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How math can help

Mareen Hallier
BTU Cottbus-Senftenberg

Workshop: "Cross-Scale Resilience in Socio-Ecological Simulations"
Lorentz Center, May 1-4, 2017

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How math can help . . . to save the world

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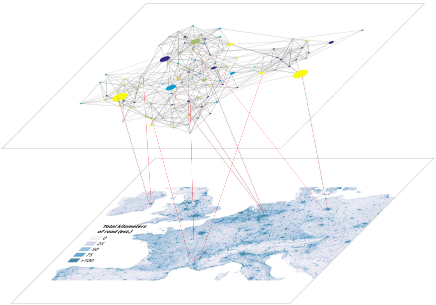
Mathematical Challenges

- Formalization
  - Concepts
  - Model
- Analysis of the System Properties
  - Regimes
  - Transition networks
  - Dominant transition pathways
- Mathematical Models of Reduced Complexity
  - Mean-field Approximation
  - Markov State Models
  - ...
- Statistical Estimation and Efficient Simulation
  - Sensitivity Analysis
  - Efficient Numerical Simulation Methods
- Visualization



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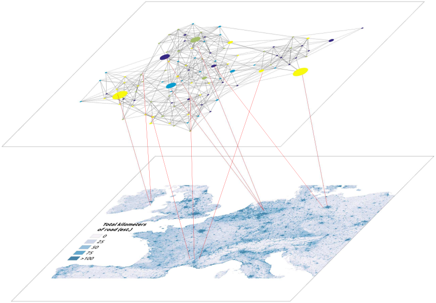

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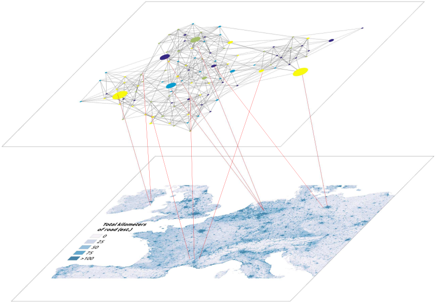

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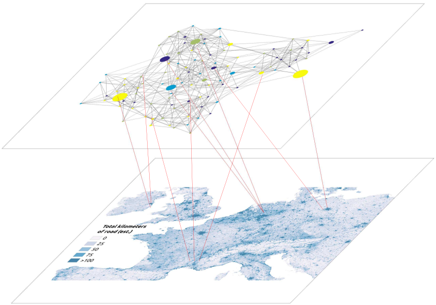

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The FAVAIA Project (EU Project ADAM)

- Formal Approaches to Vulnerability that Inform Adaptation
- Joint Research project between Potsdam Institute for Climate Impact Research and Stockholm Environment Institute
  - Jochen Hinkel, Sarah Wolf, Daniel Lincke, Sandy Bisaro: Global Climate Forum
  - Cezar Ionescu, Oxford University
  - Richard T.J. Klein, Stockholm Environment Institute
  - M.H.
- What have we done?
  - A meta-analysis of definitions of and approaches towards assessing vulnerability, adaptive capacity, risk, and resilience
  - Mathematical formalisation of the common structure found
- Literature considered
  - climate change, natural hazards, poverty
  - 50 conceptual papers and book chapters
  - 200 impact, vulnerability and adaptation case studies conducted in Europe

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How do theorectical concepts attain their meaning?

- Cultural/social convention
  - Emerge through social interaction over (long) time
  - Intuitive understanding
- Definition
  - Operational definitions
    - defined upon observable concepts
    - operations we perform for assessing the concept (methodological approach, methodology, measurement)
  - Theoretical definitions
    - defined upon other theoretical concepts
    - these other theoretical concepts must be well-defined or intuitively clear

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Findings – Theoretical Definitions

Similar theorectical definitions

- the analysis of theoretical definitions is actually quite boring
- literature elaborates endlessly with vaguely defined concepts
- discourse detached from observable reality
  - huge gap between description of cases in observable concepts and the theorectical concept such as resilience and vulnerability
  - little effort made to connect the two

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Findings – Operational Definitions

Different operational definitions

- the analysis of operational definitions is much more interesting
- one common element
  - representation of possible futures
- distinct elements
  - vulnerability: measure of harm
  - resilience: similarity criterion
- both contain a strong normative component
  - vulnerability: what does harm mean?
  - resilience: what does similar mean?
- many degrees of freedom in making concepts operational
  - different focus – social system, ecological system, both – depending on research are (flood defence, poverty)
  - theoretical definitions do not show focal points
  - focal points explain why "the word vulnerability means different things to different researchers" (O'Brien et al. 2007) despite similar definitions

