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Regime Identification and Hierarchical Pathways of Change

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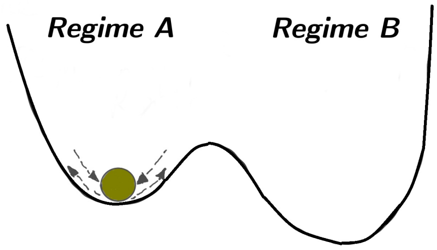
Initial Thoughts

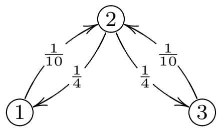
Central to the investigation of resilience in socio-ecological systems is the understanding of possible transitions between different system regimes at various scales. For this, agent-based simulations can be used to represent the complex socio-ecological system of interest. Often, these simulations are very complex themselves so that mathematical models of reduced complexity could be of help not only to see the wood for the trees but to allow the application of advanced analytic methods in order to, e.g., identify system regimes at different levels of scale or to get a quantitative understanding of the transition pathways between these regimes (on the basis of which questions of policy changes could be addressed).



Relevant Work

- Construction of Markov models of reduced complexity from simulation data (involves appropriate “coarse-graining” of state space)
- Computation of committor functions: $q_i(z)$ = probability to hit regime R_i next when being in state z
- Regime identification
- Transition networks
- Dominant pathways

Markov chain



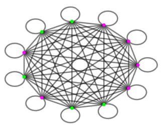
Network of transition probabilities.

$$P = \begin{pmatrix} 9/10 & 1/10 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/10 & 9/10 \end{pmatrix}$$

Transition matrix.

Regime Identification

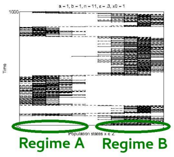
Illustrative “toy” example



Individual level

Binary choice with choice rule:

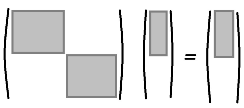
- high probability: do what majority is doing
- low probability: randomize



Population level

Regime identification via analysis of transition matrix

Structure of the transition matrix, no randomization: P has two eigenvalues 1 and two dynamically invariant regimes



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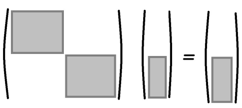
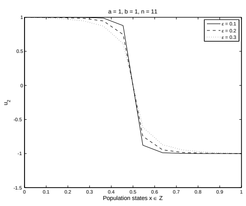




Second right eigenvector of example

Regimes are related to eigenvectors and eigenvalues of the transition matrix

- in practice: several blocks / regimes with eigenvalues close to 1, eigenvalues relate to different time scales
- regimes can be computed via sign structure of related eigenvectors

Further approaches to regime identification for Markov chains, e.g.,

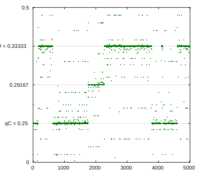
- via expected hitting times
- via Schur decomposition
- via singular vectors.

Pathways of Regime Change

Cournot Duopoly

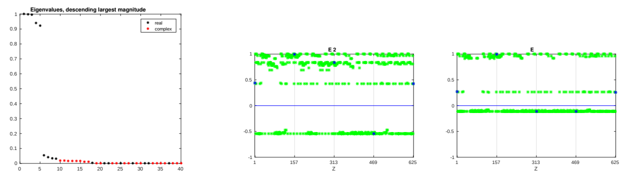
(Work in Progress...)

Sample path

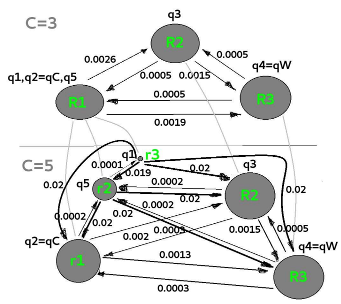


- two firms choose quantities from a grid $\{0, 0.25, 0.29, 0.33, 0.5\}$
- update choice according to a Imitate-the-Best behavioral rule with one-step memory and payoffs $u_i(q_1, q_2) = P(q_1 + q_2) - c(q_i)$, where q_i is current quantity of firm i, $P = \max(1 - q_1 - q_2, 0)$ is price funtion and $c(q_i) = 0.5q_i^2$ represents costs.
- known: $\{0.25, 0.29, 0.33\}$ visited most often
- unknown: their weight in stationary distribution; how do transitions take place?

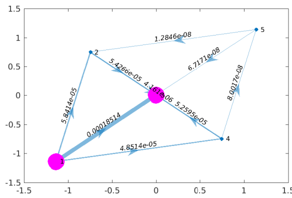
Dominant eigenvalues, 2nd and 3rd eigenvectors



Hierarchical Networks of Transitions



Dominant pathways of change



$\rightsquigarrow$ Direct pathway from Walras to Cournot state is the most typical pathway of change.

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