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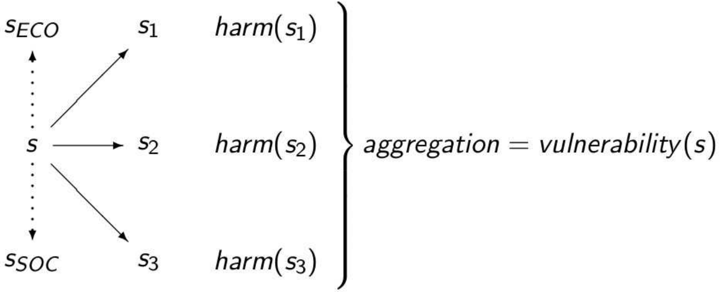
Illustration



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Recommendations

There are many good ideas around in the vulnerability and resilience approaches. However, they are hidden behind these concepts. In order to advance, we need to spell them out explicitely.

- "From practise to theory" rather than from "theory to practise"
  - ignore theorectical discourse on vulnerability and resilience
  - throw away the concepts
- carefully describe cases in less abstract, observable concepts
  - including the objective (research question, goal)
  - this automatically connects to "operational management concepts"
- carefully build theory from the cases

Math helps

narrowing down ambiguous natural language concepts

structuring complex discussions

create the starting point for serious Applied Math

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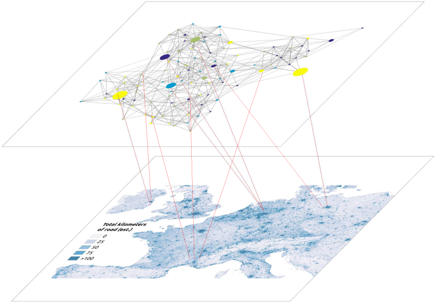

Specifications

Why is there a need for (mathematical) specifications of agent-based models?

- computer models
  - to solve well-defined problems
  - exploratory programming
    - no well-defined problem to be solved
    - Model description consists of implementation and additionally, e.g., narratives, solitary mathematical equations, rarely pseudo-code
      $\rightsquigarrow$ Problematic for model reimplementation, analysis, ...

Specifications

Functional framework for specifying agent-based models of exchange

- Based on the agent-based model by Gintis (2006).
- Motivation: several attempts to reimplement the model to get the reported results failed.

Botta, N., Mandel, A., Ionescu, C., Hofmann, M., Lincke, D., Schupp, S., and Jaeger, C. (2011). 'A functional framework for agent-based models of exchange'. Appl. Math. Comput. 218(8): 4025-4040.

Botta, N., Mandel, A., Hofmann, M., Schupp, S., and Ionescu, C. (2013). 'Mathematical Specification of an agent-based model of exchange'. In Lange, C., Rowat, C., and Kerber, M., editors, Do-Form: Enabling Domain Experts to use Formalised Reasoning (Proceedings to the Do-Form symposium at the AISB Annual Convention in April 2013), Exeter, UK. AISB. ISBN 978-1-908187-32-1.

Hallier, M. (2015). 'Formalization and Metastability Analysis of Agent-based Evolutionary Systems. PhD Dissertation, Freie Universitt Berlin.

Specifications

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Functional framework for specifying agent-based models of exchange

- Basic structure: agent-based models of exchange consist of
  - Time-discrete Markov process which models the evolution of prices via learning and which depends at each time step on the outcome of a
  - Trading game, which is (almost) a population game, and in which the agent-specific prices determine the strategies of the agents.

$\rightsquigarrow$ Exposes relationship with evolutionary game theory.

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- Applications
  - Problem formulation
    - What are "plausible rules of trading interactions that might explain how equilibrium prices become established"?
  - Model analysis and numerical investigation

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"Going beyong look-and-see analyses"

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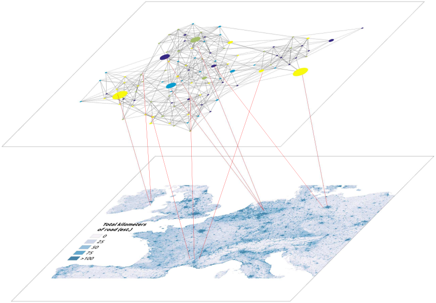
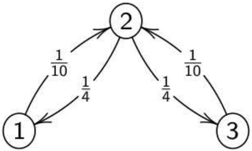


Markov Chain Analysis

Idea: Represent ABM as a Markov chain

- Markov chain:
  - the probability distribution of the next state depending only on the current state and not on the sequence of events that preceded it
  - can be represented with a “transition matrix”



$$P = \begin{pmatrix} 9/10 & 1/10 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/10 & 9/10 \end{pmatrix}$$

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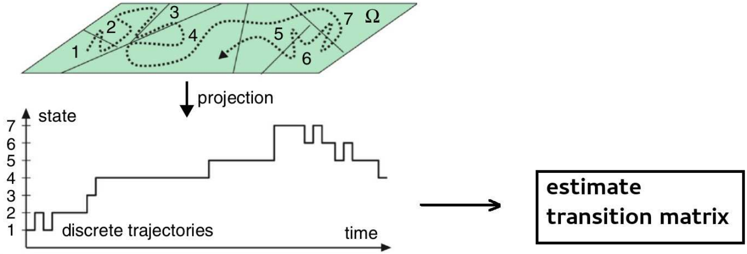
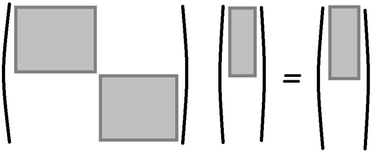
  - in practice: coarse-graining of state space necessary



Simplest case



Regime Identification

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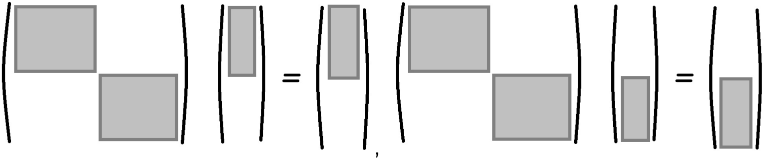

- Structure of the transition matrix: two eigenvalues 1, two dynamically invariant regimes

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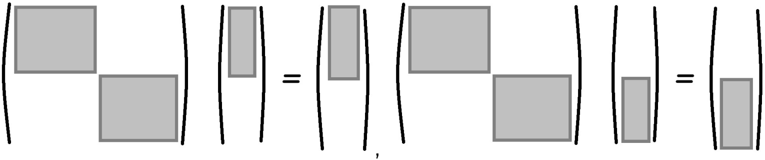
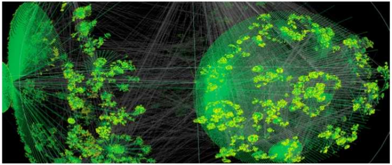


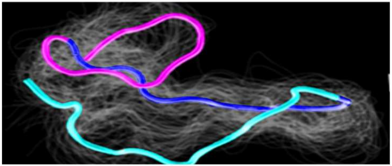
- Structure of the transition matrix: two eigenvalues 1, two dynamically invariant regimes
- in practice: several almost invariant regimes with eigenvalues close to 1, eigenvalues relate to different time scales
- regimes can be computed via sign structure of related eigenvectors

Application areas

Agenten-basiertes Modell

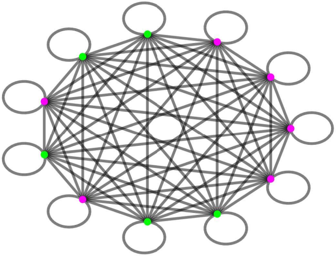


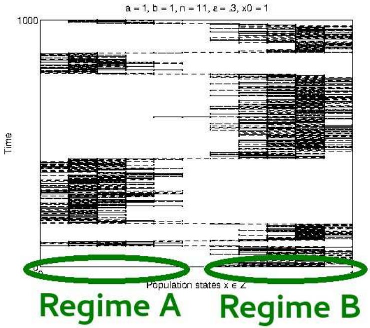
Google PageRank



Protein-faltung

Illustrative Example





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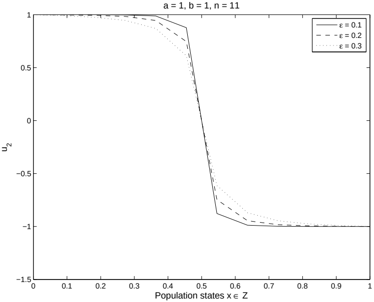
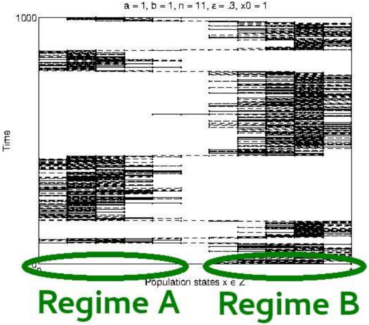




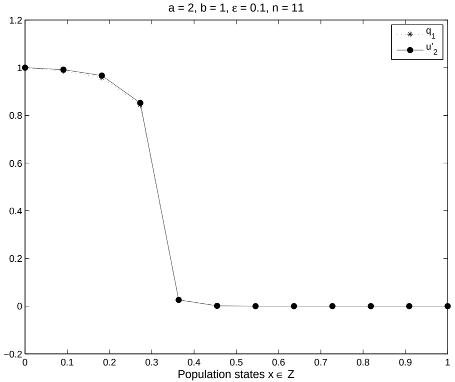
Committor functions

For C_i define committor function $q_i : Z \to [0,1]$ by

$$q_i(z) = \mathbb{P}[\tau_{C_i} < \tau_{C \setminus C_i} \mid X_0 = z],$$

the probability of hitting regime set C_i next when being in state z, where τ_A denotes the hitting time

$$\tau_A = \inf\{k' \geq 0 \mid X_{k'} \in A\}.$$



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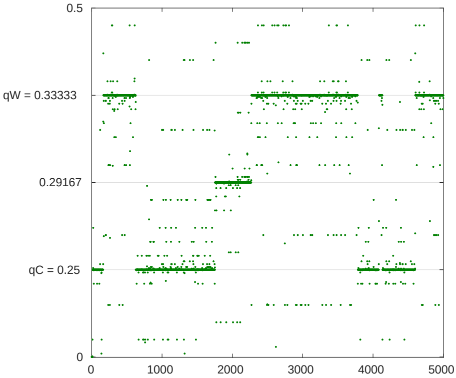
Cournot Duopoly

(Work in Progress...)

- two firms choose quantities from a grid $\{0, 0.25, 0.29, 0.33, 0.5\}$
- update choice according to a Imitate-the-Best behavioral rule with one-step memory and payoffs $u_i(q_1, q_2) = P(q_1 + q_2) - c(q_i)$, where q_i is current quantity of firm i, $P = \max(1 - q_1 - q_2, 0)$ is price funtion and $c(q_i) = 0.5q_i^2$ represents costs.
- known: $\{0.25, 0.29, 0.33\}$ visited most often
- unknown: their relative frequency in the long-run; how do transitions take place?



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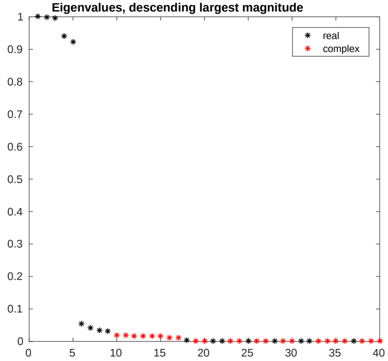

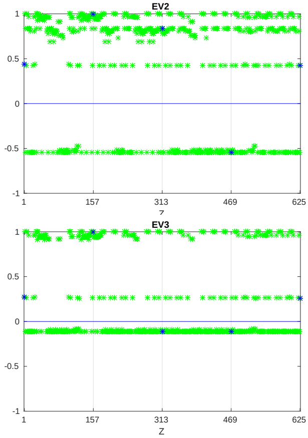

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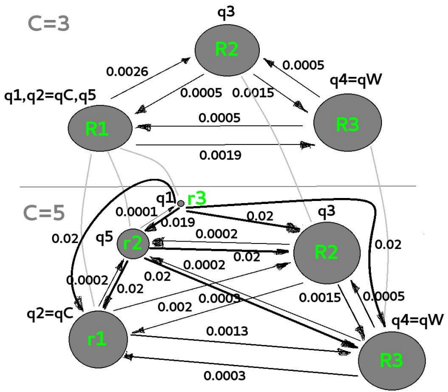
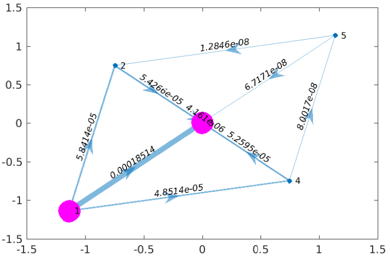
Cournot Duopoly

Hierarchical Networks of Transitions



Dominant Pathways



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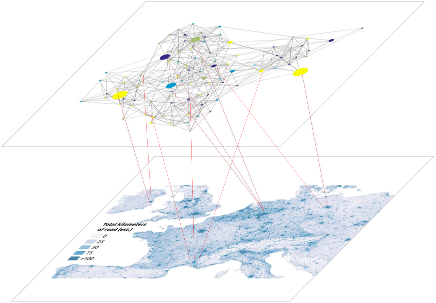

Save the world!

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Do we agree?